APPLIED PHYSICS

NOTES AND EXAMPLES IN SI UNITS FOR STUDENTS OF NAUTICAL SCIENCE

By

J. H. CLOUGH-SMITH, B.Sc. (Lond.), Extra-Master, F.R.I.N.

Revised by

R. A. DAVIES, B.Ed. (Hons), B.A.



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for Students of
NAUTICAL SCIENCE

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INTRODUCTION

FIRST EDITION

THIS book does not claim to be a general textbook on physics, or mechanics, or hydrostatics. On the contrary, it is written with a strictly limited objective in view, namely, to cover the syllabus in General Science laid down by the Merchant Navy Training Board for the training of apprentices at sea.

This purpose has been kept in mind throughout, though not to the extent of unduly restricting the scope of the book, nor so as to prevent an orderly development of each subject. It is hoped, therefore, that the book may be found useful in those schools providing courses in General Science for Cadets prior to going to sea.

In addition to covering the syllabus laid down the author had in mind the difficulties under which many apprentices study at sea, and the needs of those students have been carefully considered. Particularly in the sections on mechanics and hydrostatics, explanation has been reduced to a minimum and excessive "wordiness" avoided. There are plenty of worked examples, and other examples for the student to practise on. Thus, the book may be helpful to apprentices who are following a correspondence course with one of the nautical schools.

General Science is not a subject of the examination for the Second Mates' Certificate — at least, not under that name — and perhaps it is for this reason that apprentices and junior officers have never been over anxious to devote time to its study. There has been a tendency to pick up items of knowledge piecemeal, as some particular ship-board problem demanded, while basic principles remain neglected. This practice has very little to recommend it — in fact, a sound knowledge of basic principles is essential.

The author hopes, therefore, that the practical applications of these principles to so many shipboard problems will encourage junior officers to study this book. It should be borne in mind that the Merchant Navy Training Board syllabus was very carefully drawn up in order to provide a sound basis for the professional

examinations. It is not saying too much, therefore, when we say — a junior officer ought to know it all. There is nothing in these pages that he will not encounter at some other time or other in his examinations from Second Mate to Master, and even the student for Extra-Master may find this a useful starting point from which to proceed with his serious studies.

The author greatly appreciates the permission accorded by the Merchant Navy Training Board to include their syllabus and also to use many of the questions previously set at their examinations. He is also grateful to the Controller of H.M. Stationery Office for permission to use some illustrations from the *Marine Observer's Handbook* and other official publications. The kind co-operation of the British Thomson-Houston Co., Ltd., and of the Baldwin Instrument Co. Ltd., in supplying illustrations of their equipment is also gratefully acknowledged.

In conclusion, the author would warmly thank his colleagues in the Cardiff College of Technology and Commerce and in other Nautical Schools for their invaluable advice, so readily given. It is acknowledged with gratitude that many portions of the book are much improved as a result of incorporating the helpful criticisms and suggestions received.

J. H. CLOUGH-SMITH

CARDIFF, *June*, 1957

SECOND EDITION

A few additions have been made to the text, together with some examples for practice in Appendices II and III. However, for the benefit of Schools which may be using the book for Correspondence Courses, the page numbers and paragraph references remain almost unchanged throughout.

Grateful acknowledgement is made to the Merchant Navy Training Board for permission to include their revised syllabus in this edition, and also to my many colleagues who have been kind enough to speak of the book in appreciative terms from time to time.

J.H.C-S.

1961

THIRD EDITION — APPLIED PHYSICS

The introduction of new syllabuses by the Merchant Navy Training Board has rendered a complete revision of the text necessary, and the book now appears under a new title.

Nevertheless the original concept has been kept in mind, namely, to cover the syllabus as thoroughly as possible consistent with brevity. For the most part, the original content, although revised, has been preserved, and added to as necessary to meet the new requirements. In addition, four new chapters have been added, together with many new examples.

The book as it now stands should provide a good pre-sea course in general physics. It is also hoped it will be found suitable for continued study at sea, not only by Navigating Cadets preparing for the Merchant Navy Training Board examinations, but also by Junior Officers.

The author acknowledges with gratitude his indebtedness to many well-known works — a fuller acknowledgement appears elsewhere — and also to his many colleagues and friends who have offered advice and assistance during the work of revision.

J.H.C-S.
1967

FOURTH EDITION

In preparing this Revised Edition I have retained some of the concepts which influenced the style of its predecessors.

Firstly, one has to keep in mind the reasonably safe assumption that whilst the reader will have some, and often a quite extensive, knowledge of physics, it will possibly be a year or two since he engaged in any formal study of the subject. Hence, the basic approach to the elementary parts of the subject may not be out of place by way of revision.

There is also the factor that, because of the nature of his calling, the reader may not have the assistance of an experienced teacher readily available — in fact, he may often have to study quite alone.

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There is also the factor that, because of the nature of his calling, the reader may not have the assistance of an experienced teacher readily available — in fact, he may often have to study quite alone.

Above all, I think I would like him to be able to *understand* what I have written. If this has caused me at times to oversimplify matters, it is hoped that at least nothing actually harmful is conveyed.

At the other end of the scale, one must try to achieve a standard appropriate to the Ordinary National Certificate, itself some two years on from O. Level. Though, at the time of writing, there is no actual experience of this particular examination for the O.N.C. in Nautical Studies, it is hoped that the more advanced portions of the book will prove adequate for this purpose.

Almost all chapters, other than the most elementary, have been extended by the addition of new work, and four completely new chapters have been added, namely, chapters 5, 19, 25 and 26.

The book is written in SI units to conform with the requirements of the examination boards but reference is made to the obsolescent C.G.S. and F.P.S. systems because for some years to come these systems will continue to be found in many reference books and some knowledge of them is essential.

Also we are still, at the time of writing, in a transitional period. Ideas of teachers everywhere are taking a little time to crystalise. For example, there are some who will not have terms such as kilogramme-force (kgf), tonnes-force (tf), at any price. There are others who find them convenient, practical and not in any way distasteful.

A book such as this cannot settle issues of this nature and so I have taken account of the fact that two points of view exist. I fear this may lay me open to criticism but my back is broad and fortunately (unlike Ohm — p. 352) I have no teaching post to lose!

All answers have been checked but the book has been written under some pressure as regards time and if any errors come to light I should be most grateful to have then pointed out.

I acknowledge with gratitude my indebtedness to all those friends in the profession who have assisted with advice and information, former colleagues at Leith Nautical College, Liverpool Polytechnic, and in the office of the Principal Examiner of Masters and Mates. Perhaps I should especially thank members of my former department for their guidance so readily available. W. E. (Bill) Scobie, W. (Willem) Burger, and in particular, W. H. P. (Bill) Canner for his invaluable advice and assistance in the revision of the chapters on electricity.

I have enjoyed writing this book — for once, I have not had to do a full-time job as well!

J. H. CLOUGH-SMITH
Easingwold, *September,* 1971

FIFTH EDITION — APPLIED PHYSICS

Revising this text so long after the last edition has been a challenging task. The removal of what are now obsolete units and their replacement with SI units is but one of the many changes that have occurred. Some chapters have been almost completely rewritten whilst others have gained extra material.

Throughout the revision I have borne in mind that many of the potential readers of the text might well be studying for the Applied Science and Mathematics examinations for the Deck Officer Class 3 certificate.

These examinations are administered by SCOTVEC (Scottish Vocational Education Council) and I am grateful to this body for granting me permission to reproduce past examination questions. These are to be found amongst the questions which appear at the end of each chapter.

I am also grateful to my colleague Capt. Jeff Birrell of the Nautical Campus (Fleetwood) of Blackpool and the Fylde College for his good-humoured help and advice during the revision. Special thanks must also go to my "better half", Anne, for her patience and forbearance.

Finally I express the hope that, despite major revision, the text still retains the indelible stamp of its original author — the late J. H. Clough-Smith.

R. A. DAVIES
BLACKPOOL, *March,* 1994

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CONTENTS

Chap.		PAGE
INTRODUCTION		
BIBLIOGRAPHY AND ACKNOWLEDGEMENTS		ix
THE USE OF SI UNITS		xiii
HYDROSTATICS		
Chap. 1	Density, Relative Density, Archimedes' Principle, Flotation	1
Chap. 2	Force, Pressure, Pressure at a Depth, Thrust on a Lamina	10
STATICS		
Chap. 3	Vectors, Addition of Vectors, Parallelogram, Triangle and Polygon of Forces, Resolution and Composition of Forces	23
Chap. 4	Moments, Centre of Gravity, Centre of Parallel Forces, Movement of Masses, Equilibrium of a Rigid Body Under a System of Coplanar Forces	40
Chap. 5	Couples, Torque, Torsion, Elasticity, Stress, Strain, Bending Moments and Shearing Forces	59
HYDROSTATICS		
Chap. 6	Hydrometers, Loadlines, F.W.A.	86
Chap. 7	Atmospheric Pressure, Barometers, Errors and Corrections	98
Chap. 8	Pressure, Centre of Pressure, Boyle's Law, Manometer, Simple Hydraulic Appliances	108
DYNAMICS		
Chap. 9	Velocity, Acceleration, Rectilinear Motion Under Constant Acceleration, Velocity Time Graph, Relative Velocity, Force as the Product of Mass and Acceleration, Momentum	121
Chap. 10	Work, (Potential and Kinetic) Energy, Power, Mechanical Advantage, Velocity Ratio, Efficiency, Simple Machines	147
Chap. 11	Friction, Statical and Dynamic Coefficients, Efficiency, Machines	170
Chap. 12	Motion in a Horizontal Circle, Simple Harmonic Motion, Applications	186
HEAT		
Chap. 13	General Introduction, Transmission of Heat, General Effects, Expansion, Thermometry	200
Chap. 14	Units, Specific Heat Capacity, Heat and Mechanical Energy, Elementary Thermodynamics	222
Chap. 15	Thermal Expansion of Solids, Liquids and Gases, The Gas Laws, Isothermal and Adiabatic Expansion $PV = mrT$, with Applications	234
Chap. 16	Change of State, Latent Heat, Fusion and Vaporisation, Movement of Air Masses, Dalton's Law of Vapours, Refrigeration	256
Chap. 17	Water Vapour in the Atmosphere, Humidity, S.V.P., Dewpoint, Condensation, Cloud and Fog	269

MAGNETISM

Chap. 18 General Introduction, Nature of Magnetism, Attraction, Repulsion, Magnetic Properties of Materials (Qualitative), Terrestrial Magnetism 283

Chap. 19 Magnetic Flux, Inverse Square Law, Field Strength, Magnetic Moment, Intensity of Magnetisation, Permeability, Susceptibility, Short Bar Magnet, The Magnetic Circuit ... 303

Chap. 20 Ship Magnetism, Elementary Notion of Forces *P, Q, R*, Compass Deviation 319

ELECTRICITY

Chap. 21 General Introduction, Nature of Electricity, the Electric Current, Cells as a Source of Current, Electromagnetism and the Simple Alternator as a Source of Current, Definition and Units, Ohm's Law, Work, Energy, Power, Static Electricity, Capacitors and Capacitance 339

Chap. 22 Simple D.C. Circuits, Resistance, Resistivity, Insulation, Heating Effect, Magnetic Effect, Chemical Effect with Applications, Rectification 372

Chap. 23 Magnetic Effect, Measuring Instruments, Voltmeter and Ammeter, Electromagnet, Trembler Bell, Circuit Symbols . 412

Chap. 24 Induction (Self and Mutual), Principle of the D.C. Motor . 429

Chap. 25 Simple A.C. Theory, Peak, Instantaneous and R.M.S. Values of Current and Voltage, Addition of Alternating Quantities, A.C. Circuits-resistive, capacitive, inductive, Series Circuits, Impedance, Resonance, Power Factor, Parallel Circuit 442

Chap. 26 Electronics, Rectification, Diode, Triode, Amplifier, Loudspeaker, Semiconductors, Transistor, Oscilloscope 480

LIGHT

Chap. 27 Rectilinear Propagation, Reflection in Plane Mirrors, the Sextant, Reflection in a Curved Mirror, Refraction in the Atmosphere, Refractive Index, Critical Angle, Prisms, Formation of Images by Lenses, Optical Instruments 504

GENERAL WAVE PHENOMENA

Chap. 28 *SOUND*

Production and Propagation of Sound, Wave Motion, Velocity in Air and Water, Reflection and Refraction, Doppler Effect, Elementary Notions on General Wave Phenomena, Interference, Resonance 544

ELEMENTS OF RADIO

Principle of Radio Transmission and Reception, Modulation, Transmission, Sky Waves, Ground Waves, Aerials 560

ANSWERS TO EXAMPLES 573

INDEX 585

The International System of Units (S.I.)

Many units are used in Physics for the measurement of quantities. A coherently organised international system of units is in general use. This system is called the S.I. system (from the French Systeme International d'Unites).

In this system there exists base units, supplementary units, derived units and decimal multiples and sub-multiples of these units.

The following are base units which are used in this book. Their correct abbreviations are also shown.

BASE UNITS OF S.I.

Unit of length	metre	(m)
Unit of mass	kilogramme	(kg)
Unit of time	second	(s)
Unit of electric current ..	ampere	(A)
Unit of temperature	kelvin	(K)

SUPPLEMENTARY UNITS

In addition to the above the following are used:

Plane angle — the radian (rad)

Solid angle — the steradian (sr).

Nevertheless it is acknowledged that degrees also remain in general use for plane angles.

DERIVED UNITS

These are obtained from the base units.

For example as we shall see later the correct S.I. unit for velocity is:

$$\frac{\text{metre}}{\text{second}} \text{ or } \text{metre per second}$$

In this case we have one base unit (the metre) divided by another (the second) and there are two commonly used short forms for the resulting derived unit.

These are:

$$m/s \quad \text{or} \quad m\,s^{-1}$$

In a similar manner we shall also see that the S.I. unit for density is:

$$\frac{\text{kilogramme}}{\text{cubic metre}} \quad \text{or} \quad \text{kilogramme per cubic metre}$$

which can be written

$$kg/m^3 \quad \text{or} \quad kg\,m^{-3}$$

In this book some chapters will use the one form whilst the other form will be used in the remaining chapters.

Some derived units have been given special names usually in honour of a famous scientist.

For example we shall see that the unit of force is

$$kg\,m/s^2 \quad \text{or} \quad kg\,m\,s^{-2}$$

and that this is given the name NEWTON (abbreviation N).

The above units, and others, will be introduced in the text as we proceed, together with the standard abbreviations.

MULTIPLES AND SUB-MULTIPLES

Decimal multiples and sub-multiples are formed from the base unit by means of the prefixes given in the list below.

Basic unit	*Factor by which unit is multiplied*	*Prefix*	*Symbol*
ampere, metre second, etc.	10^{12}	tera	*T*
	10^{9}	giga	*G*
Examples:	10^{6}	mega	*M*
milliamp (mA)	10^{3}	kilo	*k*
kilometre (km)	10^{2}	hecto	*h*
microsecond (μs)	10	deca	*da*
megavolt (MV)	—	—	—
and so on.	10^{-1}	deci	*d*
	10^{-2}	centi	*c*
	10^{-3}	milli	*m*
	10^{-6}	micro	μ
	10^{-9}	nano	*n*
	10^{-12}	pico	*p*
	10^{-15}	femto	*f*
	10^{-18}	atto	*a*

For example 3000 m can be written as 3×10^3 m and this in turn can be written as 3 km.

In a similar manner 0.003 m can be written as 3 mm.

Some of the above are rarely used in ordinary work and for the purposes of this book it is recommended that the following should be memorised:

mega kilo centi milli micro nano pico

CHAPTER 1

HYDROSTATICS

Density, Relative Density, Archimedes' Principle

1.1 INTRODUCTION

Hydrostatics is the branch of applied mathematics which deals mainly with fluids at rest.

It has in common with most other branches of applied mathematics the fact that the formulae are few and generally simple. The principles and laws are almost self evident.

Yet formulae and principles must be thoroughly grasped. It is in applying these laws to practical problems that great care must be taken.

1.2 MASS

The mass of a body is the quantity of matter it contains. It is measured in kilogrammes (kg).

The household of bag of sugar one can buy at a shop has a mass of 1 kg.

Mass is used to define density.

1.3 DENSITY

A block of steel does not have the same mass as a block of wood of the same volume.

The steel block is said to be more dense than the wooden one.

Definition:

The density of a substance is defined to be its mass per unit volume.

$$\textbf{density} = \frac{\textbf{mass}}{\textbf{volume}} \quad \ldots\ldots\ldots\ldots \textbf{I}$$

The symbol used for density is the Greek letter ρ (rho).

$$\rho = \frac{m}{V}$$

The SI unit for density is kilogramme per cubic metre (kg/m³).
The density of fresh water is 1000 kg/m³ or 1 t/m³.
Other typical values are set out below:

SUBSTANCE	DENSITY (kg/m³)
Ice at 0° C	920
Gold	19300
Steel	7800 (but varies)
Air	1·26 (but varies)
Sea water	1025 (but varies)

Note that ice is less dense than water and consequently ice floats in water.
In fact it is quite common to compare the density of substances to that of fresh water as we shall see in the next section.

WORKED EXAMPLE 1

A spherical bullet 0·80 cm diameter has a mass of 3·6 g. Find the density of the metal in the bullet.
We must convert into the correct units.

$$\text{Mass} = 3{\cdot}6 \text{ g} = \frac{3{\cdot}6}{1000} \text{ kg} = 0{\cdot}0036 \text{ kg}$$

$$\text{Diameter} = 0{\cdot}80 \text{ cm} = \frac{0{\cdot}80}{100} \text{ m} = 0{\cdot}0080 \text{ m}$$

$$\text{Volume of sphere} = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi \times 0{\cdot}004^3 \text{ m}^3$$

$$= 2{\cdot}681 \times 10^{-7} \text{ m}^3$$

$$\text{Density} = \frac{\text{mass}}{\text{volume}} = \frac{0{\cdot}0036 \text{ kg/m}^3}{2{\cdot}681 \times 10^{-7}} = 13429 \text{ kg/m}^3$$

EXERCISE A

(Use the table of densities from above where appropriate).

1. Find the density of a piece of rock of mass 0·72 kg and volume 144×10^{-6} m^3.
2. Determine the mass of 8 m^3 of fresh water.
3. Find the volume of 0·8 g of gold.

1.4 RELATIVE DENSITY

This is defined to be the density of a substance relative to the density of water.

$$\textbf{Relative density} = \frac{\textbf{density of substance}}{\textbf{density of fresh water}} \quad \ldots\ldots \quad \textbf{II}$$

For example for gold

$$\text{Relative density} = \frac{19300 \text{ kg/m}^3}{1000 \text{ kg/m}^3}$$

$$\text{R.D.} = 19{\cdot}3$$

Note that the units have cancelled out.

Thus relative density has no units. It is simply a number.

As the above example demonstrates, since the density of fresh water is 1000 kg/m^3, it is possible to obtain a substance's relative density from its density by simply moving the decimal point 3 places to the left. The converse is also true.

For example the relative density of ice is 0·920.

Any substance with a relative density less than 1 will float in water.

WORKED EXAMPLE 2

Find the relative density of a piece of rock of which 1·8 m^3 has a mass of 6 tonnes.

$$\rho = \frac{m}{V}$$

$$= \frac{6000}{1{\cdot}8} = 3333 \text{ kg/m}^3$$

$$\text{R.D.} = \frac{3333}{1000} = 3{\cdot}333$$

EXERCISE B

1. Find the R.D. of a substance if 250 cm^3 of it has a mass of 1 kg.
   (Note if 1 cm = 0·01 m
   then 1 cm^2 = 0·01 × 0·01 m^2 = 0·0001 m^2
   and 1 cm^3 = 0·01 × 0·01 × 0·01 m^3 = 0·000001 m^3)
2. A tank 2·2 m by 1·3 m by 2·0 m when filled contains 5·834 tonnes of dock water. Find the R.D. of the dock water.

1.5 MASS and WEIGHT

Before dealing with the concept of flotation it is necessary to be clear regarding the use of the terms "mass" and "weight".

The weight of a body is the force it exerts vertically downwards on anything supporting it and is due to the force of attraction of the Earth upon its mass i.e. it is due to gravity. Thus if gravity varies so does the weight. However mass remains constant.

There is therefore a distinction between mass and weight and this is dealt with in more detail in Chapter 9. For the time being we shall ignore this distinction and say that if a body has a mass of, for example, 5 kg then it has a weight of 5 kilogramme-force (5 kgf) i.e. it exerts a downward force of 5 kgf.

1.6 ARCHIMEDES' PRINCIPLE

Why do ships float when they are made of materials more dense than water?

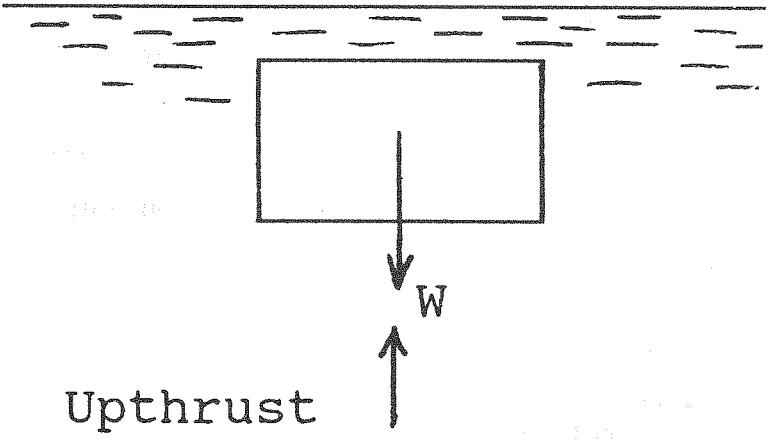
Firstly when a body is immersed in liquid it experiences an upward force or **UPTHRUST**.

Secondly if wholly immersed the body will displace a volume of liquid equal to its own volume.

Now Archimedes stated:

When a body is wholly or partially immersed in a fluid it experiences an upthrust equal to the weight of fluid displaced.

Consider, for example, a cube of volume 1 m^3 and weight W which is totally immersed in fresh water.



The cube will displace its own volume of water i.e. it will displace 1 m^3 of water.

This amount of water will have a weight of 1000 kgf.

According to Archimedes this means the upthrust on the body will also be 1000 kgf.

Three possibilities now arise:

(1) W is greater than 1000 kgf in which case the body will sink.

(2) W is equal to 1000 kgf in which case the body will float.

(3) W is less than 1000 kgf in which case the body will rise (as can be the case of a balloon in air).

WORKED EXAMPLE 3

A box-shaped barge floats in sea-water of R.D. 1·025 with 30 m^3 of its volume below the water line. Determine its weight.

Volume of water displaced $= 30\ m^3$

$$\text{R.D. of Sea water} = \frac{\text{Density of sea-water}}{\text{Density of fresh water}}$$

$$1{\cdot}025 = \frac{\text{Density of sea water}}{1000\ kg/m^3}$$

$$\text{Density of sea-water} = 1{\cdot}025 \times 1000\ kg/m^3 = 1025\ kg/m^3$$

$$\rho = \frac{m}{V}$$

$$1025 = \frac{m}{30}$$

Wt. of sea-water displaced $= 1025 \times 30 = 30\ 750$ kgf

From Archimedes upthrust $= 30\ 750$ kgf

Now if the barge is floating this upthrust must exactly equal the weight of the barge.

Therefore the weight of the barge is 30 750 kgf (30·75 tf).

1.7 LAW OF FLOTATION

The above problem is an example of the use of Archimedes Principle in the particular case of an object floating.

For the barge to float we had to say that the upthrust must equal the weight of the barge.

However the upthrust must also equal the weight of water displaced. (Archimedes Principle).

Therefore the weight of the barge must equal the weight of water displaced.

This is summed up in the law of flotation which states:

Every floating body displaces its own weight of the fluid in which it floats.

Hence a ship will float as long as it can displace its own weight of water.

WORKED EXAMPLE 4

A box shaped barge of mass 20 t floats in fresh water with half of its volume submerged. Calculate its R.D. (Density of fresh water = 1 t/m³).

Weight of barge = 20 tf
From law of flotation
Weight of water displaced = 20 tf
Mass of water displaced = 20 t
Volume of water displaced = 20 m³
Volume of barge submerged = 20 m³
Total volume of barge = 40 m³

$$\text{Density of barge} = \frac{\text{mass}}{\text{volume}}$$

$$= \frac{20 \text{ t/m}^3}{40}$$

$$= 0{\cdot}5 \text{ t/m}^3$$

Therefore R.D. of barge = 0·5

Thus, no matter what materials have been used to construct the barge, it will float because its R.D. is less than 1. This, of course, is because of the volume of very low density air contained in the barge.

WORKED EXAMPLE 5

An iceberg of R.D. 0·92 floats in sea-water of R.D. 1·024 with 10 615 m^3 immersed. Find the total volume of the iceberg. Hence determine the fraction of the iceberg below the water-line.

Volume of sea-water displaced $= 10\ 615\ m^3$
Density of sea-water $= 1024\ kg/m^3$
Mass of sea-water displaced $= 1024 \times 10\ 615\ kg$
Weight of sea-water displaced $= 10\ 869\ 760\ kgf$

Therefore by law of flotation
weight of iceberg $= 10\ 869\ 760\ kgf$

For iceberg density $= 920\ kg/m^3$

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

$$920 = \frac{10\ 869\ 760}{\text{volume}}$$

$$\text{volume} = 11\ 815\ m^3$$

Fraction of iceberg below water-line

$$= \frac{10615}{11815}$$

$$= 0{\cdot}9$$

EXERCISE C

1. A rectangular barge, 4 m by 2 m, is floating at a draught of 0·7 m in salt water of density 1024 kg/m^3. Find the upthrust exerted by the water on the bottom of the barge.
2. A load of weight W kgf causes a raft measuring 2·8 m by 2 m by 0·8 m to just submerge completely in sea-water of R.D. 1·024. Calculate W given that the mass of the raft is 1·5 t.
3. A lump of material weighs 56 kg in air and 32 kg in fresh water. Find its R.D.

TEST PAPER 1
Density, Relative Density, Archimedes' Principle

1. A box-shaped vessel of length 200 m, breadth 20 m, floats in fresh water at a draught of 12 m. Calculate the upward thrust on the bottom of the vessel in tonnes-force.
2. A box-shaped vessel of length 70 m and breadth 17 m displaces 5355 t of salt water. Find the draught of the vessel, assuming the R.D. of the salt water is 1·025.
3. A log measuring 4 m × 1 m × 0·7 m, having a R.D. of 0·75, is floating in fresh water with its short side vertical. What additional load will it support on its upper surface?
4. A cylinder of length 3·5 m and diameter 0·4 m, weighing 1200 kg, is lowered into salt water of R.D. 1·024. Find the stress on the gear when the cylinder is completely immersed.
5. A closed rectangular steel tank of length 5 m, breadth 3·2 m and depth 2·8 m and weighing 50 tf is to be salvaged from a sunken vessel. What is the stress in the gear before the tank breaks surface? (R.D. of sea water = 1·025)
6. A cylindrical tube floats upright in fresh water with 22 cm of its length immersed. What length would be immersed when floating in sea water of R.D. 1·024?

CHAPTER 2

HYDROSTATICS

Force, Pressure, Thrust

2.1 FORCE

FORCE is that which causes, or tends to cause, motion. As a general description of force, the above will do. Broadly speaking, we are all familiar with the concept of force — we have all experienced force at times and in different ways.

But scientifically speaking, it is necessary to be much more precise.

Forces which "tend to cause motion"

In other words, although a force is present, the system is at rest.

An example of such a system would be a brick of mass 5 kg resting on a table. We saw in section 1.5 of Chapter 1 that the brick exerts a downward force, called its weight, on the table and that this weight is expressed in kgf, tf, etc.

But it must be emphasised that these units are not S.I. units. Nor have they anything to recommend them when systems are in motion. In fact, they are useful — something to keep in the back of one's mind — but capable of only strictly limited application.

Forces which "cause motion"

In S.I. units, the unit of force is the newton (N) and is that force which when applied to a mass of 1 kg imparts to it an acceleration of 1 metre per second per second (1 m/s^2).

At this juncture, this definition will not mean a great deal to us and we shall return to it in more detail later. For the present, let us look at it like this.

A stone, released from a height, will fall to earth with an acceleration due to the force of gravity. The symbol for the acceleration due to gravity is g, and its value in S.I. units is 9·81 metres per second per second (m/s^2). This means that as the stone falls its velocity increases by 9·81 m/s every second.

Now our 5 kg brick is exerting a force of 5 kgf on the table top. If it were pushed over the edge, it would immediately experience an acceleration of g, that is, the force acting on it is equivalent to $5g$ N, i.e. $5 \times 9{\cdot}81$ N.

(Why? See Chapter 9, where this point is developed more carefully).

So 5 kgf is equivalent to $5g$ N $= 49{\cdot}05$ N.

This gives us some concept of how much force a newton is — a 1 kg bag of sugar would exert a weight of $1g$ N $= 9{\cdot}81$ N (approximately 10 N) on anything supporting it.

A fuller development of this subject is given in Chapter 9. But for the present, it will be sufficient for our purpose if we note the relationship between the quantities, i.e.,

A mass of M kilogrammes will exert a force of M kilogrammes-force or of Mg newtons.

And without trying to understand too deeply what a newton is, let us gain some familiarity with it in use.

2.2 PRESSURE AND THRUST

Definition:

Pressure is force per unit area ..... **I**

$$\text{Pressure} = \frac{\text{Force}}{\text{Area}}$$

Since the unit of force in S.I. units is the newton, and of area the square metre, pressure in S.I. units is expressed in

newtons per square metre (N/m²)

Since this is liable to lead to a numerical answer which may be inconveniently large, a further (and larger) unit is acceptable, namely, the bar, which is 10^5 newtons per square metre. Hence,

a pressure of 1 bar = a pressure of 10^5 N/m²

Quite often the term "thrust" is used instead of force.

Thrust is the total force on a surface ..... **II**

Thus, if a plate is 48 m² in area and the pressure on it is 100 N/m² then obviously, the thrust on it is 48×100, that is 4800 N.

In general, we may write,

Thrust = Pressure × Area ..... **III**

(Always remembering to keep our units right).

WORKED EXAMPLE 1.

A cylindrical boiler radius 3 m length 10 m contains steam at a pressure of 1500 N/m². Find total thrust on the internal surfaces of the boiler.

$$\text{Area of ends} = 2 \times \pi r^2 = 2 \times \pi \times 3 \times 3 \text{ m}^2$$
$$= 56{\cdot}55 \text{ m}^2$$
$$\text{Curved area} = 2\pi r \times L = 2 \times \pi \times 3 \times 10 \text{ m}^2$$
$$= 188{\cdot}5 \text{ m}^2$$
$$\text{Total internal area} = 56{\cdot}55 + 188{\cdot}5 \text{ m}^2 = 245{\cdot}05 \text{ m}^2$$
$$\text{and thrust in N} = 245{\cdot}05 \times 1500 \text{ N}$$
$$367575 \text{ N or } 3{\cdot}68 \times 10^5 \text{ N}$$

WORKED EXAMPLE 2.

Two tonnes of sea water are contained in a rectangular tank whose base is 2·4 m by 1·6 m.

Calculate the pressure on the tank bottom.

$$\text{Thrust on base} = 2 \text{ tf}$$
$$= 2000 \text{ kgf}$$
$$= 2000 \times 9{\cdot}81 \text{ N} = 19620 \text{ N}$$
$$\text{Pressure} = \frac{19620}{2{\cdot}4 \times 1{\cdot}6} = 5109 \text{ N/m}^2$$

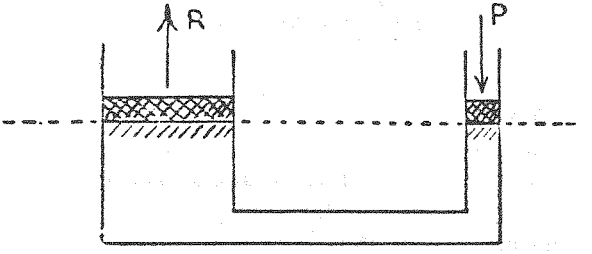
EXERCISE A

1. Two tonnes of water are contained in a rectangular tank whose base is 4 m by 2 m. Find the pressure on the bottom.
2. If the steam pressure inside a boiler is 18,060 N/m² find the thrust on one end of the boiler, given that the ends are circular and 2 m in diameter.

2.3 TRANSMISSION OF FLUID PRESSURE

This is a very simple principle. It means that **any pressure applied to a fluid is transmitted uniformly throughout the fluid.**

This principle is used in the hydraulic press.



Principle of hydraulic press.

Suppose the pipes are circular, and their areas are 1 cm^2 and 40 cm^2 respectively.

Now, if an effort of 1 kgf is applied at P (called the plunger or inlet) the pressure there is 1 kgf per cm^2.

But (by the principle above) *this pressure is transmitted throughout the fluid*, so the pressure at R (called the ram) is also 1 kgf per cm^2.

And since the area of the ram is 40 cm^2, the *thrust* exerted by the ram will be 40 kgf.

Thus, an effort of 1 kgf on the plunger will raise a load of 40 kgf on the ram.

This is the principle of all hydraulic presses, and by its use, quite a moderate effort can be made to raise a very big load.

Also, this concept (of getting "more out than is put in") is called the **Mechanical Advantage** of the press.

By definition:

$$\text{M.A.} = \frac{\text{load}}{\text{effort}}$$

Load and effort are, of course, both forces and therefore are both given by force = pressure × area.

In the case of the hydraulic press we have

$$\text{M.A.} = \frac{\text{load}}{\text{effort}} = \frac{\text{pressure on ram} \times \text{area of ram}}{\text{pressure exerted by plunger} \times \text{area of plunger}}$$

and since the pressure is the same throughout the press

$$\textbf{M.A.} = \frac{\textbf{area of ram}}{\textbf{area of plunger}} \quad \ldots\ldots\ldots\ldots\ldots\ldots \textbf{IV}$$

So in the example above, the M.A. is 40, that is, the effort is 1/40th the load.

WORKED EXAMPLE 3.

In a hydraulic press, the diams of inlet and ram are 1·5 cm and 24 cm respectively. Find the effort to raise a load of 9 tonnes.

We must first find the M.A. of the press.

For this press (from IV).

$$\text{M.A.} = \frac{\pi \times 12 \times 12}{\pi \times \frac{3}{4} \times \frac{3}{4}} = \frac{12 \times 12 \times 4 \times 4}{3 \times 3} = 256$$

That is, the effort is $^1/_{256\text{th}}$ of the load.

$$\text{So effort required} = \frac{9 \times 1000}{256} = 35{\cdot}2 \text{ kgf (or } 35{\cdot}2g \text{ N)}$$

Note how this type of problem does require one to be quite clear on this idea of mechanical advantage.

EXERCISE B

1. In a hydraulic press, the area of the inlet is 1 cm^2 and of the ram 50 cm^2. Find what load will be raised by the ram when a pressure of 64 kN/m^2 is applied to the inlet.
2. In a hydraulic press the plunger is 1 cm in diam and the ram 20 cm in diam. When a force of 100 N is applied to the plunger, find (*a*) the pressure in the press (*b*) the thrust exerted by the ram.

2.4 ATMOSPHERIC PRESSURE

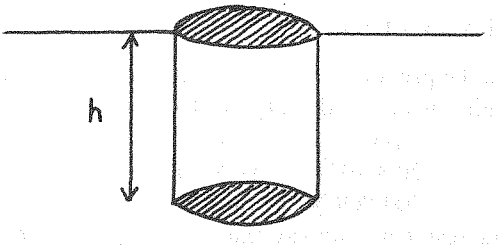
Air is a fluid and exerts a pressure on the earth's surface just like any other fluid. Air is relatively light of course — but then there is also a lot of it.

The average pressure of the atmosphere in S.I. is 101325 N/m^2 — not a convenient number! It may be shortened to 101·33 kN/m^2 or for approximate work, to 101 kN/m^2, or even to 100 kN/m^2.

2.5 PRESSURE AT A DEPTH IN A LIQUID

By this we mean the pressure at a point in the fluid. Pressure is the same in every direction around that point and, in fluids at rest, always acts normally to any surface exposed to such pressure.

By "normally" we mean at right angles to the surface. Consider a column of liquid as shown below, of height h metre, cross-sectional area A m^2 and density ρ kg/m^3.



Volume of liquid in the column = h.A

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

$$\text{mass} = \text{density} \times \text{volume}$$

Mass of liquid in the column = h.A.ρ

Weight of liquid in the column = h.A.ρ.g

The pressure in the liquid at a point h metre below the liquid surface will be due to this weight of liquid above it plus the pressure of the air above the surface.

However let us confine ourselves to the pressure due to the weight of liquid. This will be given by

$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

In this case the force is the weight of the liquid. Thus we have

$$P = \frac{h.A.\rho g}{A}$$

$$\mathbf{P = h.\rho g} \quad \ldots\ldots\ldots\ldots\ldots \mathbf{V}$$

Having used S.I. units throughout this pressure will be in N/m^2.

If we wanted to find the **TOTAL** pressure at a point in the liquid then we would have

$$\text{Total } P = h.\rho.g + \text{air pressure}$$
$$= h.\rho.g + 101\,325\ N/m^2.$$

Note that the pressure is not affected by the area A of the liquid.

Thus the pressure 10 m down a ship's sounding pipe (filled to deck level) is exactly the same as that at a depth of 10 m in the middle of the Atlantic Ocean.

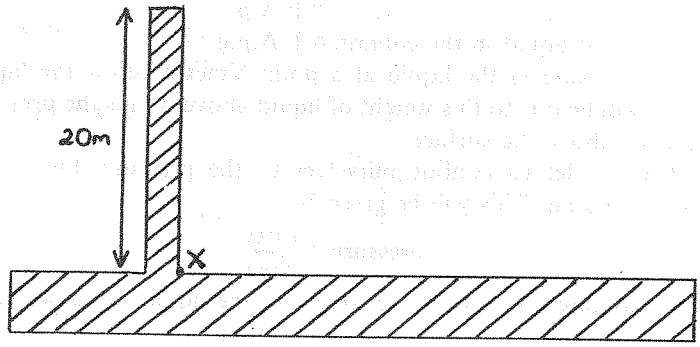
(Provided that we have the same density water at the same temperature).

WORKED EXAMPLE 4.

Calculate the pressure due to the water at a point 20 m deep in a dock containing water of density 1024 kg/m³.

$$p = h.\rho.g.$$
$$= 20 \times 1024 \times 9{\cdot}81 \text{ N/m}^2$$
$$= 200\,909 \text{ N/m}^2 \text{ (approx } 201 \text{ kN/m}^2\text{)}$$

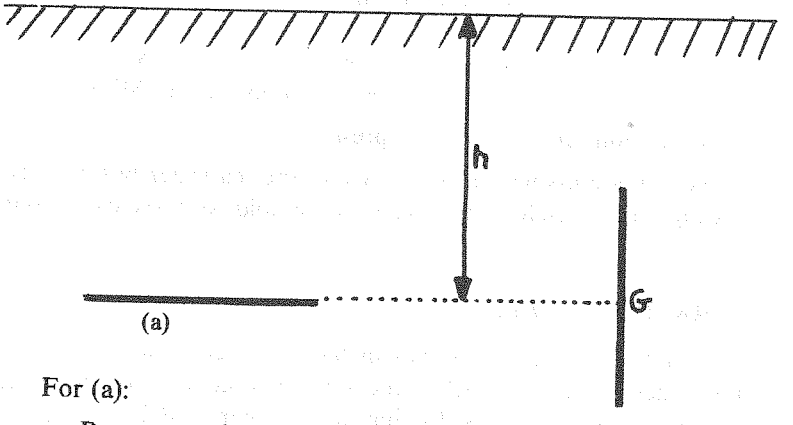
Using this result we can say that if a ship's double bottom tank were filled in that dock and the water overflowed on deck, say 20 m above the tank top, then the pressure at point X on the diagram below would also be 201 kN/m².



This pressure would be the same all over in the inside of the tank top.

2.6 THRUST ON A SURFACE

Diagram (a) below shows a plane surface (e.g. a shell plate) horizontal in some water. Diagram (b) shows the same plate vertical. The plate has an area A.



For (a):

Pressure at depth h is $h\rho g$
Thrust = pressure × area
and so the total thrust on the plate = $h\rho gA$

For (b) the situation is different. The pressures will not be the same all over the plate's surface because it is vertical. The bottom part of the plate will experience greater pressure than the top part.

In this case to find thrust we have to say

Thrust = pressure × area

The mean (or average) pressure is the pressure at the Centre of Gravity G of the plate.

On diagram (b) above G is shown also to be at a depth h below the surface.

Therefore once again we have Thrust = $h\rho gA$

WORKED EXAMPLE 5.

Find the thrust on a ship's bottom plate measuring 8 m by 3·5 m if the ship's draught is 4·16 m in sea water of R.D. 1·024.

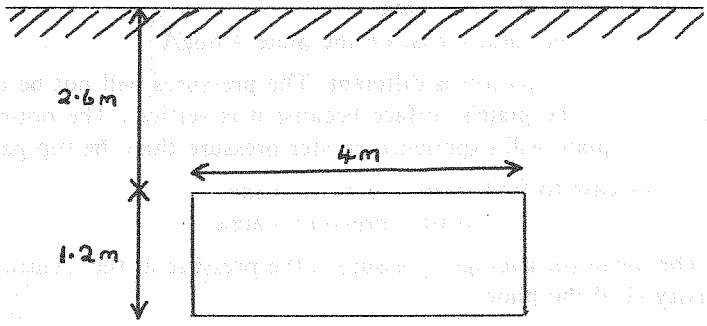
R.D. of sea water = 1·024
Therefore $\rho = 1024$ kg/m^3
Thrust = $h\rho gA$
= 4·16 × 1024 × 9·81 × 8 × 3·5 N
= 1 170 092 N (approx 1·17 MN)

What about atmospheric air pressure?

Well, it is pressing on the surface of the sea water but it is also pressing on the inside of the plate, in the hold, so these cancel out.

WORKED EXAMPLE 6.

A plate on a ship's side is 4 m by 1·2 m, as shown below, the upper edge being 2·6 m below the surface. Calculate the thrust on the plate when the ship is floating in sea water of R.D. 1·025.



Thrust = $h\rho gA$
and in this case h = 3·2 m as we can see from the diagram.
Thrust = 3·2 × 1025 × 9·81 × 4 × 1·2 N
= 154 449 N (approx 155 kN)

WORKED EXAMPLE 7.

A double bottom tank measures 14 m by 7·6 m by 1·4m. The level of the dock water (R.D. 1·025) on the ship's side is 2·2 m above the tank top. When the tank is allowed to run up till level with the water outside find the thrust on (a) the tank top, (b) the tank bottom.

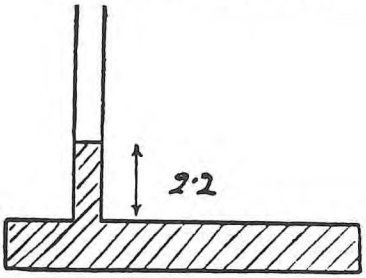
We do not need to think about the *quantity* of water at all.

In each case, the thrust is equal to pressure × area, and the pressure is solely dependent on the "head" of water (i.e. the depth of the plane below the surface).

In short it is just $h\rho gA$ in each case.

(*a*) Thrust = 2·2 × 1025 × 9·81 × 14 × 7·6 N
= 2 353 733 N (approx 2·354 MN)

(*b*) Thrust = 3·6 × 1025 × 9·81 × 14 × 7·6 N
= 3 851 563 N (approx 3·852 MN)



It is worth reflecting for a moment that if the tank was *just* full, and no water in the pipe at all, then the thrust on the tank top would be *zero*. So the small quantity of water 2·2 m up the pipe (perhaps a couple of bucketfuls!) causes a thrust of 2·354 MN on the tank top.

EXERCISE C.

1. A ship's keel plate is 5·2 m by 1·8 m. Find the thrust on the plate when the ship is in F.W. at a draught of 6·13 m.
2. A cylindrical water tank, 1·82 m diam, 3·2 m long, stands on one of its circular ends with its axis vertical. It is full of sea water, R.D. 1·040. Find (*a*) the thrust on the circular base (*b*) the thrust on the curved surface.
3. The same tank as in Question 2 closed at both ends, and mounted with axis horizontal. Find the thrust on one end and on the curved surface.
4. A shell plate 4·18 m by 1·62 m is vertical, with the upper edge 0·56 m below the surface of sea water R.D. 1·024. Find the thrust on the plate (the long edge is horizontal).
5. A fresh water tank is a cylinder 1·88 m high, and stands on one end which is 1·06 m in diameter. Find the pressure on the base when the tank is full of F.W. (Ignore atmospheric pressure).

TEST PAPER 2

Pressure, Thrust, Pressure at a depth

1. In a hydraulic press the plunger and ram are of circular cross section and are 0·056 m and 0·448 m respectively in diameter. Find (*a*) the mechanical advantage of this press.
   (*b*) the thrust exerted by an effort of 16 kgf.

2. In a hydraulic press an effort of 12 kgf on the plunger, which is of circular cross section and 2·5 cm radius, raises a load of 1200 kgf on the ram. Find the diameter of the ram. Find also the mechanical advantage of the press.

3. The crown of a water ballast tank has an area of 164 m^2 and is 7·24 m below the surface of the water. Find the thrust on the crown of the tank when the filling cock is open and the tank full. (R.D. of the water is 1·025).

4. Referring to the previous problem the filling cock is now shut. The surface of the water in the air pipe will be at sea level, which is ascertained to be 2·17 m below the deck. The diameter of the pipe is 4·8 cm. Water is now poured into the pipe up to deck level. Find (*a*) the mass of water poured into the air pipe. (*b*) the increase in the thrust on the tank top.

5. A diver can work at a total pressure of 324 kN/m^2. What is the maximum depth at which he can work in salt water of R.D. 1·024? (Assume atmospheric pressure to be 101 kN/m^2).

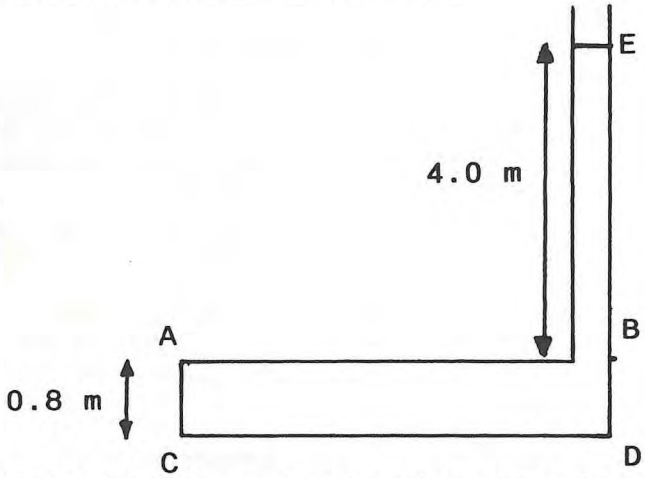
6. A submarine can stand a pressure of 1400 kN/m^2 above atmospheric pressure. Find the maximum depth at which she can work in sea water of R.D. 1·025.

7. If 0·975 m^3 of sea water weigh 1 tonne force, find the depth at which the pressure is 800 kN/m^2 greater than atmospheric.

8. A ship's side has been holed. The hole has an area equal approximately to that of a circle of radius 0·8 m, the centre of which is 5·2 m below the water line. What thrust must a temporary patch be capable of withstanding to keep the ship watertight? (R.D., of sea water 1·024).

9. In water of relative density 1·030, at a place where the atmospheric pressure is 1040 millibars, calculate
(*a*) the pressure due to the water at a depth of 4·8 m.
(*b*) the depth below the surface at which the total pressure is atmospheric pressure multiplied by three.
(*c*) the thrust due to the water on a horizontal circular hatch of radius 0·65 m at a depth of 2·6 m below the surface.
(SCOTVEC Dec 1990)

10. The diagram below shows a vertical section through a double-bottom tank with a vertical filler pipe.
CD is the ship's bottom. *AB* is the tank top, 0·8 m above *CD*.
The top and bottom of the tank are horizontal.
The tank is filled with sea water of relative density 1·030 to point *E* in the filler pipe, 4 m above *AB*.



Calculate (*a*) the pressure on the tank top due to the water in the tank;
(*b*) the thrust on the tank top due to the water in the tank if the tank top is a rectangle 8·0 m by 6·5 m;
(*c*) the thrust on the end of the tank due to the water in the tank if the end is 6·5 m by 0·8 m.
(SCOTVEC March 1990)

CHAPTER 3

STATICS

Addition of Vectors

3.1 DEFINITION OF A VECTOR

A vector quantity has both magnitude (size) and direction.

Force is an example of a vector — its direction as well as its magnitude must be specified.

Another example of a vector is velocity — we might say the wind velocity is 3 metres per second from the south.

Any vector can be represented in magnitude and direction by a straight line. The length of the line (to scale) represents the magnitude of the vector and the direction of the line coincides with the direction of the vector.

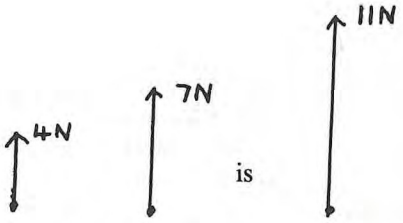
3.2 RESULTANT VECTOR

This is the single vector which would have the same effect as two or more other vectors, i.e. it is the result of adding the vectors.

3.3 VECTORS ACTING IN THE SAME DIRECTION

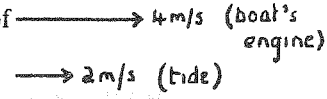
To find the resultant of vectors which all act in the same direction is a relatively simple operation — we simply add the magnitudes.

For example

i) The resultant of 

is



ii) The resultant of 

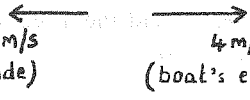
is 

3.4 VECTORS ACTING IN THE OPPOSITE DIRECTION

Again this is a simple operation — we subtract the magnitudes. For example

i) The resultant of 

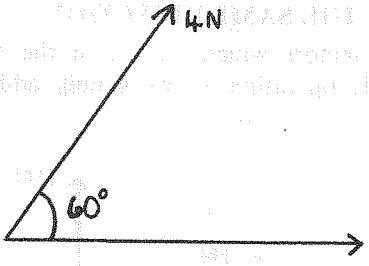
is 

ii) The resultant of 

is 

3.5 VECTORS AT AN ANGLE TO EACH OTHER

This is not as straight-forward. Consider the following two vectors.



How do we find their resultant?

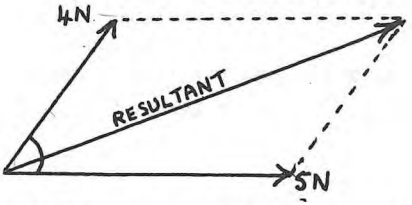
How do we take account of the angle?

We use what is known as the **PARALLELOGRAM OF FORCES.**

This states:-

If two vectors acting at the same point are represented in magnitude and direction by two straight lines drawn from that point, then the resultant vector is the diagonal of the completed parallelogram drawn from the same point.

Using this for the above example we obtain:



If the above parallelogram was drawn to scale carefully we would find that the magnitude of the resultant is approximately 7·8 N and that it is acting at an angle of approximately 26½° to the 5 N vector.

Of course the bigger the scale drawing used the more accurate is our resultant.

It should be noted that an alternative to scale drawing is to use trigonometry. This takes longer but is more accurate.

EXERCISE A

1. Find the magnitude of the resultant of:

a) 5 N and 12 N at 90° to each other,

b) 30 N and 30 N at 30° to each other,

c) 18 N and 30 N at 60° to each other,

2. Two stays are attached to an eyebolt, the tension in one is 7 tf and in the other 11 tf, the angle between them is 120°.

Find by scale drawing the force on the eyebolt.

3. Forces of 2 kgf, 3 kgf, 4 kgf and 5 kgf act on a ringbolt in directions of North, East, South and West respectively. Determine the magnitude and direction of the resultant force on the ringbolt.

3.6 FORCES

Since the force is the most common example of a vector, I shall use forces in the remainder of the Chapter. However, the discussion is still valid for all vectors.

3.7 EQUILIBRANT

This is the name given to a particular single force.

We have seen that the resultant is a single force which would have the same effect as two given forces.

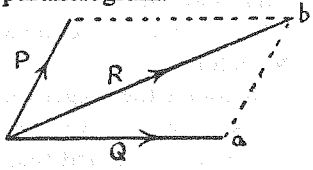
The EQUIBLIBRANT is equal and opposite to the resultant, that is, it would exactly balance the two forces.

The whole process (that is of combining two forces by the parallelogram law to find their resultant or equiblibrant) is referred to as the **COMPOSITION OF FORCES.**

3.8 THE TRIANGLE OF FORCES

This is obtained by using half of the parallelogram.

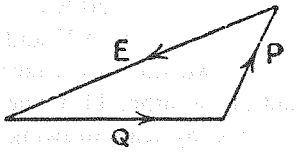
Take two forces P and Q and combine them by the parallelogram law to give the resultant R.



Once again R is the *one* force which has the same effect as P and Q together. And if the arrow of R were pointing the opposite way, it would signify the **equilibrant,** i.e., the one force would exactly balance P and Q.

Also it is obvious that side ab is equal to P.

If we make these changes we get this triangle.



Triangle of forces.

This is the **triangle of forces.**

Definition: If three forces are acting on a body in equilibrium, then any triangle which has its sides parallel to the lines of action of the three forces, taken in order, shall have the lengths of those sides proportional to the magnitude of the forces ..... **V**

Note:

Equilibrium. This means the forces balance and their resultant is zero.

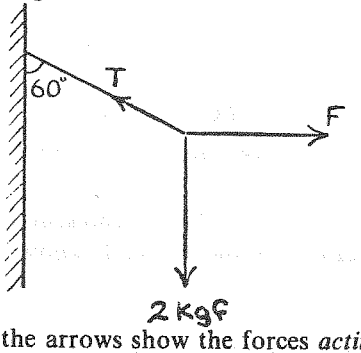
Parallel. The sides of the triangle do not have to coincide with the actual wire or rod along which the force is acting. It is enough to be parallel to the wire or rod.

Taken in order. The arrows in the triangle must run the same way, that is, in the direction of the forces concerned.

Finally, of course, the triangle must be drawn to scale, so *one of the forces must be known.*

WORKED EXAMPLE 1

A 2 kg mass hangs by a cord on a wall. The mass is pulled away from the wall by a string held horizontally until the cord makes an angle of 60° with the wall. Find the tension in each.



The forces acting on the mass are as shown.

Very Important:

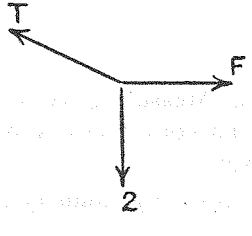
Before going on to the problem, note particularly the direction of the arrows. How are these decided? One must fix one's mind firmly *on what it is the forces are acting on.* In this case, the arrows show the forces *acting on the mass.*

Alternatively, the force acting *on the wall* would show the arrow in T pointing the other way.

There is nothing wrong with that, really. The cord is in a state of tension, trying to pull the wall down, the weight up.

So it is not enough to start by saying "what are the forces acting?". We must *always* ask ourselves, "what are the forces acting *on the mass*?" or whatever it is.

Well now for the problem.



The forces acting on the mass are as shown. (A diagram like this simply showing the arrangement of the forces acting is sometimes called the **space diagram**).

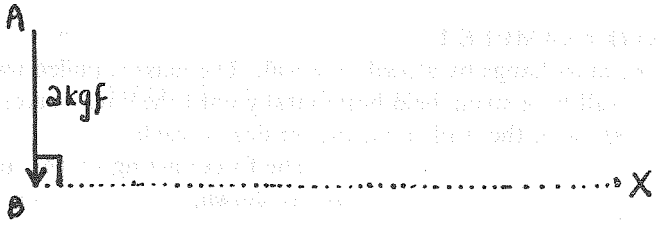
These three forces are in equilibrium.

So we want some triangle whose sides are parallel to the forces, and the arrows have to run the correct way round.

Method.

The magnitude of only one force is known — the 2 kgf weight.

Thus we can draw a line AB to scale represent this.

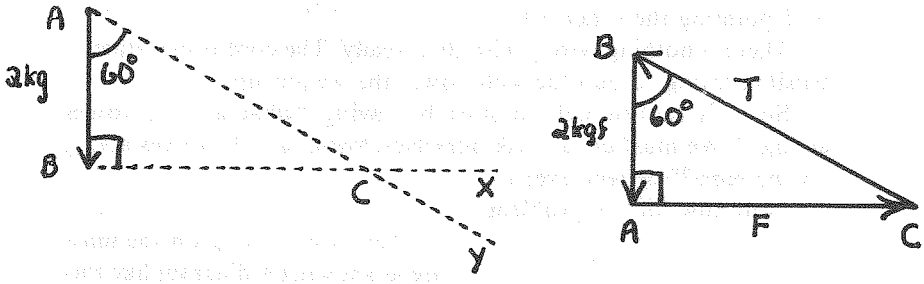


We can now draw a line in the direction BX to represent F.

Note we do not know how long to draw this line but we do know it is at 90° to AB.

Similarly we can draw a line in a direction AY to represent T. Again we do not know how long to draw this line but we do know it will be at 60° to AB.

These lines intersect at C to give us a representation of F and T.



We now have a FORCE TRIANGLE. Measuring AC and CB will give us the tensions in the string and cord respectively. We should obtain approximately 3·5 and 4 kgf.

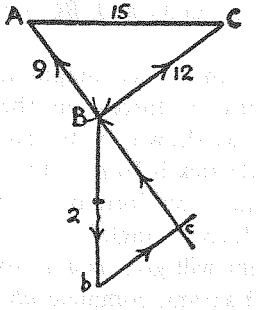
Note we could obtain these tensions using trigonometry rather than scale drawing.

WORKED EXAMPLE 2

Two wire spans 9 m and 12 m long are attached to points 15 m apart, on the same level. The other ends of the spans are shackled together and carry a load of 2 tonnes.

Find the tension in each span.

The forces acting on the shackle are as shown.



At the moment we do not know any angles — just lengths of sides.

Method

We construct the space diagram to scale and so obtain the correct directions for the sides of the force diagram. To some convenient scale make $AC = 15$ units.

With dividers set to 9 units draw in an arc near B.

With dividers set to 12 units draw in an arc to cut this.

Join AB, CB.

We can now either measure the angles and draw the force diagram. In this case we can make use of the space diagram.

From B, draw the vertical to scale to represent 2 tf.

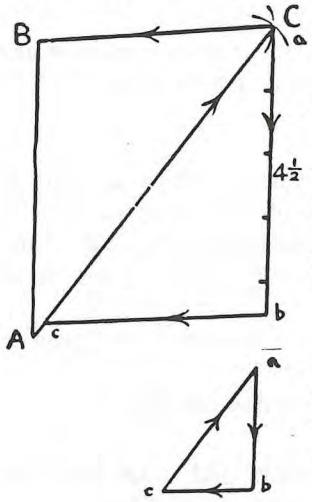
Produce AB to give side cB.

Draw bc parallel to BC, and the force triangle is complete. Measure bc and cB.

They are found to be 1·2 and 1·6 tf respectively.

WORKED EXAMPLE 3

A 10 m derrick is supported by a span 6 m long attached to a vertical mast at a point 8 m above the heel of the derrick. Find the forces in derrick and span when a load of 4½ tonnes is hanging vertically from the derrick head.



Draw AB to scale equal to 8 units.

Cut in $AC = 10$ units and $BC = 6$ units.

Then ABC is the space diagram, and the directions of forces on the derrick head are as shown. Note that the force on the derrick is from A to C in order to maintain equilibrium.

Where is the force triangle?

A little thought will give us abc, so using the space diagram, complete abc as shown. Measure bc ($= 3{\cdot}4$ tf the stress in the span) and ca ($= 5{\cdot}6$ tf, the stress in the derrick).

Work through this carefully.

Note that in this example, and in the ones that follow, the weight of the derrick is ignored.

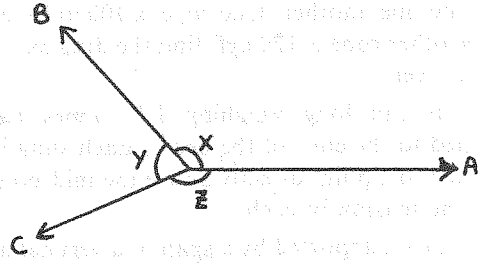
EXERCISE B

1. Two spans 6 m and 8 m long are attached to points on the same level, 10 m apart. The other ends are shackled together and carry a load of 2·5 tonnes. Find the tension in each span.
2. A boat is moored in a river 200 m wide by two ropes at right angles attached to trees on opposite banks and directly opposite one another. One rope is 100 m long. If the tension on the other rope is 120 kgf, find the drag exerted on the boat by the river.
3. A beam 3 m long weighing 1·5 tonnes hangs on slings attached to the ends of the beam. Each sling is 2 m long and attached to a point directly above the mid-point of the beam. Find the tension in each.
4. A derrick is supported by a span to a vertical mast. The angle between derrick and mast is 50° and the span is at right angles to the derrick. Find thrust on derrick and tension in the span when a load of 6 tonnes hangs from the derrick head.
5. A 12 m derrick is held by a span 6 m long attached to the derrick heads and to a point on a vertical mast 10 m above the heel of the derrick. Find the tension in the span and thrust in the derrick when a load of 2 tonnes is hanging from the derrick head.

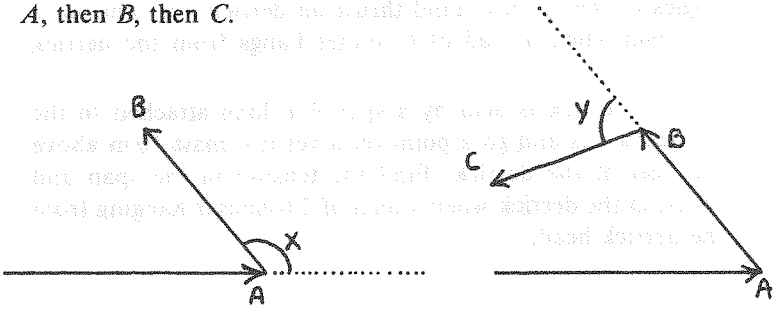
3.9 POLYGON OF FORCES

In the previous section we considered three forces in equilibrium, i.e. their resultant was zero and we were able to draw a triangle. We shall now consider how to find the resultant of three forces which are not in equilibrium.

Consider for example the following system of forces:

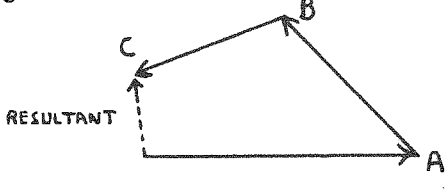


Suppose we attempt to draw the triangle of forces, starting with *A*, then *B*, then *C*.



The fact that we do not obtain a triangle tells us that the forces are not in equilibrium.

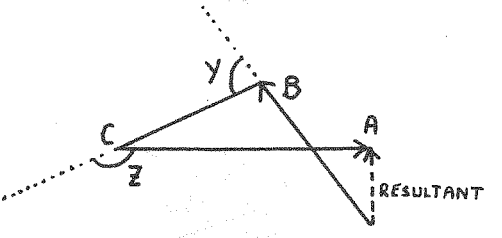
To obtain the resultant we simple draw a line from the STARTING POINT to the FINISHING POINT. The resulting figure is a polygon.



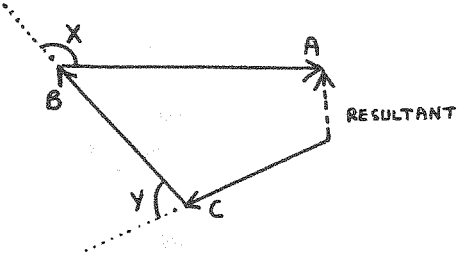
Of course the equilibrant would be exactly the same line as the resultant, but in the opposite direction.

NOTE: We need not have started with A, but whichever force we started with we would have obtained the same resultant.

Starting with B, then C, then A (i.e. going anti-clockwise).



Starting with C, then B, then A (i.e. going clockwise).



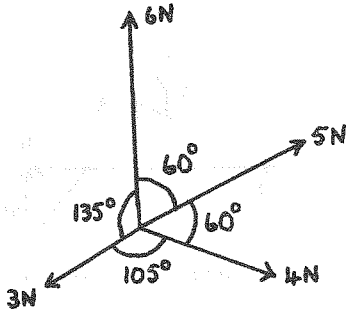
The above method can be extended to deal with a system of any number of forces.

We can start by drawing, to scale, any force we care to chose, but then we must proceed either clockwise or anti-clockwise until all the forces have been drawn.

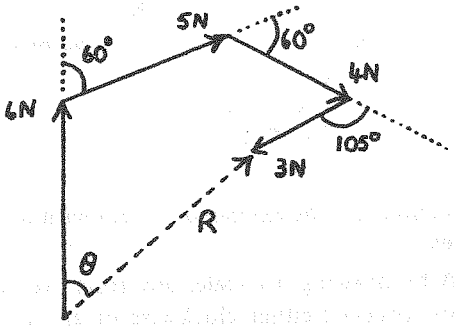
Great care must be taken to draw the forces at the correct angle to each other.

WORKED EXAMPLE 4

Determine if the following system of forces is in equilibrium. If not, determine the magnitude and direction of its resultant.



Starting with the 6 N force and going clockwise, we obtain:

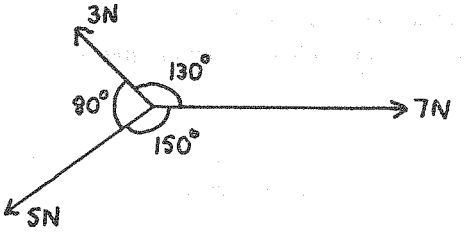


We find that R is approximately 7·2 N and θ is approximately 52°. (Or R is 38° to the horizontal).

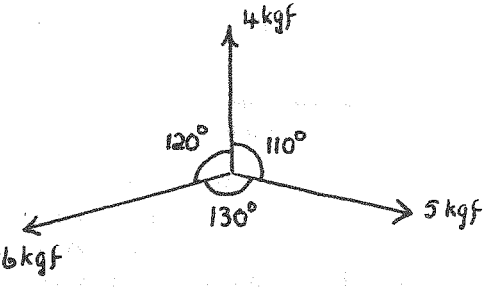
EXERCISE C

Find the magnitude and direction of the resultant of each of the following systems of forces.

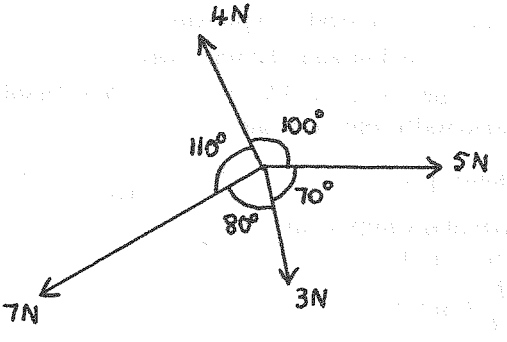
1.



2.



3.

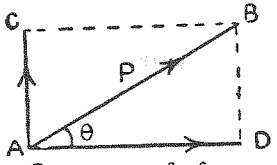


3.10 THE RESOLUTION OF FORCES

This is the reverse operation to finding the resultant of two forces, that is, given a single force, find the two forces that would have the same effect as the single one.

The two forces are then called the **components** of the original one.

In general, this is done graphically in our work. There is one exception, namely, *when the components are at right angles*. It can then be done quite easily by trigonometry.



Components of a force.

Let force P be represented by the vector AB.

Then clearly, by constructing the rectangle, components AC and AD would have the same effect as P (because if we reversed our steps, starting with AC and AD we should get AB as the resultant).

AC is called the **vertical component**.

AD is called the **horizontal component**.

Splitting a single force up like this is called "**resolving** (i.e. **resolution) horizontally and vertically**".

Finally, if we know the direction of AB, then $\frac{AD}{AB} = \cos\theta$, that is

the horizontal component $= \mathbf{P} \cos\theta$

And (since $BD = AC$)

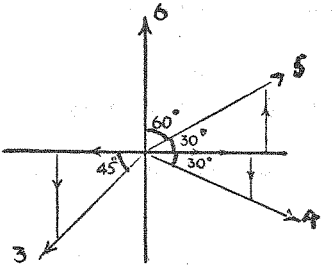
$$\frac{BD}{AB} = \sin\theta, \text{ that is}$$

the vertical component $= \mathbf{P} \sin\theta$

WORKED EXAMPLE 5

Forces of 6, 5, 4, 3 kgf act in directions 000°, 060°, 120°, 225°, respectively. Find the resultant force by resolving into vertical and horizontal components.

N.B. *Work through the following, checking every step.*



The forces are as shown (not to scale).

We must first combine all the vertical components, so as to obtain a *single vertical force.*

Similarly we must obtain a *single horizontal force.*

We then combine these two forces to obtain the resultant, which is thus the resultant of the whole system.

Resolving vertically (call upwards +, downwards −) we have,

$6 + 5 \sin 30 - 4 \sin 30 - 3 \sin 45.$

i.e., $6 + 2{\cdot}5 - 2 - 2{\cdot}12 = 8{\cdot}5 - 4{\cdot}12$

$= + 4{\cdot}38$, that is, a single component of 4·38 kgf acting vertically upward.

Resolving horizontally (to the right +, to the left −) we have,

$5 \cos 30 + 4 \cos 30 - 3 \cos 45.$

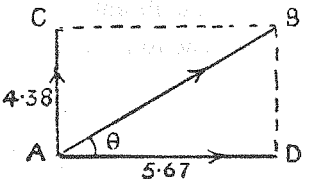
i.e., $4{\cdot}33 + 3{\cdot}46 - 2{\cdot}12 = 7{\cdot}79 - 2{\cdot}12$

$= + 5{\cdot}67$, that is, a single component of 5·67 kgf acting to the right.

We have now reduced it to two forces as shown here.

To find the magnitude of the resultant R we can use Pythagoras

$$(AB)^2 = (BD)^2 + (AD)^2$$
$$= (4{\cdot}38)^2 + (5{\cdot}67)^2$$
$$R = 7{\cdot}16 \text{ kgf}$$



To find the direction of R

$$\tan \theta = \frac{BD}{AD}$$
$$= \frac{4{\cdot}38}{5{\cdot}67} \text{ kgf}$$
$$\theta = 37{\cdot}69°$$

Hence the resultant is 7·16 kgf in a direction 37·69° above the horizontal.

EXERCISE D

Rework the problems at Exercise C using resolution.

TEST PAPER 3
Composition and Resolution of Forces

1. Four forces act on a ringbolt as follows:
   160 kgf in direction 090°, 200 kgf at 000°, 300 kgf at 315°, 120 kgf at 240°.
   Calculate the magnitude and the direction of the resultant force on the ringbolt.

2. A derrick, inclined at 40° to a vertical mast, is supported by a span which is horizontal. A load of 4 tonnes hangs from the derrick head. Calculate the tension in the span and the thrust in the derrick.

3. Two wires with loads of 20 t and 8 t respectively are attached to a ringbolt, the angle between their lines of action being 48°. Calculate by resolution of forces the magnitude of the resultant stress on the ringbolt.

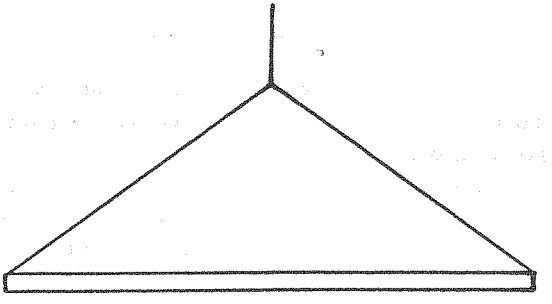
4. A 200 kg mass hangs by a chain from a ringbolt in a wall. The mass is pulled away from the wall by a horizontal wire until the chain makes an angle of 60° with the wall. By resolving horizontally and vertically calculate the force on the ringbolt.

5. (*a*) For a set of forces acting on a body, define the terms:
   (i) resultant force;
   (ii) equilibrant.
   (*b*) The following four horizontal forces act on a bollard:

   2200 N in direction 040°T
   1800 N in direction 165°T
   1600 N in direction 210°T
   800 N in direction 325°T

   By means of a scale diagram, find the resultant force and the equilibrant.
   (SCOTVEC December 1989)

6. The diagram shows a uniform beam of length 2·4 m and weight 1200 N suspended by a two-legged sling, each leg of the sling being of length 1·85 m. Find the tension in each leg of the sling:
(*a*) by calculation; (*b*) by scale drawing.
(SCOTVEC July 1991)



CHAPTER 4

STATICS

Moments, Centres of Gravity

4.1 THE CENTRE OF GRAVITY of a body is that point through which the whole weight of the body may be assumed to act vertically downwards.

Thus for example, if a uniform beam is 10 m long and weighs 1 tonne then we can regard this as if a single force of 1 tf were acting vertically downwards through a point 5 m from either end. This is the point at which the beam would balance, if supported by a single prop. And the prop would have to exert a force of 1 tf *upwards*.

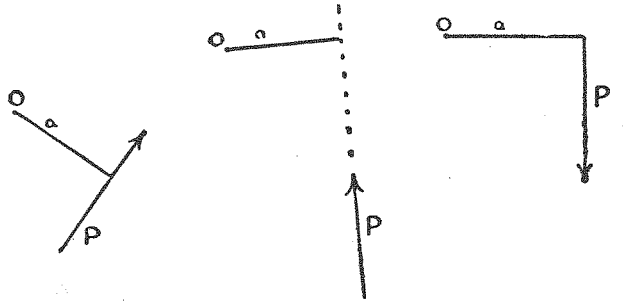
Thus we get the idea of the "equilibrant" again. The "resultant" force on the beam is 1 tf acting downwards, and the equilibrant is 1 tf acting upwards.

4.2 MOMENT OF A FORCE. This is a measure of the turning power exerted by a force.

Definition: The moment of a force about a point is the product of the force and the perpendicular distance between the point and the line of action of the force.

Or in short,

$$\text{Moment} = \text{Force} \times \text{Arm} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad \text{I}$$



Moment of a force.

E.g., in the three cases shown, if P is any force, then the moment of P about O is $P \times a$.

Strictly speaking, rotation can only take place about an axis (not about a "point"). So in the above diagram, O is the point in which an axis normal to the paper intersects the plane of the paper. In the interests of economy of words, it is usual to speak of the "moment about O", instead of the "moment about the axis normal to the plane of the paper and intersecting the paper in O.

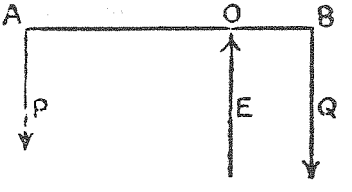
Since the S.I. unit of force is the newton and of length the metre, the **unit of moment in S.I. is the newton metre (Nm).**

The reader may meet other units, for example in naval architecture, but in physics, this is the unit we shall try to use at all times.

To Find the Equilibrant of Two Like, Parallel Forces.

(Like, i.e., acting in the same direction, and parallel to each other).

For example, let us find the equilibrant of two forces, P and Q, acting at opposite ends of a light rod, AB, as shown in the diagram below.



Equilibrant of two parallel forces.

Clearly, E will be at some point O, nearer to B than A.

Remember that E is one single force that will exactly balance the two forces P and Q. In other words, if there were two forces (weights for example) P and Q acting on the rod, then O is the point about which the rod would balance.

If we think of the rod as being in equilibrium under *three* forces P and Q downwards, E upwards, then for this state of equilibrium to be possible, we must have two conditions.

They are:

1. That the sum of all upward forces equals the sum of all the downward forces.

Hence, $P + Q = E$ II

2. That if moments are taken about any point, then the sum of all the clockwise moments equals the sum of all the anti-clockwise moments.

This is known as the Principle of Moments.

Hence (taking moments about O) P × AO = Q × BO III

These two principles, (1) and (2), are all that is required to solve any problem of this type.

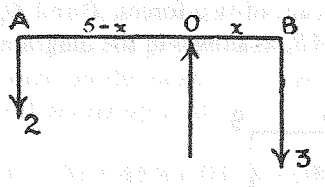
WORKED EXAMPLE 1.

A rod 5 m long supports masses of 2 kg and 3 kg as shown. Find the point at which the rod would balance and the load on the pivot.

Then clearly, from **II**.

load on pivot $= 2g + 3g = 5g$ N

where g is the acceleration due to gravity.



Now, let the pivot be x m from end B.

Then since moments clockwise = moments anti-clockwise, we have,

$3gx = 2g(5 - x)$

Divide both sides by g

$3x = 2(5 - x)$

$3x = 10 - 2x)$

$5x = 10$

$x = 2$ m

Hence, pivot should be 2 m from end B.

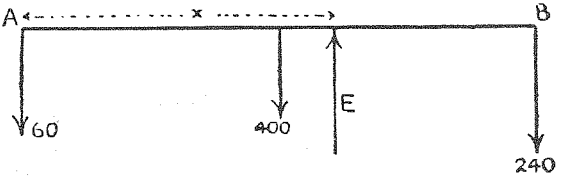
It is also important to note here that if O is the point at which it will balance, then O is the **Centre of Gravity of the arrangement.**

The foregoing method is suitable only for the simplest cases, such as Example 1. Normally, we shall be dealing with more complex systems and in all such, it is preferable to take moments about an axis at one or other extremity of the system (or even about an axis outside the system altogether).

An example or two will make this clear.

WORKED EXAMPLE 2.

A beam AB, 12 m long weighs 400 kg. At A it carries a load of 60 kg and at B a load of 240 kg. Find the C.G. of the loaded beam.



The forces acting on the beam are as shown, viz., its own weight, acting through its C.G., at the mid-point, and 60 and 240 as shown.

Let E be the equilibrant.

Since sum of upward forces = sum of downward forces,

$E = 60 + 400 + 240 = 700$ kgf.

Moments about A.

Clockwise = anti-clockwise.

i.e., $(400 \times 6) + (240 \times 12) = 700 \times x$. (Cancelling g throughout).

(The 60 kg force passes *through* A and so has no moment).

$$700x = 2400 + 2880$$

$$700x = 5280$$

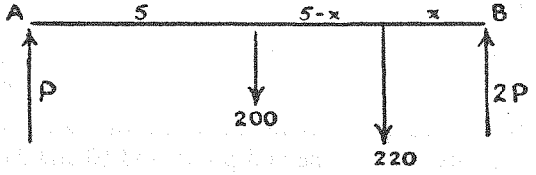
$$x = 7{\cdot}54 \text{ m}$$

i.e., C.G. is 7·54 m from A.

WORKED EXAMPLE 3.

A beam mass 200 kg and 10 m long is supported on two props at the ends. Find where a mass of 220 kg must be placed for the load on one prop to be double the load on the other.

Conditions are as shown.



Forces up = forces down

$P + 2P = 200 + 220$

$3P = 420$ and $P = 140$ kgf

so $2P = 280$ kgf

To find where the 220 kg mass is placed, we want x.

Take moments about B.

Clockwise = anti-clockwise, i.e.,

$(140 \times 10)g = (200 \times 5)g + (220 \times x)g$

$1400 = 1000 + 220x$

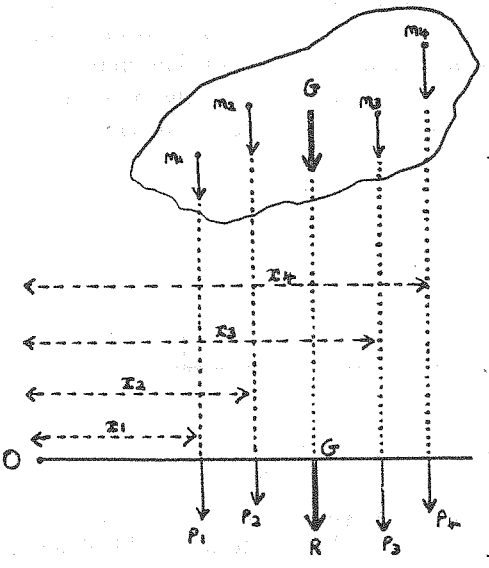
whence $x = 1{\cdot}82$ m

i.e., mass must be placed 1·82 m from heavy end.

EXERCISE A

1. A girder AB 6 m long weighing 120 kg is supported by a vertical wire at each end. A man weighing 84 kg stands 1·4 m from end A and 26 kg is placed 2·2 m from end B. Find the tension in each wire.
2. A beam AB 4 m long, weighing 20 kg is supported on two props M, N, so that $AM = 0{\cdot}6$ m and $BN = 1{\cdot}2$ m. A mass of 4 kg is hung from A and 12 kg from B. Find the load on each prop.
3. A rod 1·4 m long weighing 8 kg is laid on a table with 0·4 m projecting over the edge. What is the greatest mass that can be hung on the end without capsizing it?
4. Masses of 3, 5, 7, 8 kg are suspended at equal distances of 1 m from a light rod. Find the C.G. of the arrangement.

4.3 CENTRE OF PARALLEL FORCES



Consider a mass M as shown, composed of heavy particles m_1, m_2, m_3, ... Then the mass M of the whole body $= m_1 + m_2 + m_3 + \ldots$ i.e., Σm*, and the position through which the whole weight of the body acts is the position of the resultant of all these parallel forces.

Projecting these along their line of action and calling the forces $P_1 P_2 P_3 \ldots$ (that is, $P_1 = m_1 g$, and so on), clearly, to find the position of the resultant is just a problem in moments.

From first principles,

$$R = P_1 + P_2 + P_3 + \ldots = \Sigma P$$

and taking moments about the usual axis through O, we have, since **the sum of the moments of components about any point = the moment of the resultant about the same point,**

$$P_1 x_1 + P_2 x_2 + P_3 x_3 + \ldots = R \times OG$$

whence

$$OG = \frac{P_1 x_1 + P_2 x_2 + P_3 x_3 + \ldots}{R}$$

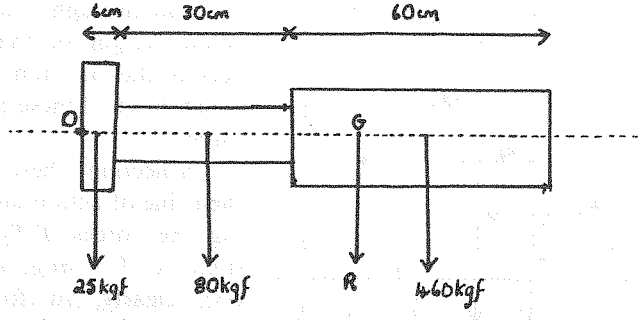
$$OG = \frac{P_1 x_1 + P_2 x_2 + P_3 x_3 + \ldots}{P_1 + P_2 + P_3 + \ldots} = \frac{\Sigma P x}{\Sigma P} \quad \ldots\ldots\ldots \quad \textbf{IV}$$

The position through which the resultant R of all these parallel forces acts is called the **Centre of Parallel Forces**. It will be seen, on reflection, that it is only an alternative concept of the Centre of Gravity — though one having many useful applications.

* Σm, i.e., Sigma m, meaning "the sum of all the quantities like m".

WORKED EXAMPLE 4

The diagram below shows a shaft which is made up of three parts. It has a flange which is 6 cm thick and has a mass of 25 kg joined to a length of 30 cm which has a mass of 80 kg. This in turn is joined to a 60 cm length of mass 460 kg. Each of the 3 parts is cylindrical and of uniform cross-sectional area. Determine the position of the shaft's centre of gravity.



The above diagram shows the weight of each individual part acting from the midpoint of the part. R is the total weight of the shaft acting from its centre of gravity whose position we have to find.

Then $R = 25 + 80 + 460 = 565$ kgf.

Taking moments about O we can say that

Moment of whole = sum of moments of components

Thus, working in newton metre, we have

$$565g \times OG = (25g \times 0{\cdot}03) + (80g \times 0{\cdot}21) \times (460g \times 0{\cdot}66)$$

cancelling out g

$$\begin{aligned} 565 \times OG &= (25 \times 0{\cdot}03) + (80 \times 0{\cdot}21) + (460 \times 0{\cdot}66) \\ &= 0{\cdot}75 + 16{\cdot}8 + 303{\cdot}6 \\ &= 321{\cdot}15 \end{aligned}$$

$$\therefore \quad OG = \frac{321{\cdot}15}{565} \text{ m}$$

$$OG = 0{\cdot}568 \text{ m}$$

Therefore the C.G. of the shaft is 56·8 cm from O.

Note it is sometimes the case that the masses of the various portions are not given but we are given their dimensions.

In such cases the portions are assumed to be uniform and we say that each mass is directly proportional to its volume (or area for a thin lamina).

Thus when calculating moments we can use volumes (or areas) instead of masses.

Thus for Worked Example 4 above suppose we are not given the masses but are told that the radius of the flange is 11·5 cm, that of the middle shaft is 9·2 cm and that of the remaining shaft is 15·6 cm.

Before taking moments we calculate the volume of each portion. Since they are all cylinders we use the formula $V = \pi r^2 h$ in each case:

For the flange $V = \pi \times 11{\cdot}5^2 \times 6 = 793{\cdot}5\,\pi$ cm

For the middle shaft $V = \pi \times 9{\cdot}2^2 \times 30 = 2539{\cdot}2\,\pi$ cm

For the other shaft $V = \pi \times 15{\cdot}6^2 \times 60 = 14601{\cdot}6\,\pi$ cm

Therefore total volume of the whole shaft $= 17934{\cdot}3\,\pi$ cm

Now we take moments about O as before:

Moment of whole = Sum of moments of components

$$17934{\cdot}3\pi \times OG = (793{\cdot}5\pi \times 3) + (2539{\cdot}2\pi \times 21) + (14601{\cdot}6\pi \times 66)$$

We can divide throughout by π

$$17934{\cdot}3 \times OG = 2380{\cdot}5 + 53323{\cdot}2 + 963705{\cdot}6$$

$$OG = \frac{1019409{\cdot}3}{17934{\cdot}3}$$

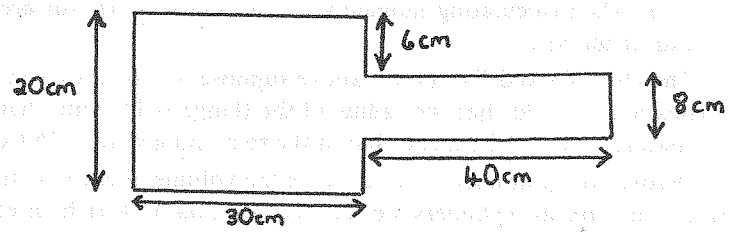
$$OG = 56{\cdot}8 \text{ cm}$$

which is what we obtained before!

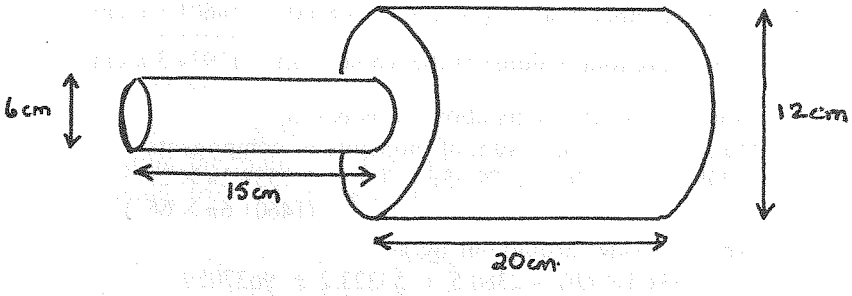
However it is apparent that NOT being given the masses and having to calculate volumes has made the solution more long-winded.

EXERCISE B

1. Find the position of the C.G. of a thin lamina which consists of two rectangles as shown below.



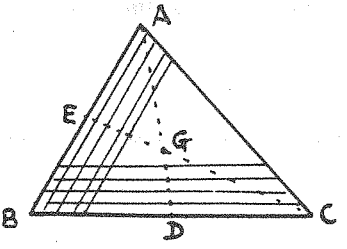
2. A solid object is composed of two cylinders as shown below. Find the position of its C.G.



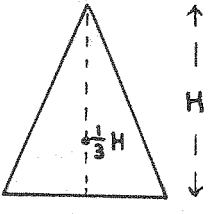
4.4 C.G.'s IN PARTICULAR CASES

1. **Symmetrical figure** (square, circle, sphere, **H** section girder, for example) — the C.G. lies at its geometrical centre.

2. **Triangular area (or lamina).**

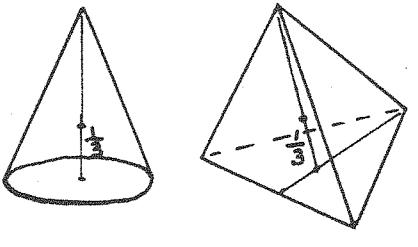


Imagine it divided into strips. The C.G. of each strip is at its mid-point. Therefore the C.G. must lie on the median AD. Similarly it must lie on the median CE. Hence, the C.G. must lie at G, the intersection of the medians. And since the medians of any triangle trisect each other, we have that **the C.G. of any triangular lamina lies $^1/_3$ of the way along any median**, from the side towards its opposite apex.



For a lamina in the shape of an isosceles triangle, this just becomes $^1/_3$ of the perpendicular height.

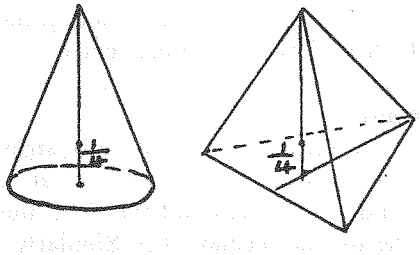
3. **Hollow cone or pyramid** (without proof).



The C.G. lies $^1/_3$ up the line drawn from the C.G. of the base to the apex.

N.B. *Although we have referred to a "base", there is no actual base in a "hollow cone". If there is a base, the C.G. must be found by moments.*

4. Solid cone or pyramid (without proof).



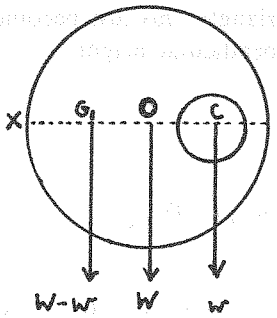
The C.G. lies $^1/_4$ way up the line joining the C.G. of the base to the opposite apex.

Note that in the case of the pyramid, hollow or solid, any plane face may be regarded as the "base".

4.5 MOVEMENT OF MASSES

When a weight, or a portion of the body itself, is removed, the centre of gravity moves to a new position *away from* the removed weight.

If a weight is added, the centre of gravity moves *towards* the added weight.



For example, consider the circular plate illustrated, of mass W, from which a smaller circular portion of mass w has been removed. The centre of gravity of the plate, originally at O, will move to some new position G_1.

To find it, take moments. We may do this about any point, but let us take moments about O.

We can say

Moment of portion remaining = Moment of portion removed

Then clearly, $(W-w)OG_1 = w \times CO$

i.e. $$OG_1 = \frac{w \times CO}{W-w}$$

This has many applications on board ship, in connection with loading and discharging cargo.

Taking moments about O is an alternative to taking moments about, for example, X.

In this latter case we would say

Moment of whole before removing portion = Sum of moments of portions remaining and removed

i.e. $W \times OX = (w \times CX) + (W - w) \times G_1X$

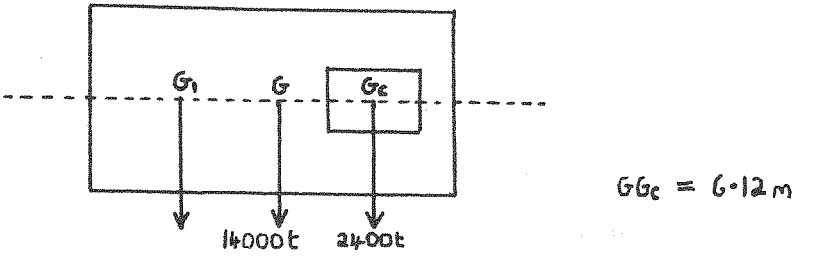
We can re-arrange this to find G_1X.

WORKED EXAMPLE 5

A vessel of 14 000 t displacement and centre of gravity at G discharges 2400 t of cargo from No. 4 hold, the C.G. of the discharged cargo being 6·12 m abaft G. Find the movement of the C.G. of the vessel.

On the following diagram the positions of the centres of gravity are:

for the cargo G_c, for the loaded vessel G and for the unloaded vessel G_1.



Note that on discharging the cargo the C.G. must move forward.

Taking moments about G

moment of load remaining = moment of load removed

$$11600 \times G_1G = 2400 \times 6{\cdot}12$$

$$G_1G = \frac{14688}{11600}$$

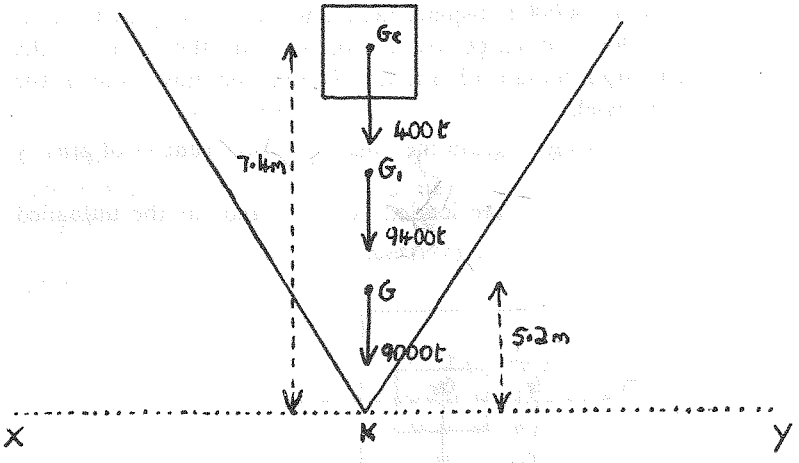
$$= 1.266 \text{ m}$$

Thus the C.G. has moved 1·266 m forward.

WORKED EXAMPLE 6

A vessel of 9000 t displacement with centre of gravity 5·2 m above the keel loads 400 t of cargo in the 'tween deck. The centre of gravity of this cargo is 7·4 m above the keel. Find the position of the new centre of gravity above the keel.

On the following diagram the positions of the centres of gravity are:
for the vessel G , for the cargo G_c , for the new C.G. G_1.



We shall consider two alternative solutions.

Solution 1

Take moments about G

$$\text{moment of loaded vessel} = \text{moment of cargo}$$

$$(9000 + 400) \times GG_1 = 400 \times 2{\cdot}2$$

$$9400 \times GG_1 = 880$$

$$GG_1 = \frac{880}{9400}$$

$$GG_1 = 0{\cdot}094 \text{ m}$$

The new C.G. is therefore 5·294 m above the keel.

Solution 2

Take moments about line XY

moment of loaded vessel = moment of vessel + moment of cargo

$$9400 \times KG_1 = (9000 \times 5{\cdot}2) + (400 \times 7{\cdot}4)$$
$$= 46800 + 2960$$
$$= 49760$$
$$KG_1 = \frac{49760}{9400}$$
$$KG_1 = 5{\cdot}294 \text{ m}$$

The new C.G. is 5·294 m above the keel as found previously.

More generally for a vessel of displacement W to which w_2 is loaded and w_3 is discharged taking moments about the keel gives $(W + w_2 - w_3) \times KG_1 = (W \times KG) + (w_2 \times KG_2) - (w_3 \times KG_3)$ where G_2 is the C.G. of the loaded cargo and G_3 is the C.G. of the discharged cargo.

Thus
$$KG_1 = \frac{(W \times KG) + (w_2 \times KG_2) - (w_3 \times KG_3)}{W + w_2 - w_3}$$

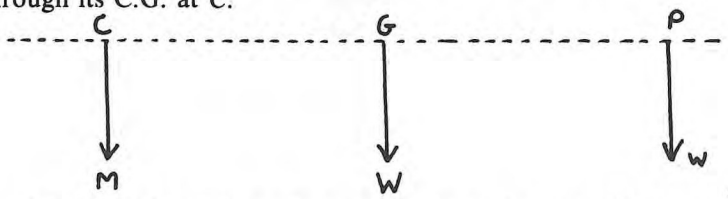
EXERCISE C

1. A vessel of displacement 16 000 t discharges 1800 t of cargo. The C.G. of this cargo is 7·2 m forward of the C.G. position of the loaded vessel. Find how far the vessel's C.G., moves after the discharge.
2. 500 t of cargo are loaded into the 'tween deck of a vessel of displacement 12 000 t. Before loading the C.G. of the vessel is 6 m above its keel. If the C.G. of the cargo is 8 m above the keel find the position of the C.G. of the loaded vessel above the keel.

Important Note

The case of a cargo being moved to some new position, as distinct from being discharged or added, calls for a little more thought.

Let us consider a ship whose total displacement is W and let us regard this as composed of, say, w in No. 5 hold acting through its own C.G. at P and also the remainder of the vessel M acting through its C.G. at C.



G is the C.G. of the entire vessel. Also, of course, $W = w + M$

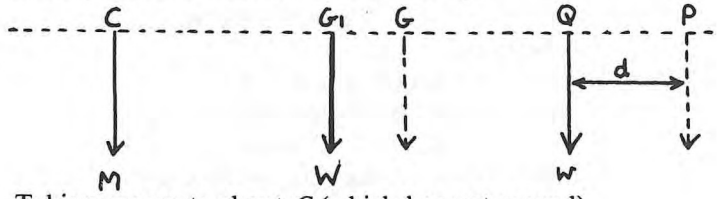
Taking moments about C

moment of vessel = moment of cargo

$$W \times GC = w \times PC \quad \ldots\ldots\ldots\ldots (1)$$

Now suppose w is moved from No. 5 hold to No. 4 hold, a distance between C,G,'s of d.

Clearly this will cause G to move to G_1.



Taking moments about C (which has not moved)

moment of vessel = moment of cargo

$$W \times G_1C = w \times QC$$
$$W\ (GC - GG_1) = w\ (PC - \mathrm{d})$$
$$W \times GC - W \times GG_1 = w \times PC - wd$$

Substituting for the first term from (1)

$$w \times PC - W \times GG_1 = w \times PC - wd$$
$$-\ W \times GG_1 = -\ wd$$
$$W \times GG_1 = wd$$

$$\text{i.e., } GG_1 = \frac{w \times d}{W}$$

To apply this:

In a vessel of 6600 t displacement, 600 t is transferred from No. 5 hold to No. 4, a distance of 7·2 m. Find the movement of the C.G. of the vessel.

$$GG_1 = \frac{600 \times 7{\cdot}2}{6600} = 0{\cdot}655 \text{ m}$$

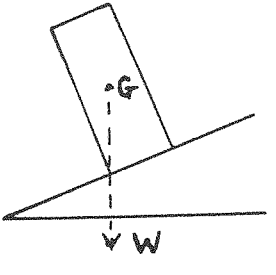
G moves 0·655 m forward.

4.6 EQUILIBRIUM — THREE STATES

Stable equilibrium. A body is said to be in stable equilibrium if, when slightly displaced from the position of equilibrium, it will return to that position. An example is provided by a cylinder standing on one of its plane faces on a level table.

Neutral equilibrium. A body is in neutral equilibrium if, when slightly displaced, it will remain at rest in the new position. A cylinder on its curved surface on a level table provides an example.

Unstable equilibrium. A body is in unstable equilibrium if, when slightly displaced from the equilibrium position, it will tend to move still further away from original position. A somewhat theoretical example is provided by a cone balanced on its apex — if one can imagine such a thing. However, very near approximations to unstable equilibrium are found in the study of ship stability.



Further note. A body is said to have reached the point of **limiting equilibrium** when it is about to capsize. This will be when the vertical down through the C.G. first falls outside the point of support. The sketch illustrates this condition for a cylinder on an inclined plane. If the inclination of the plane is still further increased, the cylinder will capsize.

4.7 EQUILIBRIUM OF A RIGID BODY

We have considered the equilibrium of a body acted upon by vertical forces only. If there are also horizontal forces acting, the system is governed by what are termed the General Conditions of Equilibrium of a rigid body. These are as follows:

If any system of coplanar forces acting on a body is in equilibrium, then:

(*a*) the algebraic sum of all vertical components is zero, i.e. **sum of upward forces = sum of downward forces.**

(*b*) the algebraic sum of all horizontal components is zero, i.e. **sum of forces to the left = sum of forces to the right.**

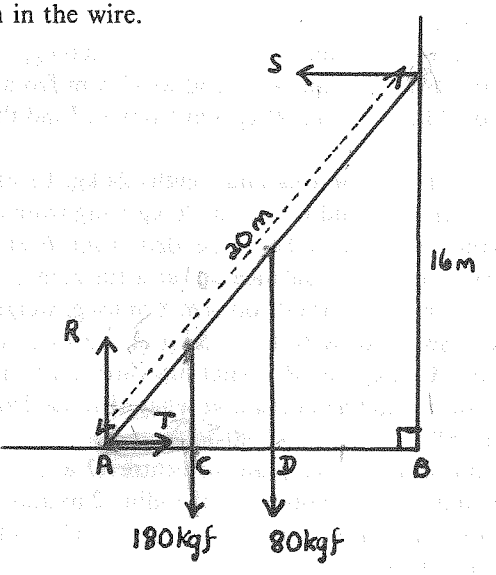
(*c*) the algebraic sum of the moments of the forces about any point in their plane of action is zero, i.e. **sum of moments clockwise = sum of moments anti-clockwise.**

These are not really very difficult in application. Note that although for convenience the terms "vertical" and "horizontal" have been used here, strictly speaking, the laws apply in any two directions mutually at right angles.

Note also that the forces must be coplanar, that is, all acting in the same plane.

WORKED EXAMPLE 7

A uniform girder 20 m long and weighing 80 kg rests against a wall at a point 16 m above the ground. A mass of 180 kg is suspended from a point quarter way up the girder, and the foot of the girder is secured by a horizontal wire. If all surfaces are smooth, find the tension in the wire.



The forces acting on the girder, all in kgf, are as shown.

Note that a "smooth" surface can only exert a reaction at right angles to the surface. R and S are these reactions.

Resolving vertically (forces up = forces down)

$R = 180 + 80 = 260$ kgf ..... (1)

Resolving horizontally (forces to left = forces to right)

$T = S$ ..... (2)

Taking moments about A.

(AB can easily be found, and = 12 m and hence $AC = 3$ m and $AD = 6$ m)

Clockwise moments = anti-clockwise moment

we have $(180 \times 3) + (80 \times 6) = S \times 16$ ..... (3)

Note distances from A are perpendicular distances.

$$S = \frac{1020}{16} = 63\tfrac{3}{4} \text{ kgf}$$

From (2), $S = T$, i.e. tension = $63\frac{3}{4}$ kgf.

TEST PAPER 4

Moments and Centre of Gravity

1. AB is a horizontal rod of negligible weight, and masses of 20 kg and 14 kg are suspended 3 m and 7 m respectively from A. Find the moment about A of each mass and the position of the C.G.
2. A uniform beam AB weighing 200 kg, 12 m long, is supported on props at A and at C, 9 m from A. It carries a load 32 kg at B and 60 kg 4 m from A. Find the load on each prop.
3. A rod AB is 6 m long and weighs 24 kg. Its centre of gravity is 2 m from A and masses of 20 kg hang from each end. What further mass must be suspended from B in order that the arrangement will balance about a fulcrum 2 m from B?
4. A uniform horizontal rod AB, 8 m long, weighs 3 kg. Masses are suspended as follows: 5 kg at A, 4 kg at a point 3·5 m from A, 2 kg at B. What upward force is necessary to produce equilibrium, and at what distance from A must it be applied?
5. A uniform circular plate of centre O and radius 6 m has a circular portion removed, of radius 2 m and whose centre is 2 m from O. Find the position of the C.G. of the remainder relative to O.
6. A uniform lamina consists of a square of 1 m side together with an equilateral triangle, of 1 m side, attached to one of the sides of the square. Calculate the position of the C.G.
7. A vessel of 12,000 t displacement and a height of C.G. above the keel (KG) 4·82 m loads 460 t of cargo in the 'tween deck, KG 6·6 m, and discharges 1200 t from the lower hold, KG 5·22 m. Find the height of the new C.G. above the keel.
8. A uniform beam 4 m long, mass 75 kg, is hinged at one end and is supported horizontally by a chain attached to the other end and to a point 7 m vertically above the hinge. Determine the tension in the chain and the reaction at the hinge. (Note that the reaction at the hinge will have a vertical and a horizontal component).

CHAPTER 5

STATICS

Couples, Stress, Strain, Bending Moments and Shearing Forces

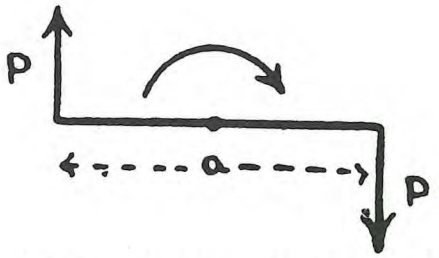
5.1 COUPLES

Definition: Two equal, parallel, and opposite forces are said to form a couple.

The familiar example of a couple is a man using a corkscrew. Another is two men turning a capstan by means of capstan bars diametrically opposite to each other.

A couple, unlike a single force, has no power of translation; that is, it cannot push things about. It can only exert a **turning effect**, and couples are measured by the amount of their turning effect, or in other words, by the *moment* of the couple.

Moment of a Couple.



Definition: The moment of a couple is the product of one of the forces and the perpendicular distance between the forces, i.e.,

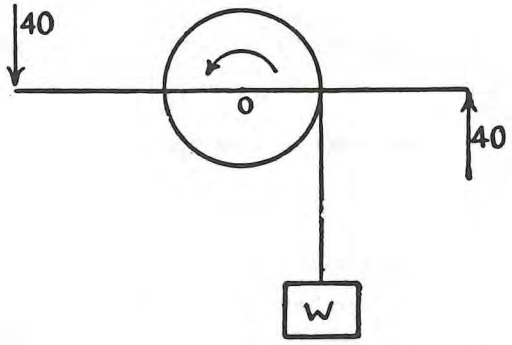
$$\text{Moment} = P \times a \quad \text{I}$$

Thus, if two forces P, equal to 7 N each, form a couple as shown, and $a = 0{\cdot}5$ m then

$$\text{moment} = 7 \times 0{\cdot}5 = 3{\cdot}5 \text{ newton metre (Nm).}$$

WORKED EXAMPLE 1.

A capstan is 0·8 m diam. and is pushed round by two capstan bars opposite to each other, the length of bar projecting being 1·6 m. One man at the end of each bar exerts a force of 40 kgf. Find the load the capstan will move, ignoring friction.



The arrangement is as shown. Let the load be W kg.

The distance between the 40 kgf forces is $(2\times 1{\cdot}6)+$ diameter of capstan

$$\text{Distance} = (2\times 1{\cdot}6)+0{\cdot}8 \text{ m}$$
$$= \quad 4 \text{ m} \quad \text{"}$$

$$\text{Moment of couple} = 40\times g\times 4 \text{ Nm} = 160g \text{ Nm}$$

and this must equal the moment of the mass W about the pin O. So

$$W\times g\times 0{\cdot}4 = 160\,g$$

$$W = \frac{160}{0{\cdot}4} = 400 \text{ kg}$$

EXERCISE A.

1. In a capstan as above, diam. 0·55 m, length of bar projecting being 1·7 m, find the force each man would have to exert to raise a coil of rope weighing 210 kg, ignoring friction.
2. Two men are tightening up a bottle screw by means of a rod pushed right through, and projecting 0·64 m each side from the central line of the screw. An effort of 8 kgf by each just moves the screw. What effort would one man have to exert, using a marline spike in the usual way, projecting 0·5 m from the central line?

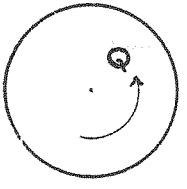
5.2 ESSENTIAL PROPERTIES OF COUPLES

These must be understood, otherwise the introduction of a couple into a problem will always cause difficulty.

1. A couple cannot be replaced by a single force.

This is because a single force applied to a body whilst it may of course cause rotation, will also cause, or "tend to cause" translation, i.e., movement of the centre of gravity of the body.

A couple on the other hand causes pure rotation without translation. For example, on a flywheel or shaft subjected only to a perfect couple of moment Q, there would be no translational force on the centre bearing at all.



It follows from the above that a couple can have no resultant, i.e., cannot reduce to a single force. Nor can a single force ever equate to a couple.

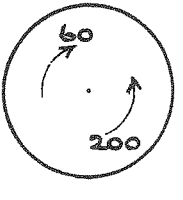
In passing, the term **torque** is often used in connection with the rotation of shafts and it should be noted that this is just another name for the couple acting on the shaft.

A torque is just a couple. A torque causes pure rotation and is measured in the same units as couples.

2. The moment of a couple about any point in its plane is constant.

In effect, this means that we do not need to be told where the couple is acting. (Compare this with a force, where we must know not only its magnitude and direction, but also its point of application).

A couple, however, is fully defined by its moment, and any other couple having the same moment is completely equivalent. Like, or unlike, couples are combined algebraically, i.e., we can compound couples by the algebraic sum of their moments.



For example, if a shaft has a driving torque of 200 Nm and a braking torque of 60 Nm is applied, the resulting driving torque (or couple) is 140 Nm.

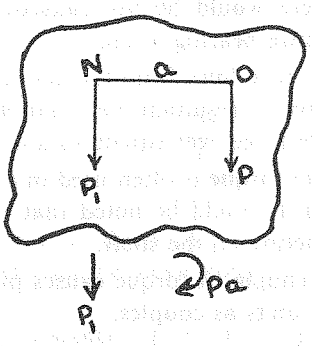
3. Important theorem

We have said that a single force cannot equate to a couple. However, a single force *can* be replaced by a force plus a couple, as given in the following theorem, which we shall state without proof.

Theorem:

Any single force can be replaced by an equal, parallel force acting along some new line, together with a couple whose moment is equal to the moment of the original force about the new line of action.

i.e., a single force can be equated to a force and a couple.



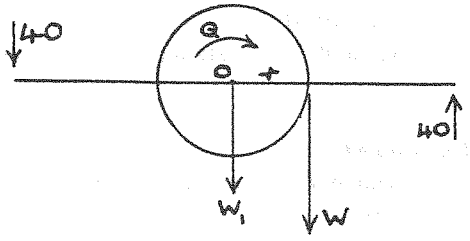
In symbols, if P is any force acting on a body at O, then P is equivalent to an equal, parallel force P_1 at N, + a couple of moment $P \times a$, i.e.,

$$\mathbf{P = P_1 + \text{couple } Pa} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \mathbf{II}$$

The converse is also true, and if a system of forces reduces to a couple + a force, then it will always be possible to reduce this still further, to a single resultant force.

WORKED EXAMPLE 2. As an example of the use of this theorem, consider again the capstan in Worked Example 1.

What is the load on the pin?



A study of the diagram in Worked Example 1 seems to indicate something wrong with it. The two men are providing a *couple*. The only other thing shown is a single *force* due to W.

These cannot be in equilibrium; a couple cannot be balanced by a force, nor conversely.

The additional force required to produce equilibrium is provided by the pin.

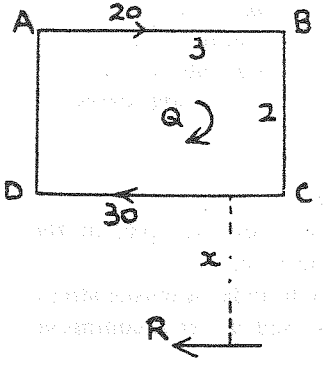
To find this force we use the above theorem to replace W by a single force W_1 ($= W$) TOGETHER WITH a couple Q whose moment must be $W \times r$. (see diagram).

For equilibrium

(1) the couple exerted by the men on the capstan will be equal to that of Q.

(2) the single force W_1 must be balanced by the force on the pin i.e. the load on the pin is simply W the load being moved.

WORKED EXAMPLE 3. A rectangular plate $ABCD$ in which $AB = 3$ m and $BC = 2$ m is acted upon by forces of 20 N along AB and 30 N along CD. Find the resultant force acting on the plate.



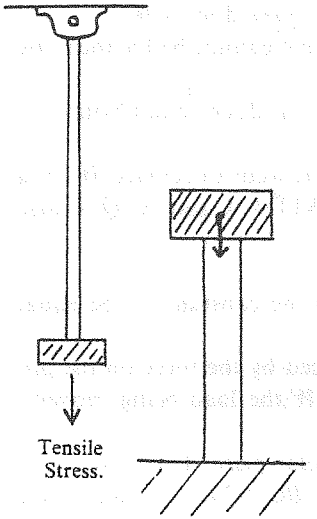
Transfer the 20 N force to DC. This leaves us with a force of 10 N along CD and a clockwise couple Q of moment 20×2 (the "moment of the original force about its new line") i.e., 40 Nm.

Where is the resultant of this force of 10 N along CD+ the couple Q? Well, to begin with, from the theorem, the resultant must be equal, and parallel. Therefore it is a force of 10 N. And its line of action must be such that $10\ \text{N} \times x = 40\ \text{Nm}$, i.e., $x = 4\ \text{m}$.

Answer: Resultant is a single force of 10 N, parallel to CD and acting in the same sense, and at a distance of 4 m from CD.

5.3 ELASTICITY OF MATERIALS

When a given piece of material is acted upon by an external force it is said to be in a state of stress.

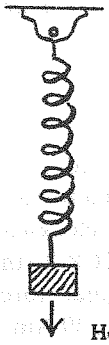


Stretching of wire or thin rod

It is familiar to most of us that a piece of elastic will stretch when a load is suspended from it. Equally, if the load is removed, that the elastic will return to its original length, provided that the limit of elasticity has not been exceeded, in which case the material would be permanently deformed.

Although the stretching of a rod is not so apparent to the eye, it is nevertheless a fact that such a rod will stretch, if subjected to a stretching force, or **tensile stress.**

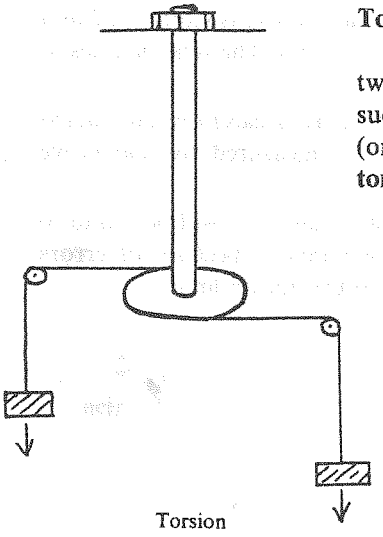
Conversely, a rigid bar subject to thrusting or pushing forces on its ends will undergo shortening of its length, and is said to be in a state of compression, or under a **compressive stress.**



Helical spring

This is used, for example, in the domestic spring balance.

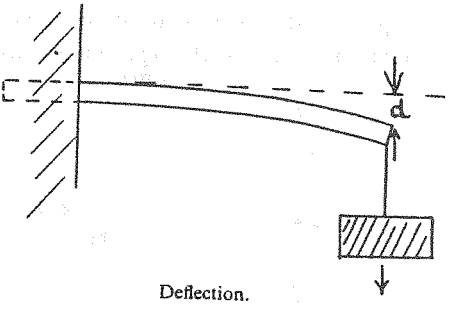
It will stretch under a tensile stress, and be compressed under a compressive stress.



Torsion

Torsion in a rod

A rod which is subjected to a pure twist (no pushing, pulling or bending) such as would be provided by a couple, (or torque) is said to be in a state of **torsion.**



Deflection.

Deflection of a beam

A beam subjected to a load as shown will undergo deflection, and the amount of deflection is defined simply as a unit of length, the measure of the distance the beam has (in this case) been lowered by the load.

5.4 HOOKE'S LAW

All the foregoing forms of stress on a material have one important principle in common, which is summed up in Hooke's Law.

This states, in simple terms:

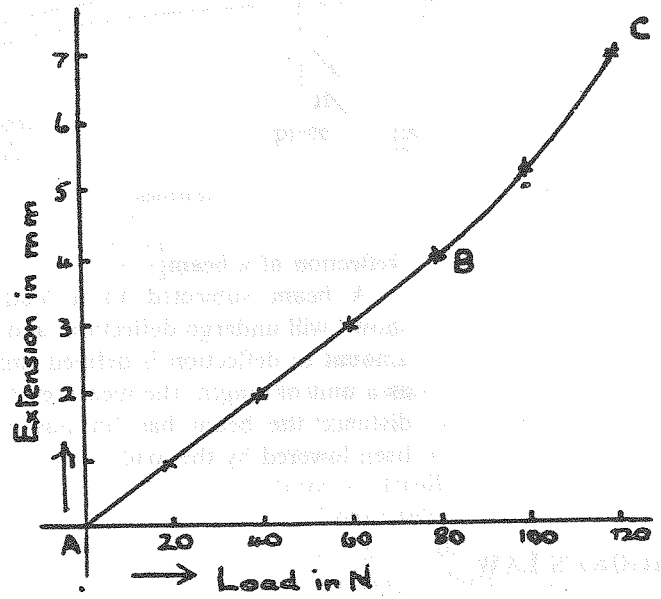
Extension is proportional to load (Hooke's Law) . . . III

Thus, whether in tension, compression, torsion or deflection, the deformation is directly proportional to the load — always provided that the elastic limit is not exceeded.

In even simpler terms, this means that if a rod is stretched 2 mm by a load of 5 N, then 10 N will stretch it 4 mm. The same applies to the other forms of stress described.

Hooke's Law may easily be verified by a carefully conducted experiment in which the deformation is measured for successive loads.

A graph is then drawn for the data obtained. It will be found to approximate to a straight line ("approximate" because of errors inherent in any such experiment) up to the elastic limit.



An example of such a graph is illustrated for a wire under tensile stress, where the extensions have been plotted against loads. The graph is a straight line up to loads of 80 N.

From that point, it will be seen that the extension is no longer directly proportional to the load. In fact, for each successive increment (of 20 N) in load, the extension increases by a greater amount, until finally at C, it will break.

The loading when this occurs is called the **ultimate strength** of the specimen.

The point where the graph ceases to be a straight line gives us the elastic limit. This is the point *B*.

From *A* to *B*, the graph is straight. This proves Hooke's Law. At any loading from *A* to *B*, the specimen will return to its original length if the load is removed. Beyond *B*, it will not return — it is permanently strained.

5.5 TESTING OF MATERIALS

All types of steel supplied for use in shipbuilding must, amongst other things, have certain specified ultimate strengths and properties of ductility as shown under tensile stress. Specimens are tested to fracture and the methods used do not differ greatly in principle from the simple test on a wire described on the opposite page. However, it goes without saying that on larger sections, sophisticated testing machines are used in order to provide a much greater pull than could ever be provided manually.

And, of course, on a completed job — say a weld or joint, non-destructive testing by electronic means is the usual method adopted.

In the simple experiment described, loads of 20 N each time were added, one may assume, with care, and the system allowed to settle. A load which remains steady on a structure is called a **dead load.**

Alternatively, if one had by some means dropped the 20 N each time on to the load already in position, it is clear that the instantaneous load — the jerk — given to the structure, would have been very much greater. It might even have broken the wire. Such a load, coming and going suddenly, is called a **live load.**

It is found that under repeated and variable live loads a structure is liable to break under only a fraction of the load it takes to break it when applied steadily.

For this reason, in all structural work, a **Factor of Safety** must be employed, often of 4 or 5 or more. This means, in general terms, that the structure must be 4 or 5 times as strong as the normal working load it is expected to bear. Or more precisely.

$$\textbf{Working Stress} = \frac{\textbf{Ultimate strength}}{\textbf{Factor of safety}} \quad \ldots\ldots\ldots \textbf{IV}$$

5.6 STRESS AND STRAIN

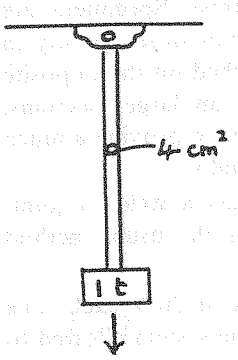
We have spoken of these in general terms. It is now necessary to define them more precisely. Both are used a good deal, incorrectly, in everyday life.

Stress in a material must be defined as the force on a unit area, when it is correctly known as the stress intensity, i.e.,

Definition: Stress Intensity = force per unit area,

or in other words,

$$\text{Stress Intensity} = \frac{\text{Total load}}{\text{Area of cross section}} \quad \ldots\ldots \text{V}$$



This applies whether a material is under tension or compression.

For example, if a load of 1 tf is suspended from a rod as shown, of cross section are 4 cm², then

$$\text{Stress Intensity} = \frac{10^3 \times 9{\cdot}81 \text{ (newtons)}}{0{\cdot}0004 \text{ (metre}^2\text{)}}$$

$$= 2{\cdot}45 \times 10^7 \text{ N/m}^2 \text{ (approx.)}$$

It will be seen that the unit of stress intensity is exactly the same as the unit of pressure.

Definition: Strain is the extension per unit of natural length, i.e.,

$$\text{Strain} = \frac{\text{Extension}}{\text{Natural length}} \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{VI}$$

Thus, if a rod 6·00 m long increases to 6·024 m under tension, then

$$\text{Strain} = \frac{0{\cdot}024}{6} = 0{\cdot}004$$

Note that strain is a pure number, i.e., it is the same, numerically, whatever units are employed — for the same specimen, of course.

The above statement means that the rod stretches "0·004 of a metre for each metre of its length". Since it is 6 m long, the total extension is 6 × 0·004, i.e., 0·024 m, as we saw at the beginning.

Similarly, for instance, it would stretch 0·004 of a foot for each foot of its length.

5.7 HOOKE'S LAW REVISITED

The statement of Hooke's Law made in section 5.4 is in fact a simplified version. A more comprehensive statement is:

> Tensile (or compressive) strain produced is directly proportional to the applied tensile (or compressive) stress **provided the elastic limit is not reached.**

In mathematical terms we can say

$$\text{strain} \propto \text{stress}$$

which can be written as

$$\text{constant} \times \text{strain} = \text{stress}$$

$$\frac{\text{stress}}{\text{strain}} = \text{constant}$$

This constant is given the name Young's Modulus and we can denote it by Y.

$$Y = \frac{\textbf{stress}}{\textbf{strain}} \qquad \text{VII}$$

Some typical values for Y are set about below.

Material	Y (N/m^2)
Steel	$2{\cdot}1 \times 10^{11}$
Copper	$1{\cdot}3 \times 10^{11}$
Glass	$0{\cdot}7 \times 10^{11}$
Rubber	$0{\cdot}0005 \times 10^{11}$ (approx)

5.8 FORMULAE

Suppose a stretching force F acts on a wire of cross-section area A and original length L thus causing an extension of e. We have:

$$\text{stress} = \frac{F}{A} \qquad \text{strain} = \frac{e}{L}$$

Now

$$Y = \frac{\text{stress}}{\text{strain}}$$

$$Y = \frac{F}{A} \div \frac{e}{L}$$

$$Y = \frac{F}{A} \times \frac{L}{e}$$

$$\mathbf{Y = \frac{FL}{Ae}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad \textbf{VIII}$$

WORKED EXAMPLE 4

A load of 15 N attached to the end of a wire 2·9 m long and cross-sectional area 2×10^{-7} m^2 stretches it by 2 mm. Calculate:
(a) the applied stress (b) the strain (c) Young's modulus

(a) $$\text{stress} = \frac{F}{A}$$

$$= \frac{15}{2\times 10^{-7}} \text{ N/m}^2$$

$$= 7{\cdot}5\times 10^7 \text{ N/m}^2$$

(b) $$\text{strain} = \frac{e}{L}$$

$$= \frac{2\times 10^{-3}}{2{\cdot}9}$$

$$= 0{\cdot}0006897$$

(c) $$Y = \frac{\text{stress}}{\text{strain}}$$

$$= \frac{7{\cdot}5\times 10^7}{0{\cdot}0006897} \text{ N/m}^2$$

$$= 1{\cdot}087\times 10^{11} \text{ N/m}^2$$

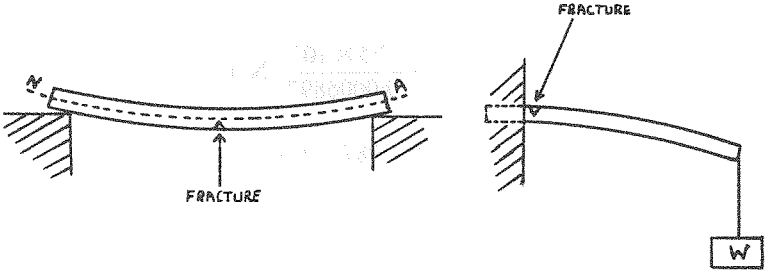
EXERCISE B

1. A mild steel tie-bar 0·04 m in diameter and 4 m long is under a tensile load of 8 tf when the extension is observed to be 0·004 m. Calculate the stress and strain in the bar. Calculate also Young's modulus for the bar.
2. Young's modulus for a certain wire of length 5·2 m and cross-sectional area 0·0004 m^2 is $2{\cdot}1 \times 10^{11}$ N/m^2. When subjected to a tensile force the strain produced is 0·005. Calculate
   (a) the stress.
   (b) the actual extension of the wire.
   (c) the tensile force.

5.9 BENDING OF BEAMS

When a beam is freely supported, it will tend to change its form by bending. This is true whether the beam is carrying isolated heavy loads — so heavy, shall we say, that the weight of the beam itself may be neglected in relation to the load — or whether the beam is just bending under its own weight.

Examples of this are illustrated.



(*a*)—Uniform, heavy beam (*b*)—Simple cantilever with isolated load

In (*a*) the upper layers of the beam are in compression, the lower layers in tension. This would correspond to the situation in a ship poised on wave crests near its ends. About half thickness of the beam, there is a layer which is neither in compression nor tension. This is called the **neutral axis** (*NA*).

In (*b*) a girder like this, held rigidly at one end and having no other form of support, is called a **cantilever**. Clearly, the upper layers are in tension and the lower layers in compression. Although the analogy is not quite an exact one, a ship poised on a wave amid ships would bend in this fashion.

Since mild steel is stronger in compression than in tension, if there is to be a fracture it will occur in the positions indicated in the opposite page.

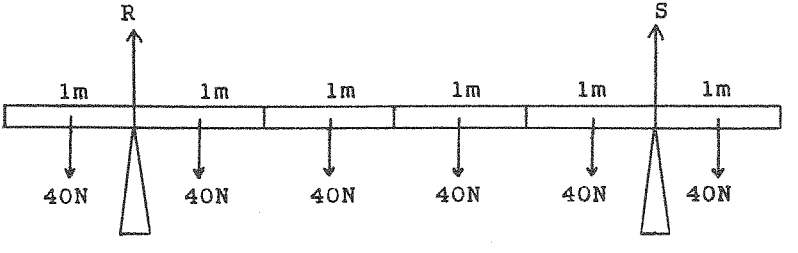
This should not be confused with buckling or bending, which will often occur, for example, in the deck plating of a ship bending as in (*a*). In the same circumstances, because of the enormous strength of the double bottom construction, a fracture in the ship as a girder is unlikely to occur as in (*a*).

Actual fracturing of the ship girder, when it occurs, is most common across the deck plating, in the circumstances shown in (*b*). Fortunately, such cases, though by no means unknown, are very rare.

5.10 SHEARING FORCE AND BENDING MOMENT

The bending of a beam can be described in terms of these two quantities.

To understand these quantities consider the example of a beam 6 m long which has a weight of 40 N per metre length and which is supported at the 1 m and 5 m positions.



R and *S* are the reactions at the supports.

In this example it is relatively simple to determine the values of *R* and *S*:

The total weight of the beam is $6 \times 40\ \text{N} = 240\ \text{N}$.

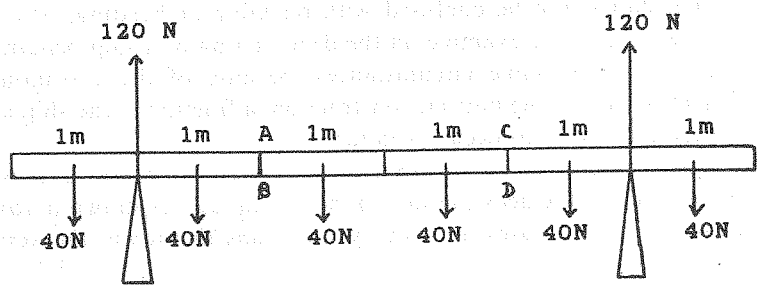
If the beam is in equilibrium then the forces down must be balanced by the forces up.

Therefore: $R + S = 240\ \text{N}$

Now since R and S are equidistant from the ends of the beam they will have the same value,

i.e. $R = S = 120\ \text{N}$

Thus we now have



Suppose we now look, for example, at the section AB of the beam. The portion of the beam to the left, or right, of it is kept in equilibrium by the combined effect of

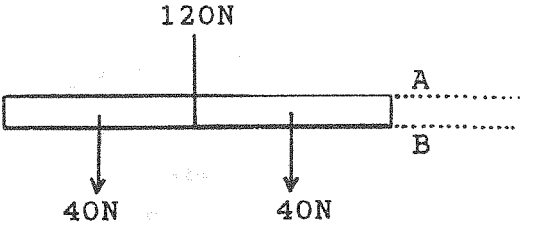
(1) a vertical force — the SHEARING FORCE.

(2) a couple derived from the forces of compression and tension and whose moment is called the BENDING MOMENT.

5.11 DEFINITION OF SHEARING FORCE

The shearing force at any section is the algebraic sum of the vertical forces either to the left or to the right of the section.

For AB on the above diagram if we look to the left we have



Forces up $= 120$ N.

Forces down $= 40\text{ N} + 40\text{ N} = 80$ N.

Therefore algebraic sum of vertical forces $= 120\text{ N} - 80\text{ N}$.

$= 40$ N (upwards)

If we consider the Shearing Force at another section, for example CD, and again look to the left we have:

Forces up $= 120$ N.

Forces down $= 160$ N.

Shearing Force $= 160\text{ N} - 40\text{ N}$ (downwards).

To distinguish between upward and downward Shearing Forces we employ a sign convention:

Looking to the LEFT an upward Shearing Force is POSITIVE.

Hence for AB Shearing Force $= +40$ N.

and for CD Shearing Force $= -40$ N.

EXERCISE C

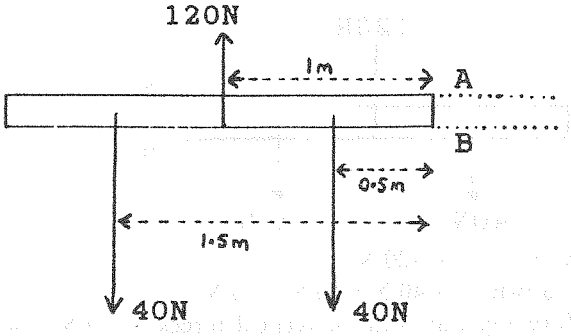
For the above beam calculate the Shearing Force at the following distances from its left-hand end:

(*a*) 0 m (*b*) 1 m (*c*) 3 m (*d*) 5 m (*e*) 6 m

5.12 DEFINITION OF BENDING MOMENT

The Bending Moment at any section is the algebraic sum of all the moments either to the left or to the right of the section

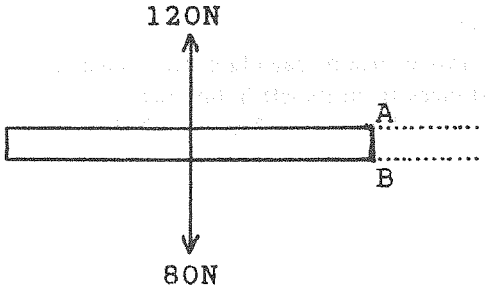
Again for AB looking to the left we have



Taking moments about AB

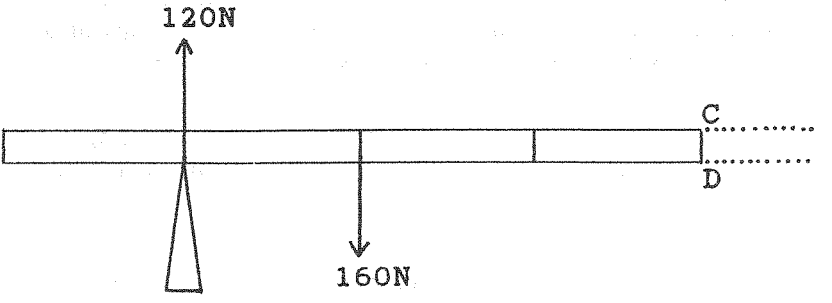
Clockwise moment	$= 120 \times 1$ Nm	$= 120$ Nm
Anticlockwise moment	$= (40 \times 0{\cdot}5) + (40 \times 1{\cdot}5)$	Nm
	$= 20 + 60$	$= 80$ Nm
Therefore Bending moment	$= 120$ Nm $- 80$ Nm	$= 40$ Nm (clockwise)

Note that we could have simplified the calculation by using one value for the weight of the portion of the beam under consideration. Since we are considering a portion 2 m long its weight will be $40 \text{ N} \times 2 = 80$ N. Thus we could have used the following:



Taking moments we will of course obtain the same Bending Moment.

In the case of the other section we considered, *CD*, we have



Taking moments about *CD*

Clockwise moment	=	120 × 3 Nm	= 360 Nm
Anticlockwise moment	=	160 × 2 Nm	= 320 Nm
Therefore Bending Moment	=	360 Nm − 320 Nm	= 40 Nm (anticlockwise)

To distinguish between clockwise and anticlockwise Bending Moments we employ a sign convention:

Looking to the left an anticlockwise Bending Moment is POSITIVE.

Hence for *AB* Bending Moment = − 40 Nm.

CD Bending Moment = + 40 Nm.

EXERCISE D

For the above beam determine the Bending Moment at the following distances from the left-hand end:

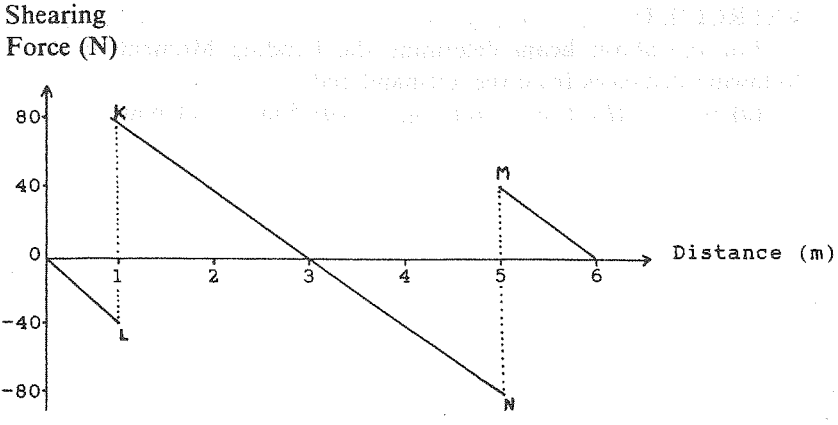
(*a*) 0 m (*b*) 1 m (*c*) 3 m (*d*) 5 m (*e*) 6 m

5.13 DIAGRAMS

We are now in a position to draw the Shearing Force and Bending Moment diagrams for the beam using the results of our calculations which are tabulated below:

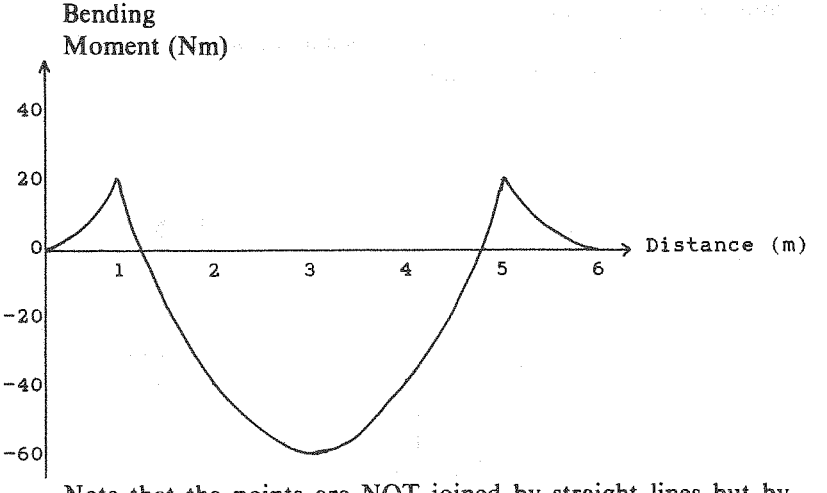
Distance from the left-hand end (m)	Shearing Force (N)	Bending Moment (Nm)
0	0	0
1	−40 rising to +80	+20
2	+40	−40
3	0	−60
4	−40	−40
5	−80 rising to +40	+20
6	0	0

Firstly the Shearing Force diagram:



Note that the gap KL corresponds to the reaction $R = 120$ N.
Similarly the gap MN corresponds to the reaction $S = 120$ N.

The Bending Moment diagram is as set out below:

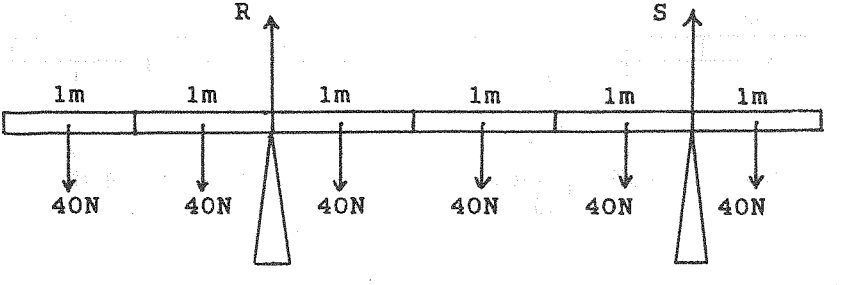


Note that the points are NOT joined by straight lines but by smooth curves. That this is the case could be demonstrated by the reader by calculating the Bending Moments at some intermediate points e.g. at 0·5 m, 1·5 m etc.

Also, by choosing our sign convention the way we did, we obtain a Bending Moment diagram that resembles the actual sagging of the beam under its own weight.

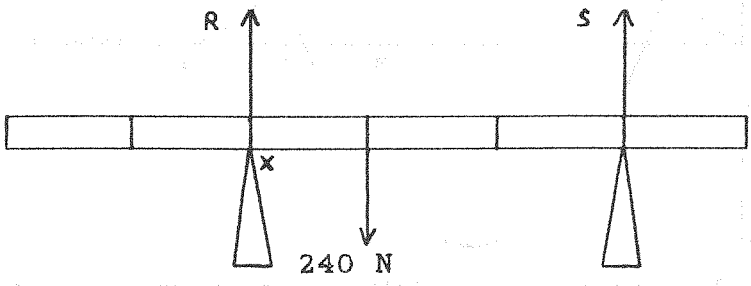
WORKED EXAMPLE 5

A beam 6 m long has a weight of 40 N per metre. It is supported at the 2 m and 5 m positions. Calculate the Shearing Forces and Bending Moments at 1 m intervals. Hence draw the Shearing Force and Bending Moment diagrams.



This of course is the same beam which we have been considering above. However since it is now supported in a different position we must re-calculate the reactions.

To calculate the reactions we consider the whole weight of the beam to act from its mid-point.



$R + S = 240$ N

Taking moments about X

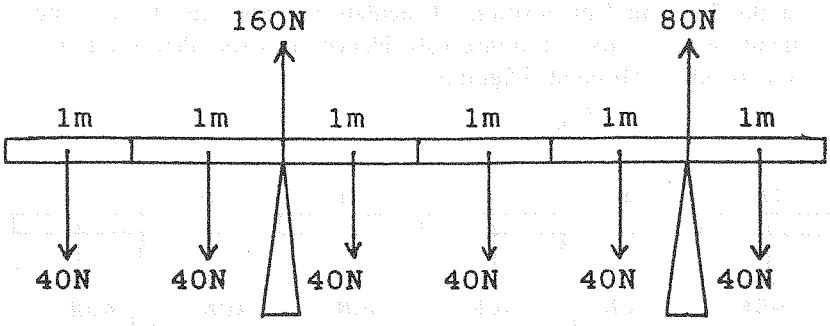
clockwise = anticlockwise

$240 \times 1 = S \times 3$

$S = 80$ N

$R = 240$ N $- 80$ N $= 160$ N

Thus we now have

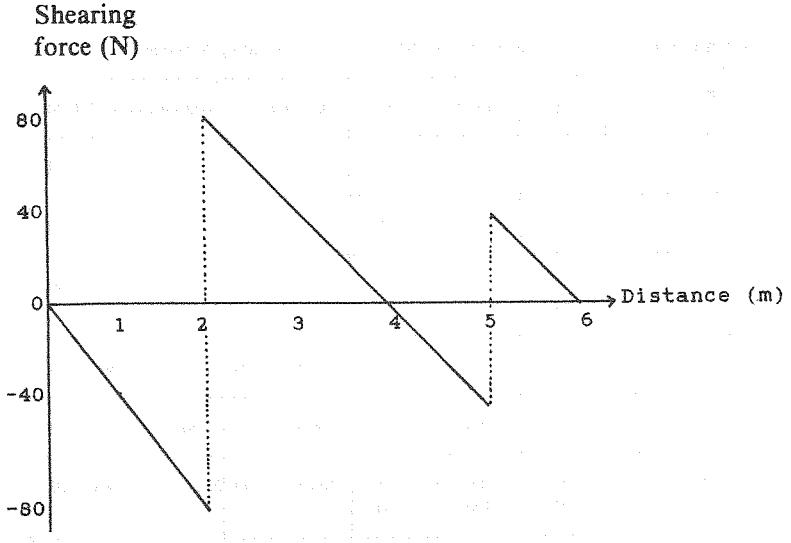


The calculations of the Shearing Forces and Bending Moments are presented in the following table and require careful study.

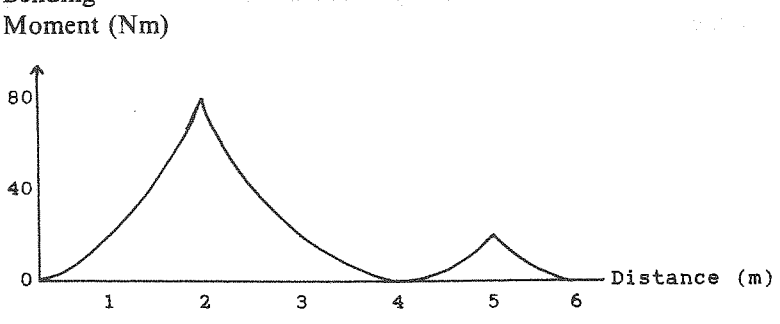
Distance from Left-hand end (m)	Shearing Force (N)			Bending Moment (Nm)		
	Up	Down	S.F.	Clockwise	Anticlockwise	B.M.
0	0	0	0	0	0	0
1	0	40	−40	0	$40\times0{\cdot}5=20$	+20
2	0 160	80 80	−80 +80	0	$80\times1=80$	+80
3	160	120	+40	$160\times1=160$	$120\times1{\cdot}5=180$	+20
4	160	160	0	$160\times2=320$	$160\times2=320$	0
5	160 240	200 200	−40 +40	$160\times3=480$	$200\times2{\cdot}5=500$	+20
6	240	240	0	$(160\times4)+(80\times1)=720$	$240\times3=720$	0

Note from the table that at each of the two support positions TWO Shearing Force values have been quoted — one without the reaction at that support being counted and one with it being counted.

SHEARING FORCE DIAGRAM



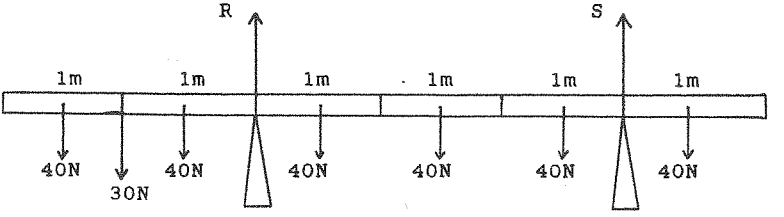
BENDING MOMENT DIAGRAM



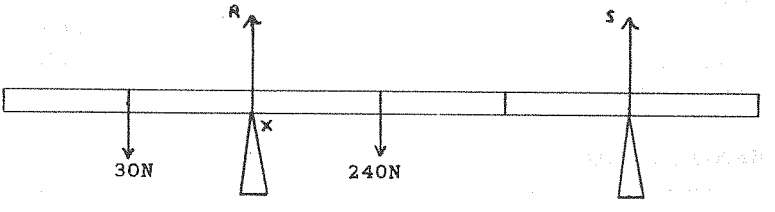
Note that exactly the same methods apply if the beam has point loads attached to it. However the calculations can be somewhat tedious.

WORKED EXAMPLE 6

Suppose a point load of 30 N is attached to the beam considered above in Worked Example 5. Calculate the S.F. and B.M. for the 3 m position.



Again we have to re-calculate the reactions to R and S; again we consider the weight of the whole beam to act from its mid-point.

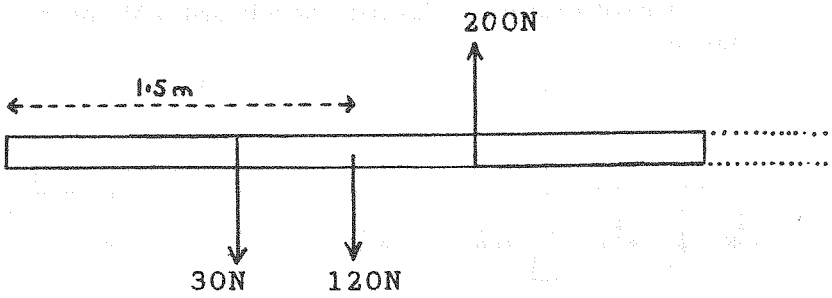


$$\text{Forces up} = \text{Forces down}$$
$$R + S = 240\text{ N} + 30\text{ N} = 270\text{ N}$$

Taking moments about X

$$\text{clockwise} = \text{anticlockwise}$$
$$240 \times 1 = (S \times 3) + (30 \times 1)$$
$$240 = 3S + 30$$
$$210 = 3S$$
$$S = 70\text{ N}$$
$$R = 270\text{ N} - 70\text{ N} = 200\text{ N}$$

Looking to the left of the 3 m position we have:



Note the weight of the 3 m portion of the beam has been drawn from its mid-point.

SHEARING FORCE

Force up	$= R$	$= 200$ N
Forces down	$= 30$ N $+ 120$ N	$= 150$ N
Therefore S.F.	$= 200$ N $- 150$ N	$= + 50$ N

BENDING MOMENT

Clockwise	$= 200 \times 1$	$= 200$ Nm
Anticlockwise	$= (30 \times 2) + (120 \times 1{\cdot}5) = 60 + 180$	$= 240$ Nm
Therefore B.M.	$= 240 - 200$	$= + 40$ Nm

EXERCISE E

For the beam in Worked Example 6 above calculate the S.F. and B.M. at 1 m intervals. Hence draw the S.F. and B.M. diagrams.

TEST PAPER 5

Couples, Stress, Strain, Bending and Shearing

1. A capstan barrel is 0·75 m in diameter and has two bars each projecting 1·75 m from the barrel. Two men are operating the capstan. Find the force each would have to exert when moving a load of 400 kgf, and the force on the capstan pin at that instant.
2. A square plate $ABCD$ of 2 m side is acted upon by 12 kgf along the diagonal AC, in that sense, and also by a couple of moment 98·1 Nm acting in the plane of the plate and in the sense $ABCD$. Find the single resultant force acting on the plate.
3. A load of 7 tonnes hangs from a tie-bar of 6 cm square section and initial length 4·26 m. The length is now observed to be 4·28 m. Calculate (*a*) the stress (*b*) the strain (*c*) Young's Modulus for the bar.
4. A tie-bar, 4·5 m long, 3 cm in diameter, has a Young's Modulus of $6{\cdot}766 \times 10^{10}$ N/m^2. When a tensile force is applied to the bar its length is observed to change to 4·506 m. Calculate (*a*) the strain (*b*) the stress (*c*) the tensile force.
5. A wire 0·4 cm in diameter and 5 m long stretches 2 mm under a tensile force. Given that Young's Modulus for the wire is $2{\cdot}410 \times 10^{11}$ N/m^2 calculate the tensile force.
6. A uniform beam 10 m long and of weight 120 N per metre run is freely supported at its ends. Calculate the S.F.s and B.M.s at 2 m intervals. Hence draw the S.F. and B.M. diagrams.
7. A uniformly loaded beam AB, weight 200 N per metre, length 12 m, is freely supported at its ends. Calculate the S.F. and B.M. at points 3 m and 6 m from end A. Draw the S.F. and B.M. diagrams.
8. Repeat Question 7 but this time the beam has a point load of 800 N suspended 9 m from A.

CHAPTER 6

HYDROSTATICS

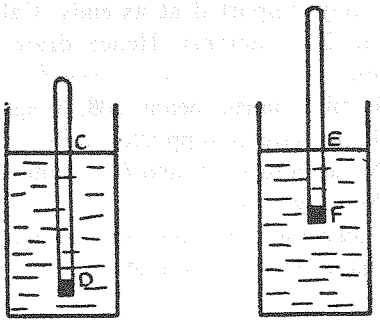
Hydrometers, Loadlines

6.1 PRINCIPLE OF THE HYDROMETER

The Law of Flotation states that every floating body displaces its own weight. This law is also the principle of the hydrometer, used for finding the relative density or the specific gravity of liquids.

Consider the piece of wood shown in diagrams (a) and (b), weighted at the bottom so that it always floats vertical. It might be called the simplest form of hydrometer.

The essential thing to remember about the piece of wood is that **its weight is constant,** and that it will **always displace that weight of liquid.** Thus, the column of liquid displaced between *C* and *D* equals the weight of the piece of wood.



(a) Liquid not so dense as *(b)* *(b) Dense liquid.*

Principle of the hydrometer.

Now suppose the R.D. of liquid *(b)* is twice of *(a)*, i.e., bulk for bulk, it weighs twice as much as *(a)* and so, of course, the column

EF will only need to be half as long as *CD*, to be equal to it in weight.

This might be summed up in the following law:

For floating bodies whose underwater section is uniform, the draught is inversely proportional to the density of the liquid, that is,

$$\frac{\text{Draught}_1}{\text{Draught}_2} = \frac{\text{Density}_2}{\text{Density}_1} \quad \text{I}$$

WORKED EXAMPLE 1

A box shaped barge floats at a draught of 3·86 m in dock water R.D. 1·016. What would be its draught in sea water, R.D. 1·025?

The formula is,

$$\frac{\text{Sea draught}}{\text{Dock draught}} = \frac{\text{Dock density}}{\text{Sea density}}$$

(Note that in using this formula, the quantity we **asked to find** should always be put down **first** i.e., as the numerator on the left hand side).

Now, if its R.D. = 1·016, its density is 1016 kg/m³ and similarly for the sea water.

Substituting in the formula,

$$\frac{\text{Sea draught}}{3{\cdot}86} = \frac{1016}{1025}$$

$$\text{Sea draught} = \frac{3{\cdot}86 \times 1016}{1025}$$

$$\text{Sea draught} = 3{\cdot}83 \text{ m}$$

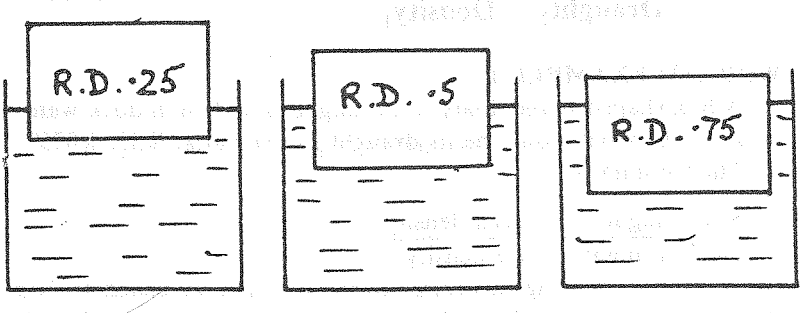
EXERCISE A

A stick weighted to remain vertical is immersed for 0·22 m of its length when placed in F.W. What length would be immersed in oil, R.D. 0·7? What would be the R.D. of another liquid if the stick floated in it with 0·18 m immersed?

6.2 IMPORTANT NOTE

Before we leave our piece of wood, we might think of it from another point of view, namely, the R.D. of the wood itself.

If we had some material whose R.D. was exactly equal to that of F.W., i.e. unity, then it would remain floating in F.W. *just* immersed.



Bodies of different relative densities.

If its R.D. were ¼ that of F.W., it would be ¾ immersed; and if ½ that of F.W. then it would be ½ immersed, and so on.

This is often useful in problems, but remember it only applies to **box sided objects** (or as we said in I., bodies with a uniform underwater section), and **floating in F.W.**

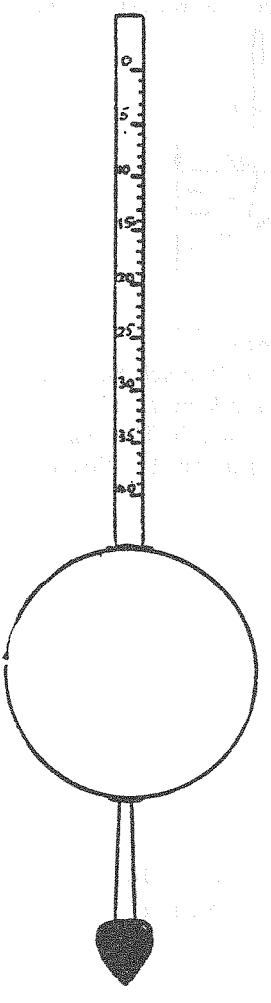
WORKED EXAMPLE 2

A uniform rectangular block of wood, of R.D. 0·7, is floating in F.W. with its large face horizontal and a 0·62 m edge vertical. What is its draught? What would be its draught in sea water, R.D. 1·025?

In F.W., draught will be
0·7 of 0·62 m = 0·434 m.
In sea water, we have,

$$\frac{\text{Sea draught}}{\text{Fresh draught}} = \frac{\text{fresh density}}{\text{sea density}} \quad \text{that is,}$$

$$\text{Sea draught} = \frac{0{\cdot}434 \times 1000}{1025} \quad 0{\cdot}424 \text{ m.}$$



The Hydrometer.

6.3 THE HYDROMETER

On board ship we are dealing with liquids (dock waters) whose relative densities only vary by small amounts. Hence the piece of wood shown would not be sensitive enough.

For this reason the hydrometer is given a large bulb and a thin stem, as in diagram.

Remember we cannot use formula I. on a body with an underwater shape like this.

However, having a comparatively thin stem, this instrument will show quite a big change in the length of stem immersed even for only small variations in Relative Density.

A common type is graduated as shown, from 0 to 40, and one must add 1000 to the reading in order to obtain the density.

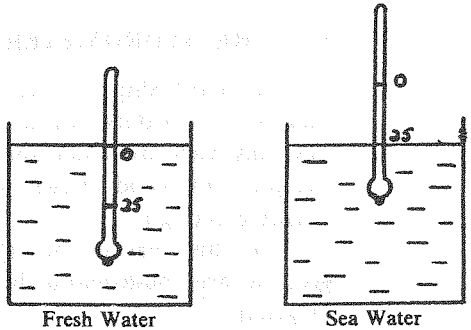
Thus, a statement like "hydrometer reads 16", means density 1016 kg/m^3 or R.D. 1·016.

Note that the scale readings increase as we go down the stem — the more dense the liquid the higher does the hydrometer float in it.

WORKED EXAMPLE 3

A hydrometer has a mass of 0·4 kg and its stem has a cross-sectional area of 0·5 cm^2. Determine the distance on the stem

between the marks 0 (F.W.) and 25 (average sea water) on this hydrometer.



Fresh Water Sea Water

Let the volume of the hydrometer up to the 25 mark be V m³. Let the additional length of stem up to the 0 mark be L m.

The solution uses the Law of Flotation (section 1·7). This tells us the hydrometer will displace 0·4 kg of any liquid in which it floats.

Sea Water

$$\rho = \frac{m}{V}$$

$$\rho V = m$$

$$V = \frac{m}{\rho}$$

$$V = \frac{0{\cdot}4}{1025}\ \text{m}^3$$

$$= 3{\cdot}902 \times 10^{-4}\ \text{m}^3$$

Fresh water

$$\text{Volume displaced} = V + (0{\cdot}5 \times 10^{-4} L)$$

$$= 3{\cdot}902 \times 10^{-4} + (0{\cdot}5 \times 10^{-4} L)\ \text{cm}$$

$$= 10^{-4}\,(3{\cdot}902 + 0{\cdot}5\ L)\ \text{m}$$

$$\rho = \frac{m}{V}$$

$$1000 = \frac{0{\cdot}4}{10^{-4}\,(3{\cdot}902 + 0{\cdot}5\ L)}$$

$$1000 \times 10^{-4}\ (3{\cdot}902 + 0{\cdot}5L) = 0{\cdot}4$$

$$0{\cdot}3902 + 0{\cdot}05L = 0{\cdot}4$$

$$0{\cdot}05L = 0{\cdot}0098$$

$$L = 0{\cdot}196\ \text{m}$$

$$L = 19{\cdot}6\ \text{cm}$$

WORKED EXAMPLE 4

When a given hydrometer floats in fresh water $^{9}/_{10}$ of its volume is immersed; in milk $^{90}/_{103}$ of its volume is immersed.

Find the R.D. of milk.

Let the volume of the hydrometer be V m^3.

Fresh water

Volume displaced $= {}^{9}/_{10} \times V$ m

$$\rho = \frac{\text{mass}}{\text{volume}}$$

density $\times$ volume $=$ mass

mass displaced $= 1000 \times {}^{9}/_{10}\ V$

By the law of flotation this is equal to the mass of the hydrometer

Milk

mass displaced $=$ density $\times$ volume

$= \rho \times {}^{90}/_{103}\ V$

But again the mass displaced is equal to the hydrometer's mass.

$$1000 \times {}^{9}/_{10}\ V = \rho \times {}^{90}/_{103}\ V$$

Dividing both sides by V gives

$$1000 \times {}^{9}/_{10} = \rho \times {}^{90}/_{103}$$

$$\frac{900 \times 103}{90} = \rho$$

$$\rho = 1030$$

Thus the R.D. of milk $= 1{\cdot}03$

EXERCISE B

1. A hydrometer weighs 0·12 kg. What volume is immersed when placed in water of R.D. 1·024? What volume is immersed when placed in fresh water?

   If the cross section area of the stem is 0·50 cm^2 what is the length of stem between the marks 0 and 24?

2. A cylindrical tube is weighted so that it floats upright. When placed in F.W. its draught is 0·18 m. What is the R.D. of a liquid in which 0·14 m is immersed? What length would be immersed in dock water R.D. 1·012?

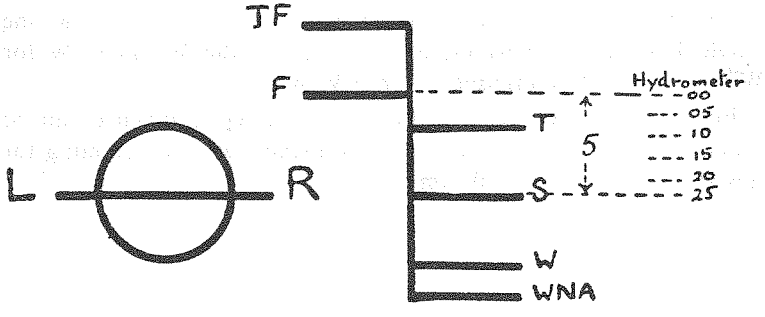
3. A hydrometer weighs 7·2 g. What volume of it will be immersed when set to float in a liquid of R.D. 0·84?

4. A given hydrometer floats in F.W. with 0·7 of its volume immersed. When set to float in another liquid it has 0·8 of its volume immersed. Find the R.D. of the liquid?
5. A stick weighted so that it floats vertically weighs 0·18 kg and is 4·5 cm^2 cross section. What would be the volume immersed when floating in liquids of R.D. 0·8 and 1·2 respectively? What would be the distance between the waterlines in the two liquids?
6. A uniform cube of wood floats in F.W. with its upper face horizontal and $\frac{2}{7}$ of its volume above the surface. What fraction of its volume would be above the surface in liquid of R.D. 1·26?
7. A solid cylinder of uniform material and length of 1·40 m will float in F.W. with its axis vertical and 1·10 m of its length immersed. Find the R.D. of the cylinder and the length it would have immersed in oil, R.D. 0·9?
8. A hydrometer weighs 0·08 kg and the cross section of its stem is 0·40 cm^2. What would be the distance between the 0 and 20 marks on this hydrometer?
9. A solid wooden cylinder 1·0 m high, 1·4 m radius, R.D. 0·6 floats in F.W. with its axis vertical. What is its draught? If 300 kg is now placed on the upper surface, what is the new draught, the axis remaining vertical?

6.4 THE LAW OF FLOTATION—APPLICATION TO LOADLINES

Fresh Water Allowance

When a ship passes from a dock, where the water may be partially fresh, into the salt water of the sea, she rises. This fact has to be allowed for when loading, by submerging the loadline by the correct amount. The amount by which the loadline is submerged is called the **Allowance**, and when the ship reaches the open sea she rises by exactly this amount and so is then at her correct loadline for the commencement of the voyage.



Loadline (starboard side).

The sketch shows a ship's load line, and on the right, the corresponding hydrometer readings.

Note that: The sketch is not drawn to scale, and in particular note that subdivision into five intervals has only been chosen for convenience. This distance varies from ship to ship, and a typical value might be 250 mm. The quantity itself, i.e., from the top of the F (Fresh Water) mark to the top of the S (Summer) mark, as illustrated in the sketch, is called the Fresh Water allowance (F.W.A.) for a given ship.

Definition: The Fresh Water Allowance is the amount by which a ship may submerge her seasonal loadline when loading in completely fresh water.

The two marks on the left of the figure are used in fresh water. The letters mean Tropical Fresh, Fresh.

The four marks on the right are used in Salt water. The letters mean Tropical, Summer, Winter, Winter North Atlantic.

The **upper edge** of the line is the official loadline. All lines are 25 mm wide.

In Summer. If the dock is salt water (R.D. 1·025) then of course there is no allowance. The ship loads to the top of the S mark. (S for Summer).

In Summer. If the dock is fresh water (R.D. 1·000) then the ship would load to the top of the F mark. When she reached the open sea she would rise in water by the amount shown and so would again be exactly at her correct loadline.

In Winter. The same principle would apply, except that she would load, in fresh water, to a line above the W mark (W for Winter) by the full amount of the F.W.A.

For any intermediate densities, the required allowance can be found as follows. Note that for convenience, we are assuming for this ship a F.W.A. of 250 mm.

6.5 TO FIND THE ALLOWANCE IN A GIVEN CASE

For any density between fresh and salt, the required allowance is only a matter of proportion.

For example, in the diagram above, if the dock density were 1005 (loading in Summer) then the waterline when loaded would coincide with the 05 dotted line shown, that is, $\frac{5}{25}$ of 250 mm, i.e., 50 mm, measured down from the zero of the scale.

But the allowance is the **amount by which the loadline is submerged,** i.e., allowance = 250 − 50 = 200 mm.

In practice on board ship, it is found more convenient to do the subtracting first.

In other words the 25 mark is used as the reference rather than the 00 mark.

Thus re-working the above calculation we have, firstly, 25 − 05 = 20.

Now $\frac{20}{25}$ of 250 mm = 200 mm.

Therefore top of the S mark should be submerged by 200 mm.

WORKED EXAMPLE 5

The F.W.A. of a ship is 210 mm. State how the ship should be loaded in winter, in a dock, density of water 1008 kg/m³.

In other words,

hydrometer reading = 8

25 − 8 = 17, and $^{17}/_{25} \times 210 = 143$ mm.

Hence, top of W mark should be 143 mm submerged.

CHANGE IN DRAUGHT

We have seen that for a box-shaped body the draught is inversely proportional to the density of the liquid in which it is floating.

For a ship, this rule is not quite accurate, but is very nearly so—near enough for practical purposes, at any rate.

Thus, for a ship, we may say,

$$\frac{\textbf{Dock draught}}{\textbf{Sea draught}} = \frac{\textbf{Sea density}}{\textbf{Dock density}} \quad \ldots\ldots\ldots\ldots\ldots\ldots \textbf{II}$$

WORKED EXAMPLE 6

A ship's loaded draught in salt water is 7·62 m. What draught should she be loaded to in a dock where a hydrometer reads 16?

$$\frac{\text{Dock draught}}{7{\cdot}62} = \frac{1025}{1016}$$

$$\text{Dock draught} = \frac{7{\cdot}62 \times 1025}{1016} \text{ and}$$

working this out gives draught in dock = 7·69 m.

EXERCISE C

(Assume sea water to have R.D. 1·025 unless otherwise stated).

1. A vessel's F.W.A. is 180 mm. How should she be loaded in dock water of R.D. 1·020 in Winter?
2. A box-shaped barge floats at a draught of 3·62 m in dock water in which a hydrometer reads 04. What would be the draught on reaching the open sea?

3. The distance from the S mark to the F mark is 162 mm. Show by a careful sketch exactly where the waterline would be when this ship loaded, (*a*) in Winter (*b*) in Summer, in a dock where a hydrometer reads 08.
4. A vessel's loaded draught in salt water is 8·16 m. To what draught should she be loaded in dock water R.D. 1·020?
5. A vessel loads to a draught of 9·08 m in a dock where a hydrometer reads 12. What would be her draught on reaching the open sea?
6. A vessel draws 7·86 m forward and 8·00 m aft, in a dock where a hydrometer reads 10. Assuming there is no change of trim, what would these draughts be on reaching the open sea?
7. A box-shaped vessel has a mean draught of 8·08 m and a freeboard of 1·72 m in a dock where the hydrometer reads 12. What will be the freeboard in the open sea?

TEST PAPER 6
Hydrometers, Loadlines

1. A vessel's F.W.A. is 148 mm. How should she be loaded in dock water of density 1016 kg/m^3 so as to float at her winter mark in the open sea?
2. If the draught of a box-shaped vessel is 7·86 m forward and 8·08 m aft when in water of R.D. 1·010, find the mean draught on reaching sea water R.D. 1·025.
3. A uniform glass tube weighed with lead shot floats vertically in F.W. with 22 cm immersed. What length would be immersed in oil R.D. 0·75? What would be the R.D. of a liquid in which it floated with 20 cm immersed?
4. A hydrometer weighs 12·48 g. What volume of it will be immersed when floating in a liquid of R.D. 1·04?
5. A ship displaces 2×10^5 m^3 of fresh water. Find the mass of the ship in tonnes. What volume of sea water R.D. 1·025 will the ship displace?
6. A rectangular raft is to carry a load of 6 tonne and the sinkage is not to exceed 20 cm in sea water. (R.D. 1·025). Calculate the area of the waterplane of the raft. What is the Tonnes Per Centimetre (TPC) for this raft?

7. A closed cylindrical "camel" is 2 m in diameter and 5 m long. It is floating in F.W. with a load suspended below it so that the axis of the camel is horizontal and lies in the surface. If the camel weighs 3 t, what is the tension in the wire supporting the load?
8. Given that the F.W.A. in millimetres = D/4T, find it for a ship of displacement (D) = 8960 tonne and TPC (T) = 16 tonne. What would be the allowance for this ship in dock water in which a hydrometer reads 18?

CHAPTER 7

HYDROSTATICS

Atmospheric Pressure, Barometers

7.1 ATMOSPHERIC PRESSURE

Air is a fluid and exerts a pressure on the earth's surface just like any other fluid.

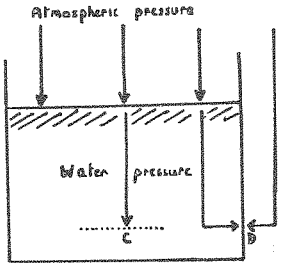
The average pressure of the atmosphere is about 101325 N/m^2 I

As an instance of the "transmission of fluid pressure", the atmosphere is an important case. Its pressure must be added on to the other fluid pressure, wherever the circumstances require it.

For example in the diagram below the pressure at a point C is clearly water pressure plus atmospheric pressure. Note that the pressure at C will act in all directions; the arrows on the diagram are merely representing one direction.

Consider D, a portion say, of the side plating. The pressure on the inside is water + atmospheric, but of course, the atmosphere is also exerting a pressure on the outside. Hence the resultant pressure at D is simply that due to water only.

It will generally be clear in any problem whether the pressure of the atmosphere should be included or not.



Total pressure at a depth.

It is also convenient sometimes to measure atmospheric pressure in terms of the "head" of water it would support.

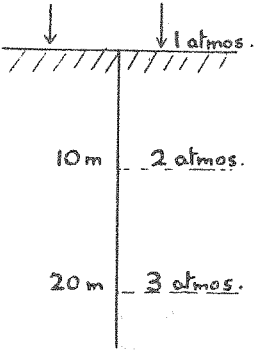
This head is 10·36 metres of fresh water or 10·06 m of salt water.

The former does not lend itself to further approximation, but for practical work the latter may be taken as 10·00 metres.

N.B. 1 atmosphere is equivalent to a head of 10 m of sea water.

This means that at a depth of 10 m in the sea, the pressure of the water is equivalent to 1 atmosphere. Since there is also 1 atmosphere pressing on the sea surface, the **actual** pressure at 10 m depth is 2 atmospheres, and so on.

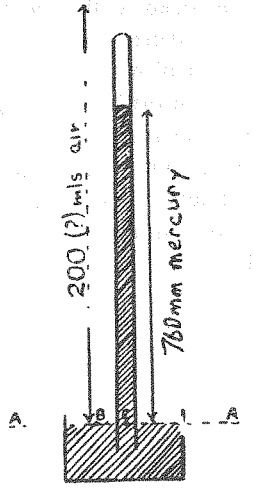
This principle must be used whenever the total pressure at a depth is required.



Pressure at a depth in "atmospheres".

7.2 MEASUREMENT OF ATMOSPHERIC PRESSURE —THE BAROMETER

In its simplest form, a mercurial barometer consists of a glass tube about 84 cm, long. If such a tube is filled with mercury, and then inverted with its open end under the surface of a trough containing mercury, the column of mercury would finally come to rest in a position such as that shown in the diagram opposite. The space above the mercury is a vacuum. The vacuum is called the Toricellian vacuum (after the inventor of the mercury barometer, Torricelli, in 1643).



Principle of mercurial barometer.

In this condition, there is a balance. The "level" or "datum", at which this balance is struck is the level of the mercury in the trough, AA; and the balance is struck between a column of air over 40 km high on one side, and a column of mercury some 760 mm high on the other.

It is very important to think once again of the atmosphere as a fluid, and we ourselves are just particles sunk to a depth of over 40 km in this fluid. By the "pressure of the atmosphere", we mean the pressure at this point where we are, immersed to a depth of over 40 km in this vast ocean which we call the atmosphere.

Such a point is B. By the "pressure of the atmosphere" we mean the pressure at B.

But the pressure at B is equal to the pressure at C, and at C, of course, the pressure is just that at a point sunk to a depth of 760 mm in mercury.

From section 2·5 $P = h\rho g$

In this case

$h = 760$ mm and $\rho = 13\,600$ kg/m^3 i.e. the density of mercury.

Therefore

$$P = 760 \times 10^{-3} \times 13600 \times 9{\cdot}81 \text{ N/m}^2$$

$$P = 101400 \text{ N/m}^2 \text{ (approx.)}.$$

Of course, as the atmospheric pressure varies, so the length of the column of mercury will vary. But this calculation shows how easy it is to get from units of length of the mercury column into true units of pressure. Occasionally this is necessary.

Whilst for scientific work barometers may still be graduated in units of length (millimetres), in meteorology, all barometers are now graduated in units of pressure.

It has already been stated (section 2.2) that the SI unit of pressure is the bar (from Greek word *baros*, weight), and by definition,

1 bar = 10^5 N/m^2 II

The bar is divided for convenience, in a vertical scale attached to the barometer, into 1000 parts, each therefore being termed a **millibar.** By means of a vernier, the barometer can be read to one-tenth of a millibar, e.g., 998·3 mb, 1012·6 mb, and so on, and **these are actual pressures,** remember.

In SI, millibar abbreviates to mbar, but the reader will find mb in common usage, especially in meteorology.

Finally, note the relationship between the three quantities discussed above.

Since 1 bar = 10^5 N/m^2, 1 mbar = 10^2 N/m^2.

So for example,

101400 N/m^2 = 1014·0 mbar = 1·014 bar
—three ways of expressing the same pressure .. III

WORKED EXAMPLE 1

The height of the mercury in a barometer was 762 mm.

Calculate the atmospheric pressure in *(a)* N/m^2.
(b) millibars.

(a) Pressure $P = h\rho g$
$= 0{\cdot}762 \times 13600 \times 9{\cdot}81$ N/m^2
$= 101663$ N/m^2

(b) 101663 N/m^2 = 1016·63 mb.

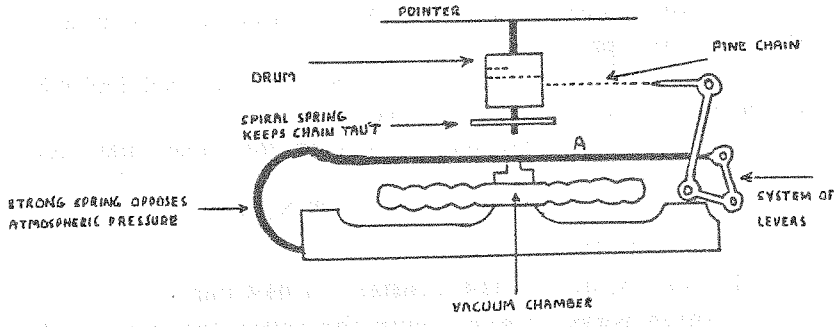
Note: Whilst mercury barometers were in wide usage on board vessels they have now been largely replaced by precision ANEROID barometers.

7.3 THE ANEROID BAROMETER

Aneroid barometers do not use a column of liquid.

The mechanism consists of a circular metal chamber with faces corrugated for strength. This chamber is partially exhausted of air, but is prevented from collapsing under atmospheric pressure by the strong leaf spring.

The varying pressure of the atmosphere causes the corrugated faces of the chamber to move in and out, and this motion is communicated through the bar *A*, which is rigidly attached to the face of the chamber, to a system of levers. These in turn operate a pointer which moves round a scale graduated to read in millibars.



Principle of aneroid barometer.

7.4 CORRECTIONS TO THE ANEROID BAROMETER READING

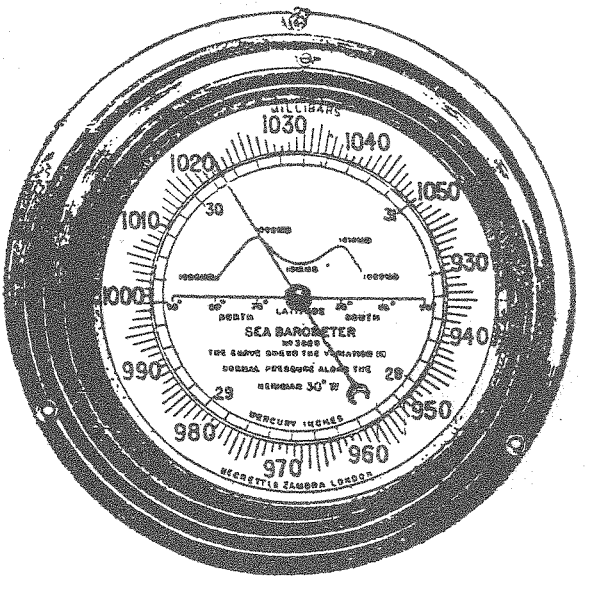
(1) TEMPERATURE — Aneroid barometers usually compensate for changes in temperature by either leaving a small amount of air in the vacuum chamber or by using a bi-metallic strip. Thus no temperature correction to the reading is necessary.

(2) INDEX ERROR — A particular aneroid barometer may have a small constant error in its readings. This is known as index error. The error may be determined by comparing the instrument's reading with that of a known accurate barometer. Since this error

can change with time, periodic comparison should occur. On some instruments an adjustment screw is provided on the back of the case by means of which index error can be corrected.

(3) HEIGHT — Air pressure falls with height above sea-level since the weight of air above us is less the higher we go. Thus if the aneroid barometer is above sea-level it will give a reading less than it would at sea-level. Obviously a height correction must be applied to the barometer reading in such cases if the air pressure at sea-level is required.

Note: The moving parts of the lever system can result in the barometer being slow to react to pressure changes; hence the need to gently tap its glass face to ensure the free movement of the pointer.



The aneroid barometer.

On the dial illustrated, which is one devised by the Meteorological Office, the curve shows the normal pressure to be expected in any latitude, along the meridian of 30° W. Thus, in any given

latitude (on or near 30° W) the observer can ascertain at a glance whether the pressure indicated by the instrument is about average, or not. If it differs greatly from the average, abnormal weather may be expected—or of course, it may point to a defect in the instrument.

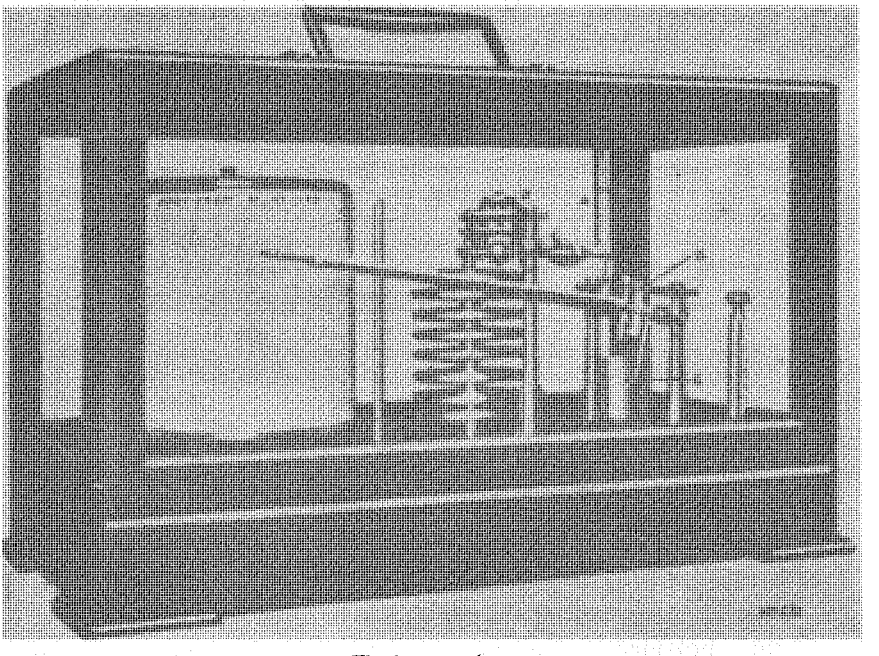
WORKED EXAMPLE 2

An aneroid barometer, index error 0·8 mb to subtract, at a height of 13·5 m, reads 996·3 mb. What is the correct sea level pressure, given that the height correction is 1 mb for every 9 m of height difference from sea level?

Reading		996·3
I. Er.	−	0·8
		995·5
Ht. corr.	+	1·5
Correct reading	+	997·0 mb

7.5 THE BAROGRAPH

The barograph is a self-recording aneroid barometer. It consists of a series of metal capsules, similar to the one in the aneroid, joined together. These expand and contract as the atmospheric pressure changes, and by means of a suitable arrangement of levers, the movement of the metal capsules is communicated to a pen which traces a graph on a paper wound round a rotating drum. In the most modern instruments the capsules are enclosed in an oil bath to reduce the effect of vibration.

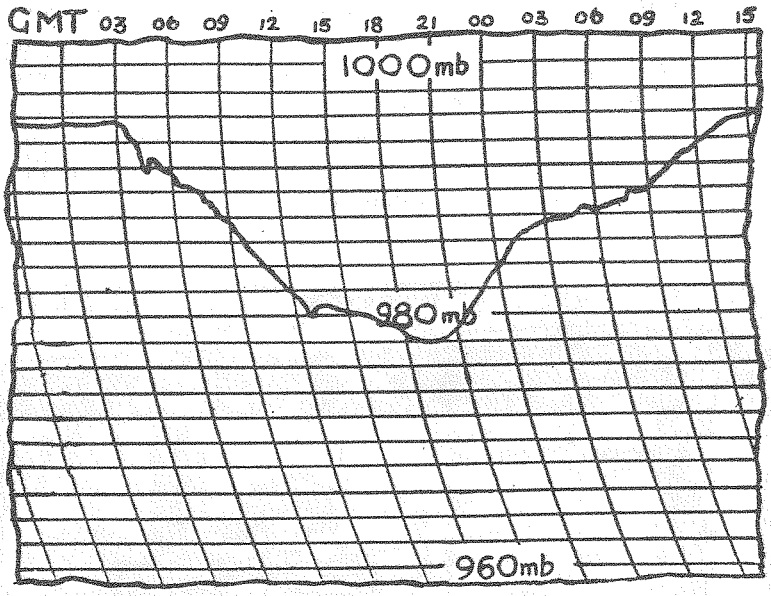


The barograph.

The drum, driven by clockwork, completes one revolution in seven days. The barogram, as the completed graph is called, must then be removed and a new one fitted. A portion of a completed barogram is illustrated in the diagram below.

The vertical (curved) lines indicate time and the day of the week; the horizontal lines pressure in millibars.

Its chief advantage is that it provides a continuous record of the pressure.



Portion of completed barogram, showing the passage of a vigorous depression. Squalls at 0430 hrs. and 1300 hrs. are clearly shown on the graph.

Thus, minor fluctuations of pressure such as might be caused by a passing squall are recorded. Like the aneroid, it is not as accurate as a mercury barometer.

Care and maintenance

Like all delicate instruments, it should be handled with care. When it is being moved, the pen should be lifted from the paper. An arm is provided for this purpose. The ink is of a special type and will not evaporate, nor congeal on the pen. The moving mechanism may require a few drops of light machine oil from time to time. A screw is provided by means of which the pen may be adjusted for index error. The clockwork will require winding each week when the barogram is changed. The clock can be regulated to keep correct time, and should always show G.M.T., the latitude and longitude being recorded daily at noon on the barogram.

The corrections to the reading are as for any other aneroid, viz., index error, which can be eliminated as mentioned above, and height above sea level. This is 1 mb for every 9 m, and normally this too would be eliminated by adjusting the pen for the present height of the instrument.

Within the limits of accuracy to be expected of this type of instrument, the trace would then show correct barometric pressure.

TEST PAPER 7

Atmospheric pressure, Barometers

(Assume density of mercury = 13 600 kg/m³ and g = 9·81 m/s²)

1. The reading on a mercury barometer is 758 mm. Calculate the atmospheric pressure in *(a)* N/m^2 *(b)* millibars.
2. The air pressure is observed to be 1015 mb. Calculate the height of mercury it would support in a barometer.
3. Correct the readings of an aneroid barometer in the following cases:
   (a) Index error 0·6 mb to add, height 12 m, reading 1012·2 mb.
   (b) Index error 0·2 mb to subtract, height 9·5 m, reading 996·3 mb.
   (c) Index error 0·5 mb to add, height 7·8 m, reading 1005·5 mb.

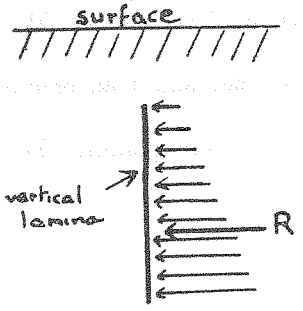
CHAPTER 8

HYDROSTATICS

Pressure, Centre of Pressure, Boyle's Law

8.1 CENTRE OF PRESSURE

The reader should now be well acquainted with the notion of pressure, which is force per unit area, and thrust, which is the whole force on an area, or lamina. In fact, we have seen how to calculate the thrust on a lamina (section 2.6) i.e., we have the *magnitude* of the force on a lamina. Through what point does it act? Where is its line of action?

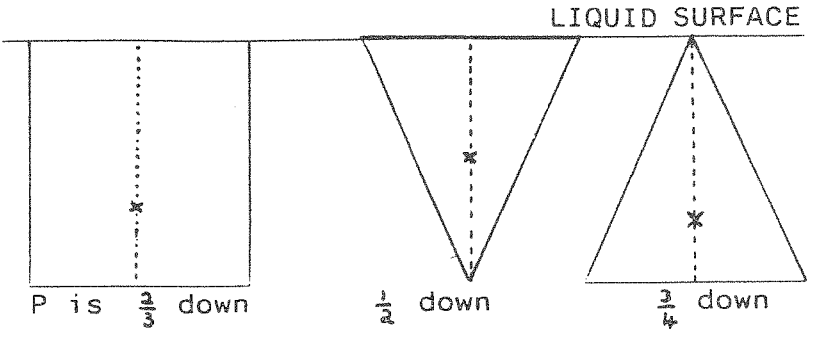


In other words, if the thrusts on a lamina are represented by the vectors shown (increasing of course with depth) where does the resultant R of the whole system of parallel forces act?

Definition: The Centre of Pressure is that point through which the whole thrust on a lamina acts I

Note that the C. of P. does not, in general, coincide with the C.G. For clearly, in the above sketch, say, of a rectangular lamina, the C.G. would be mid-way, whilst the C. of P. must lie lower down, towards the region of greater pressure and hence greater thrust.

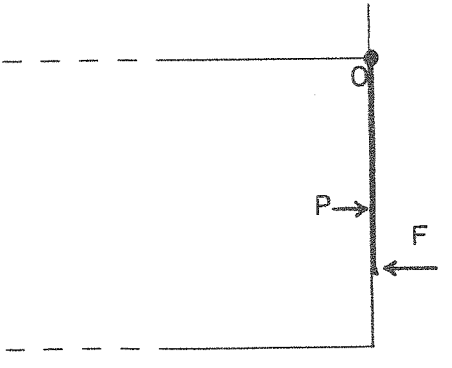
To find the Centre of Pressure of a rectangular lamina involves mathematics beyond the scope of this book. However the Centre of Pressure of various laminae are shown below:



Note that in each case the lamina has its upper edge in the surface.

WORKED EXAMPLE 1

A rectangular trapdoor 2·4 m deep by 1·6 m wide in the side plating of a tank containing sea-water has its hinge in the surface and is kept shut by bolts along its bottom edge. Find the force on the bolts.



Thrust on trapdoor
$= h\rho gA$
$= 1{\cdot}2 \times 1025 \times 9{\cdot}81 \times 2{\cdot}4 \times 1{\cdot}6$N
$= 46\ 335$ N

Note that we used a value of h halfway down the trapdoor.

The thrust calculated will act through P, the Centre of Pressure, and will give rise to an anti-clockwisee moment about O, the hinge.

This moment will be $46\ 335 \times OP = 46\ 355 \times {}^{2}/_{3}$ of 2·4 Nm
$= 46\ 355 \times 1{\cdot}6$ Nm
$= 74\ 168$ Nm

This moment is balanced by the moment of the force F on the bolts i.e. by $F \times 2{\cdot}4$ Nm.

Therefore $F \times 2{\cdot}4 = 74\ 168$
$F = 30903$ N

EXERCISE A

1. A cubical open tank of 1 m edge is full of sea-water, R.D. 1·025. One of the vertical sides is hinged at the top and is kept closed by bolts along the bottom edge. What is the force on the bolts?
2. An open drain is of equilateral triangular cross section, apex down, and of 0·6 m edge. The end can be closed by a flap with a butterfly nut at its lowest point, the flap being freely hinged at its upper edge. Find the force on the butterfly when the drain is full of fresh water.
3. A bulkhead which may be assumed rectangular has a hold flooded with sea-water on one side to a depth of 6 m, the breadth of the bulkhead being 20 m. Find the thrust on the wetted side of the bulkhead and state where a single shore should be placed to provide optimum support to the bulkhead.

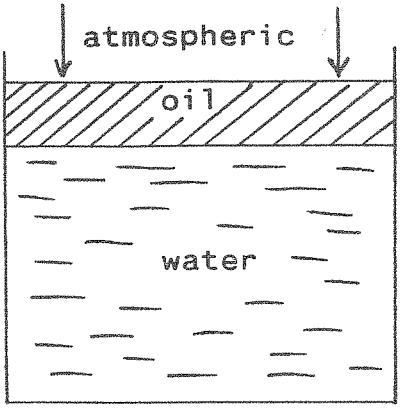
8.2 PRESSURE DUE TO MORE THAN ONE FLUID

We have already considered this in a general way when we saw that, since the atmosphere is a "fluid", overlaying and pressing down on the water surface, then the true pressure at a depth in the water is given by **water pressure plus atmospheric pressure.**

We can apply this quite easily to two liquids which do not mix, say oil and water.

WORKED EXAMPLE 2

In an open tank oil, of R.D. 0·8 and depth 0·76 m, floats on sea-water of R.D. 1·024. Find the total pressure at a point 1·84 m deep in the sea-water when the corrected barometer reading is 1016 mb.



Total pressure = pressure due to atmosphere
+ pressure due to oil
+ pressure due to water

Pressure due to oil $= h\rho g$
$= 0{\cdot}76 \times 0{\cdot}8 \times 1000 \times 9{\cdot}81 \text{ N/m}^2$
$= 5964{\cdot}48 \text{ N/m}^2$
$= \frac{5964{\cdot}48}{10^5}$ bar (from section 2.2)
$= 0{\cdot}0596448$ bar
$= 59{\cdot}6448$ mb

Pressure due to water $= h\rho g$
$= 1{\cdot}84 \times 1{\cdot}024 \times 1000 \times 9{\cdot}81 \text{ N/m}^2$
$= 18483{\cdot}6 \text{ N/m}^2$
$= 184{\cdot}836$ mb

Total pressure $= (1016 + 59{\cdot}64 + 184{\cdot}8)$ mb
$= 1260{\cdot}44$ mb

EXERCISE B

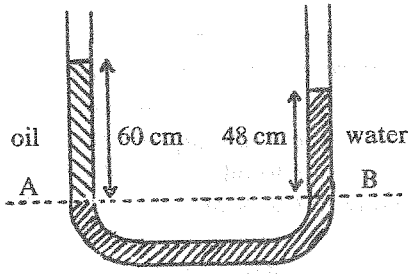
1. A trough contains 6 cm of mercury, R.D. 13·6, and 24 cm of water R.D. 1·025. Find the pressure at a point just clear of the bottom, barometer reading 1016 mb.
2. A cylindrical drum, radius 0·4 m and height 1·6 m, stands vertical half full of fresh water and half full of oil R.D. 0·8. Find the thrust on the base.

8.3 RELATIVE DENSITY BY U TUBE

This is another case of the pressure due to liquids of differing densities.

WORKED EXAMPLE 3

A U tube contained water. Oil was then poured in until the length of the oil column was 60 cm. The water was then 48 cm above the common level. Find the R.D. of the oil.



AB is the common level. The pressure of the column of oil above A must be exactly equal to the pressure of the column of water above B.

For the oil pressure at A $= h\rho g = 0{\cdot}6 \times \rho \times 9{\cdot}81\ N/m^2$
$= 5{\cdot}886\,\rho\ N/m^2$

For the water pressure at $B = h\rho g = 0{\cdot}48 \times 1000 \times 9{\cdot}81\ N/m^2$
$= 4708{\cdot}8\ N/m^2$

These pressures are equal i.e.

$$5{\cdot}886\,\rho = 4708{\cdot}8$$
$$\rho = 800\ kg/m^3$$

Therefore R.D. of oil $= 0{\cdot}8$

It will be observed that the result is the exact ratio between the two heights 48 cm and 60 cm i.e. 48:60.

This result is quite general:

For two liquids which do not mix, when placed in a U tube as shown, their relative densities are inversely proportional to the heights of columns above the common level.

8.4 MANOMETER (OR PRESSURE GAUGE)

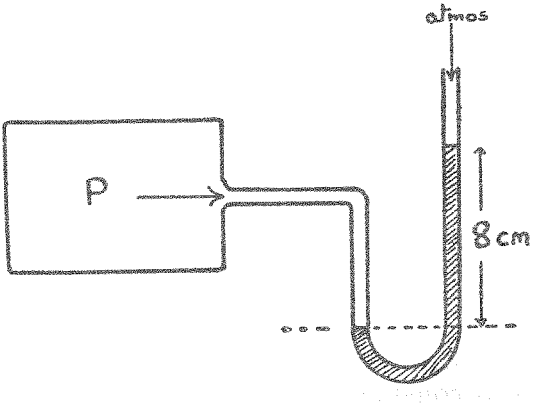
This is only the simplest form of pressure gauge and must not be confused with the more complicated gauges used on marine boilers, for instance.

The principle is the same as in the U tube.

The sketch shows a U tube (called here an "open tube manometer").

WORKED EXAMPLE 4

Initially the mercury is level in the two limbs. As a result of the pressure P in the boiler, the mercury, R.D. 13·59 is forced up the tube as shown. By measurement against an attached scale, the "head" of mercury is observed to be 8 cm. Find the pressure in the boiler, barometer reading 1020 mb.



Principle of the manometer.

Boiler pressure P = pressure due to 8 cm of mercury
+ atmospheric pressure.

$$= h\rho g + \text{atmos.}$$

$$= \frac{0{\cdot}08 \times 13{\cdot}59 \times 1000 \times 9{\cdot}81}{10^2} + 1020 \text{ mb}$$

$$= 1127 \text{ mb}$$

EXERCISE C

1. A U tube contains fresh water, and spirit is poured in until the length of the column of spirit is 7·5 cm. The water was 6·2 cm above the common level. Find the R.D. of the spirit.
2. In a mercury manometer attached to a boiler (R.D. mercury 13·6) the column of mercury stood 9·6 cm above the common level. Find the pressure inside the boiler, assuming average atmospheric pressure = 1013 mb.
3. In a mercury manometer attached to a cylinder which is partially exhausted of air, the difference between the levels in the two limbs is 8 cm. it being higher on the side communicating with the cylinder. By how much is pressure inside the cylinder less than atmospheric? (R.D. mercury = 13·6).
4. A sluice gate in the side of a F.W. reservoir is 1·4 m square, and two of its edges are vertical. It is freely hinged at the top edge and secured by bolts at the bottom. Find the force on the bolts when the water is just level with the hinge.
5. Find the pressure at a point on the bottom of a tank containing 2·0 m of F.W. and 3·0 m of oil R.D. 0·8, ht. of barometer 1013 mb.

8.5 BOYLE'S LAW

This states that if a volume of a gas (air for example) is enclosed and is then subjected to different pressures (the temperature remaining constant) then the product of the pressure and the volume is a **constant quantity**, i.e.,

$$PV \text{ is constant}$$

In problems, the application of this law is better written down as follows:

First pressure × First volume = Second pressure × Second volume

$$P_1V_1 = P_2V_2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{II}$$

This principle has many applications.

WORKED EXAMPLE 5

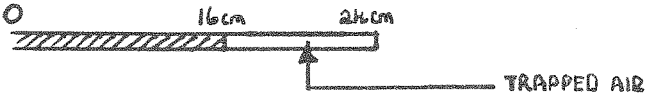
A quantity of air occupies 3·6 m³ at atmospheric pressure. What is the pressure when its volume is reduced to 2·4 m³, the temperature remaining constant? (Baro. reading 1013·0 mb).

$$P_1 = 1013 \text{ mb} \qquad P_2 = ?$$
$$V_1 = 3{\cdot}6 \text{ m}^3 \qquad V_2 = 2{\cdot}4 \text{ m}^3$$
$$P_1V_1 = P_2V_2$$
$$1013 \times 3{\cdot}6 = P_2 \times 2{\cdot}4$$
$$\frac{1013 \times 3{\cdot}6}{2{\cdot}4} = P_2$$
$$P_2 = 1519{\cdot}5 \text{ mb}$$

Note that when using Boyle's Law the pressure and volume can be in any units as long as P_1 and P_2 have the same units as each other and likewise for V_1 and V_2.

WORKED EXAMPLE 6

A glass tube 24 cm long closed at one end and internally coated with a chemical which discolours on contact with water is lowered into the sea. The sea-water traps some air in the tube. On being hauled up again 16 cm is found to be discoloured. To what depth has it been approximately?



$$P_1 = 1 \text{ atmos.} \qquad P_2 = ?$$
$$V_1 = 24 \times A \qquad V_2 = 8 \times A$$

Where A is the area of the tube's cross-section

$$P_1V_1 = P_2V_2$$
$$1 \times 24A = P_2 \times 8A$$
$$P_2 = 3 \text{ atmos.}$$

Consider what this means. When the tube was down there, the pressure on the air in it was 3 atmospheres. But 1 atmosphere is provided by the atmosphere itself, pressing on the sea surface (transmission of pressure again) so 2 atmospheres must have been provided by the water.

And 1 atmos. = 10 m of sea water, so the tube must have been to a depth of 20 m.

WORKED EXAMPLE 7

A diving bell of internal capacity 20 m^3 is lowered into the sea to a depth of 15 m. How much water will enter the diving bell?

Initially, pressure on the air inside = 1 atmos. (P_1)

Volume of air inside = 20 m^3 (V_1)

Finally, 15 m $= \frac{15}{10}$ atmospheres = 1·5 atmospheres, so pressure on air inside = (1 + 1·5) atmos. = 2·5 (P_2)

and V_2 is what we want to find.

$$P_2V_2 = P_1V_1$$

$$2{\cdot}5 \times V_2 = 1 \times 20$$

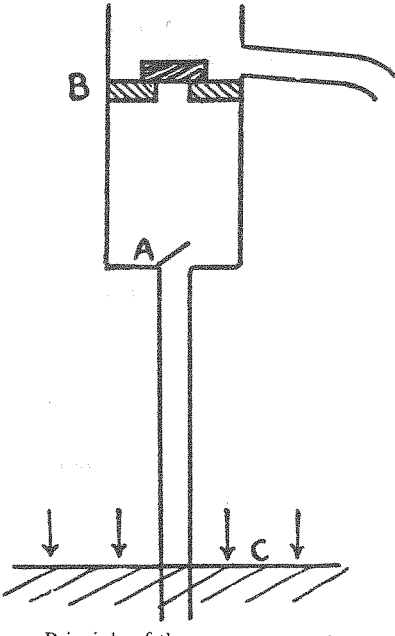
$$V_2 = \frac{20}{2{\cdot}5} = 8 \text{ m}^3 = \text{vol. of air.}$$

Volume of water entered = 20 − 8 = 12 m^3.

EXERCISE D

1. A coated tube, as in Worked Example 6, 36 cm long is lowered to the bottom of a fresh water lake and comes up with 21 cm discoloured. What is the approximate depth of the lake? (Take 1 atmos. = 10·36 m F.W.).
2. A coated tube closed at one end and 36 cm long is lowered to a depth of 50 m in sea-water of R.D. 1·025, the height of the barometer being 1018 mb. Calculate the pressure on the air in the tube when at the depth stated. What length of tube would be discoloured?
3. A closed cylinder with ends 20 cm diameter contains gas at a pressure of 14 atmospheres. It is lowered into the sea in a horizontal position to a depth of 110 m. Calculate the resultant thrust on one end of the cylinder. (Take atmos. pressure 1·013 bar and 10 m sea-water equivalent to 1 atmos.).
4. A diving bell of 12 m^3 internal volume is lowered into the sea till the level of the water inside the bell is 18 m below the surface. What is the pressure on the air inside the bell and what is its volume? (Take atmos. pressure 1·013 bar and 10 m sea-water = 1 atmos.).

8.6 THE COMMON PUMP



Principle of the common pump.

It is not proposed here to give a full description of this pump.

The general arrangement is well known, and the sketch illustrates the principle.

The piston *B* moves up and down the barrel. It has a valve which opens on a down stroke and closes on an up stroke. The valve at *A* also opens and shuts as indicated. On an up stroke, part of the air is exhausted from the pipe. The atmospheric pressure on the water surface in the cistern *C* forces water up the pipe, until finally, after a few strokes, water passes the valve *A*.

On the next down stroke of the piston it passes the valve in the piston. From then on the piston acts as a water pump, at each stroke delivering a quantity of water equal to the volume of the barrel.

Limitation

Since the action of the pump is dependent on the atmospheric pressure forcing water up the pipe to *A*, it follows that *A* must not be at a greater height above cistern level than the height of the water barometer, i.e., about 34 ft or 10·36 m.

In practice, owing to the imperfections of the pump, the actual height should be a bit less than the theoretical height.

The **Lift Pump** and the **Forcing Pump** are modifications of the common pump. Though differing slightly in design, in principle they are very similar. Instead of the water in the barrel flowing out of a spout, it is forced along a pipe line in order to deliver water, for example, to a storage tank.

Both these pumps, however, like the common pump, will not raise water more than about 10 m from the cistern to the first valve.

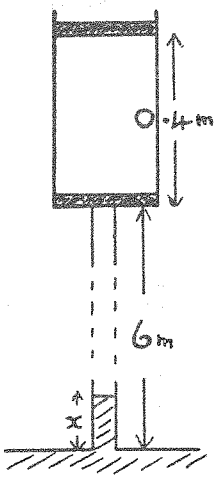
Of course, once the water has passed the first valve and enters the barrel, it may be lifted or forced to any height.

8.7 CALCULATIONS

Most problems on the common pump are solved by Boyle's Law.

WORKED EXAMPLE 8

In a common pump, the cross-sectional area of the pipe is 5 cm^2 and of the barrel, 40 cm^2. The length of the pipe from the cistern level is 6 m, and the length of stroke is 0·4 m. Find how far the water will rise up the pipe after the first stroke, the height of the water barometer being 10·36 m.



The arrangement is as shown, the piston being at the bottom, ready to start the stroke.

Considering the air inside the pump, and applying Boyle's Law to it:

(Note that 5 $cm^2 = 0{\cdot}0005\ m^2$ and 40 $cm^2 = 0{\cdot}004\ m^2$).

Initially:

Its vol. (V_1) = vol. of pipe
= $0{\cdot}0005 \times 6\ m^3 = 0{\cdot}003\ m^3$

Its press. (P_1) = atmospheric
= 10·36 m of water.

Finally, (i.e., after 1 stroke, the piston now being at the top).

Suppose the water has risen x m up the pipe.

Vol. of air, (V_2) $= (0{\cdot}004 \times 0{\cdot}4) + 0{\cdot}0005\,(6 - x)$

and this on reduction

$= 0{\cdot}0046 - 0{\cdot}0005\,x$

Press. (P_2) $= (10{\cdot}36 - x)$ m of water

and P_1V_1 $= P_2V_2$, so

$10{\cdot}36 \times 0{\cdot}003$ $= (0{\cdot}0046 - 0{\cdot}0005x)\,(10{\cdot}36 - x)$

which after reduction gives

$5x_2 - 98x + 166 = 0$

and solving this quadratic gives

$x = 17{\cdot}7$ m or $1{\cdot}8$ m.

Discarding the first answer, we have,

Ans.: 1·8 m up the pipe at the first stroke.

EXERCISE E

1. In a common pump, the length of the pipe is 7·0 m, area of pump barrel is 100 cm² and of pipe 6 cm², length of stroke 0·5 m. Find how high the water will rise up the pipe in the first stroke. (Take ht. of water baro. 10·00 m).

TEST PAPER 8

Fluid Pressure, Boyle's Law

Note: Take R.D. mercury 13·6, sea-water 1·025.

1. Find as accurately as possible the pressure at a depth of 32 m in sea-water, barometer standing at 76·0 cm.
2. A tank contains sea-water, with oil, depth 0·82 m, R.D. 0·7 floating on it. Find the pressure at a depth of 2·6 m in the sea-water. (Baro. reading 1000 mb).
3. Define a millibar. If the barometer reading is 981 mb, express this in N/m^2.
4. A diving bell has a cross-section area of 4·8 m² and an internal height of 5·6 m. It is to be lowered into the sea (10 m = 1 atmos.), but the inside pressure must not exceed 3 atmospheres. Find the depth of the top of the bell below the surface when the pressure inside is at its maximum permitted value.

5. In a mercury manometer connected to a small boiler there is 5·6 cm difference in levels in the two limbs. By how much does pressure in the boiler exceed atmospheric?
6. If the total thrust due to the atmosphere on a rectangular lamina 1·4 m by 1·2 m is 17120 kgf, what is the atmospheric pressure? What would be the height of a F.W. barometer that day?
7. A cylinder of diam. 40 cm contains gas at a pressure of 8 atmospheres. It is lowered to a depth of 132 m into the sea. Find the resultant thrust on one end of the cylinder. At what depth would the resultant thrust on one end be zero? (Take atmos. pressure 1 bar and 10 m sea-water = 1 atmos.).
8. A rectangular reservoir wall is 4 m high by 12 m long. Find the thrust on the wall when the F.W. is 1 m below the top of the wall. If the material of the wall is of density $6 \times 10^3 kg/m^3$, calculate the thickness of the wall in order that the moment of the weight of the wall may be four times the capsizing moment.

CHAPTER 9

DYNAMICS

Velocity, Acceleration, Force, Momentum

9.1 WORK

A force does work when it moves its point of application. So in mechanics, for work to be done there must be motion. Hence in this chapter we are going to learn a little about motion itself, as a preliminary to problems on work.

9.2 VELOCITY

Definition: Velocity is rate of change of position, the direction in which the change occurs also being taken into account.

Like many other definitions, this needs explaining, but in general we have been accustomed to speaking of velocity as "so many miles per hour", or knots, or "km/hr".

Of these, the knot will continue to be used at sea; in SI units however, the unit of velocity is metres per second (m/s).

The only further point to keep in mind is that velocity implies **direction** as well as **speed**. Where there is any doubt, the direction as well as the speed must be given if the velocity is to be defined fully.

Also it is necessary to be clear in one's mind between "average velocity", and "velocity at an instant". Usually this presents no difficulty.

For example: A motor car covers 300 miles in 10 hrs. Then its average speed is 30 m.p.h., but of course this does not mean it was travelling at a speed of 30 m.p.h. at every instant throughout the journey.

Again, a stone is dropped from a balloon. Experience tells us it falls faster and faster. So if we speak of its velocity after 5 seconds, clearly that can only be an instantaneous velocity.

The symbol used for the initial velocity (should there be any) is u, and for the velocity at some subsequent instant, v (both in m/s).

And as a generalisation,

$$\text{velocity} = \frac{\text{distance}}{\text{time}}$$

This is an average velocity — even over a short lapse of time, it is still an average velocity. If we want the instantaneous velocity, we must resort to the notation of the calculus. So if is δs is the distance covered in a very short interval of time δt, then

$$v = \frac{\delta s}{\delta t}$$

and taken to the ultimate limit of smallness.

$$\mathbf{v = \frac{ds}{dt}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad \mathbf{I}$$

(The reader will appreciate that a formal proof of this statement is hardly possible. All we are saying is that velocity = distance divided by time).

9.3 ACCELERATION

The symbol used for acceleration is a.

Definition: Acceleration is rate of change of velocity.

This is not quite so easy to grasp at first, "rate of change of velocity". In other words, **change in velocity in one second**. But velocity is expressed in "metres per second"; so acceleration is expressed in "metres per second per second", i.e., as a quantity of velocity, each second.

For convenience, this is generally written as "m/s^2".

$$\text{Acceleration} = \frac{\text{change in velocity}}{\text{time taken}}$$

WORKED EXAMPLE 1

A particle starts from rest with a constant acceleration and after 8 seconds is travelling at 44 m/s. Find the acceleration.

change in velocity is 44 m/s in 8s

$$\text{acceleration } a = \frac{44}{8} \text{ m/s}^2$$

$$a = 5{\cdot}5 \text{ m/s}^2$$

Acceleration can be positive, i.e., an increase in velocity, or negative, when it is sometimes referred to as a retardation.

Retardation is just negative acceleration.

The commonest acceleration is that due to the force of gravity. This is about 9·81 m/s² (approximately 10 m/s²) and the symbol for this particular acceleration is always g.

WORKED EXAMPLE 2

A stone is dropped from a balloon. Find its velocity after 5 seconds. The acceleration is g, i.e., approximately 10 m/s² each second (acceleration is just a quantity of velocity gained each second).

So after 1 second its velocity will be approximately 10 m/s,
after 2 seconds its velocity will be approximately 20 m/s,
and after 5 seconds its velocity will be approximately 50×10 mls $= 50$ m/s².

More accurately the velocities would be 9·81 m/s, 19·62 m/s ($2 \times 9{\cdot}81$) and 49·05 m/s ($5 \times 9{\cdot}81$).

This sort of argument is useful till one grasps the concept of acceleration. But of course, by the formula

$$\text{acceleration} = \frac{\text{change in velocity}}{\text{time taken}}$$

$$\text{acceleration} \times \text{time taken} = \text{change in velocity}$$

$$9{\cdot}81 \times 5 = \text{change in velocity}$$

$$\text{change in velocity} = 49{\cdot}05 \text{ m/s}$$

since the stone was initially at rest its final velocity is

$$0 + 49{\cdot}05 \text{ m/s} = 49{\cdot}05 \text{ m/s}$$

WORKED EXAMPLE 3

A bullet is shot vertically upwards with a velocity of 656 m/s; how long will it take to reach its highest point?

The bullet has an initial velocity of 656 m/s. It loses it at the rate of 9·81 each second. So the time taken to lose it all will be $656/9{\cdot}81 = 66{\cdot}9$s to the highest point.

It would then start to fall again under the same acceleration, g, and would return to earth in a further 66·9 s with a velocity on impact of 656 m/s. (We are ignoring the atmosphere).

To be able to change units is sometimes useful.

WORKED EXAMPLE 4

Convert 54 km/h to m/s.

$$\begin{aligned} 54 \text{ km per hour} &= 54000 \text{ m/hour} \\ &= \frac{54000}{60} \text{ m per minute} \\ &= \frac{54000}{60 \times 60} \text{ m per second} \\ &= 15 \text{ m/s} \end{aligned}$$

EXERCISE A

1. Convert 40 km/h into m/s.
2. A train starts from rest with acceleration 0·3 m/s^2. Find its velocity after 3 minutes. If it is now acted upon by a retardation of 0·4 m/s^2 after how long would it come to rest?
3. Compare the speeds of two particles, one of which covers 10 m in 14 seconds and the other 1·5 km in 1 hour.
4. A train 270 m long passes completely through a station 390 m long in 22 seconds. Find its speed in km/h.
5. A train travelling at 30 km/h comes to rest in 30 seconds, when the brakes are applied. Find its acceleration.
6. A body starts from rest with an acceleration of 6·5 m/s^2. In what time will it attain a velocity of 130 km/h?

9.4 GENERAL EQUATIONS OF RECTILINEAR MOTION UNDER CONSTANT ACCELERATION

Note from the heading that the equations we are about to consider may be applied only to motion *in a straight line and under constant acceleration.*

These equations use the following notation:

initial velocity ..... u
final velocity v
acceleration ..... a
time ..... t
distance ..... s

Now we have already seen

$$\text{acceleration} = \frac{\text{change in velocity}}{\text{time taken}}$$

Hence

$$a = \frac{v-u}{t}$$

$$at = v-u$$

$$\mathbf{v = u + at} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{II}$$

$$\text{average velocity} = \frac{\text{distance}}{\text{time}}$$

$$\frac{u+v}{2} = \frac{s}{t}$$

$$s = \left(\frac{u+v}{2}\right) t \quad \ldots\ldots\ldots\ldots \text{III}$$

which is useful at times, but substituting from II, we have

$$s = \frac{u+u+at}{2} \times t$$

$$\mathbf{s = ut + \tfrac{1}{2}at^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{IV}$$

a formula which gives us the distance travelled after any time t.

Finally, squaring both sides of II we have

$$v^2 = u^2 + 2uat + a^2t^2$$

$$v^2 = u^2 + 2a(ut + \tfrac{1}{2}at^2)$$

$$\mathbf{v^2 = u^2 + 2as} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{V}$$

a formula giving the relationship between v and s independently of t.

These are the fundamental formulae. To be able to use any of these equations we must be given values for at least three of the five quantities *u, v, a, t* and *s*.

Of course, if the body starts from rest, i.e. $u = 0$, and the acceleration is *g*, they become

$$\mathbf{v = gt}$$

$$\mathbf{s = \tfrac{1}{2}gt^2}$$

$$\mathbf{v^2 = 2gs, \text{ and so on.}}$$

Note that the units employed must be uniform, i.e., all in metres and seconds.

Finally note, that in these equations as stated, the + *ve* sense is regarded as "outward", "forward", progressively away from some initial point.

Hence, anything measured in the contrary sense, *must be given a — ve sign.*

WORKED EXAMPLE 5

A motor car, starting from rest and with uniform acceleration, covers 150 m from the start in 5 seconds. What is its velocity at this instant? If it continues the same acceleration for a further 4 seconds how far will it then have travelled?

For the first 5 seconds we have

$$u = 0$$
$$v = ?$$
$$a = ?$$
$$t = 5\text{s}$$
$$s = 150\text{ m}$$

This means we can use equation III

$$s = \left(\frac{u+v}{2}\right) t \quad \ldots\ldots\ldots\ldots \textbf{III}$$

$$150 = \left(\frac{0+v}{2}\right) 5$$
$$300 = 5v$$
$$v = 60\text{ m/s}$$

For a further 4 seconds we have

$$u = 0$$
$$v = ?$$
$$a = ?$$
$$t = 9\text{s}$$
$$s = ?$$

This time we are not given values for three quantities. However we do know that the motor car is continuing with the same acceleration. Thus we can use the values in the first part of the question to calculate this acceleration.

$$s = ut + \tfrac{1}{2}at^2$$
$$150 = (0 \times 5) + \tfrac{1}{2}a5^2$$
$$150 = 12{\cdot}5a$$
$$a = 12\text{ m/s}^2$$

Returning to the second part of the question

$$s = ut + \tfrac{1}{2}at^2$$
$$s = 0 + \tfrac{1}{2} \times 12 \times 9^2$$
$$s = 486\text{ m}$$

Note that in this example, everything was $+ve$, measured, or acting, progressively forward from the start.

WORKED EXAMPLE 6

A man on the top of a cliff throws a stone vertically upwards with a velocity of 30 m/s. Where will it be after 7 seconds. (Assume $g = 9{\cdot}81$ m/s^2)

$$u = 30 \text{ m/s}$$
$$v = ?$$
$$a = -9{\cdot}81 \text{ m/s}^2$$
$$t = 7\text{s}$$
$$s = 0$$

Note the acceleration is negative because the stone is travelling vertically upwards but g is acting downwards.

$$s = ut + \tfrac{1}{2}at^2$$
$$= (30 \times 7) + (\tfrac{1}{2} \times -9{\cdot}81 \times 7^2) \text{ m}$$
$$= 210 - 240{\cdot}345 \text{ m}$$
$$= -30{\cdot}35 \text{ m}$$

The minus sign means that after 7 seconds the stone is 30·35 m BELOW its starting point.

EXERCISE B

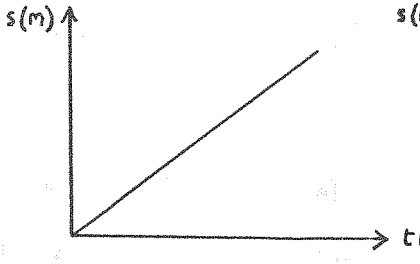
Assume where required that $g = 9{\cdot}81$ m/s^2

1. A stone is thrown vertically upwards with an initial velocity of 2000 cm/s. To what height will it rise?
2. A train travelling at 72 km/h has its brakes applied and from that instant takes 60 seconds to travel the next kilometre. Find the acceleration and the total distance run, from the time the brakes were applied, before coming to rest.
3. A man on top of a tower 36 m high throws a stone vertically upwards with an initial velocity of 30 m/s. After what time will it hit the ground?
   Hint: Consider the stone's motion in 2 parts: upwards (calculate t and s) and downwards (calculate t).
4. A launch with an initial speed of 12 m/s has its speed reduced uniformly to 4 m/s in 400 m. What was the time occupied? What was the average velocity? When and where was its velocity equal to the average?

9.5 DISTANCE-TIME GRAPHS

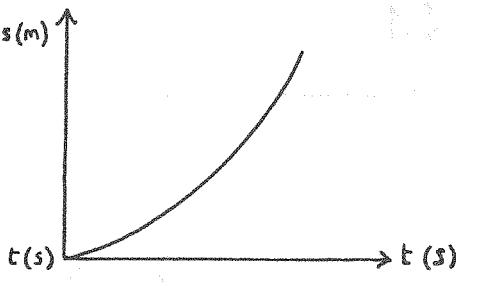
Typical distance-time graphs are shown bewlow:

Graph 1
Constant Velocity



This graph shows that in equal time intervals equal distances are travelled.

Graph 2
Constant Acceleration



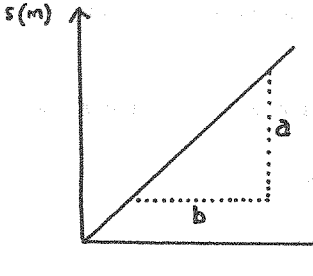
This graph shows that in equal time intervals bigger and bigger distances are travelled.

Apart from being able to read off from such a graph the distance travelled in any particular time, we can also use the graph to determine velocity at any instant.

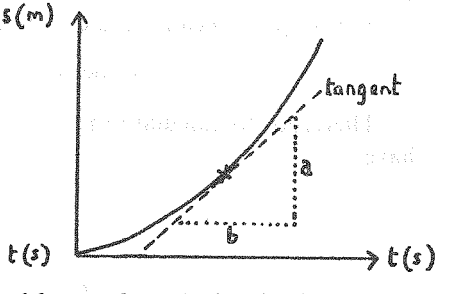
The velocity is given by the GRADIENT of the graph.

Since graph 1 above is a straight line, measuring its gradient is a relatively straightforward matter. However for graph 2 the gradient, and hence the velocity, at any instant is obtained by first drawing the tangent at that point as shown below:

Graph 1



Graph 2



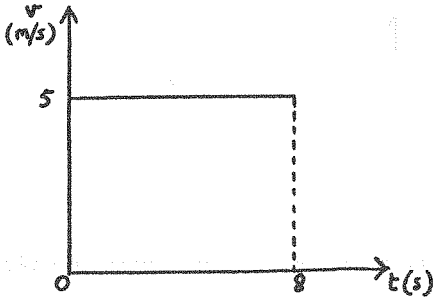
In each case the gradient, and hence the velocity, is given by

$$\frac{a}{b}$$

9.6 VELOCITY-TIME GRAPHS

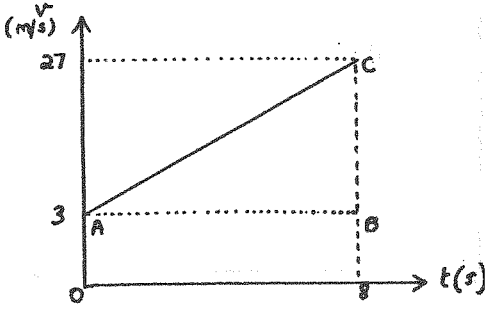
Consider the following graphs as typical examples:

Graph 1
Constant Velocity



No change in velocity occurs as time passes.

Graph 2
Constant Acceleration



Equal changes in velocity occur in equal time intervals.

Apart from being able to read off from the graphs the velocity at any instant, we can also use them to determine TWO other quantities.

(1) The acceleration can be determined from the **GRADIENT** of the graph.

For graph 1 the gradient is 0 corresponding to a body with uniform velocity having no acceleration.

For graph 2 we can measure the gradient as, for example, $BC/AB = 24/8 = 3\ \text{m/s}^2$.

(2) Velocity-time graphs provide a means of finding the distance travelled.

For graph 1 because there is no acceleration we can simply say

$$\text{velocity} = \frac{\text{distance}}{\text{time}}$$

Therefore to calculate the distance travelled in the first 8 s we have

$$5 = \frac{\text{distance}}{8}$$

$$5 \times 8 = \text{distance}$$

$$\text{distance} = 40\ \text{m}$$

Inspection of the graph shows that what we have actually found is the **AREA** of the rectangle shown.

Is the distance always the area under a velocity-time graph?

Consider graph 2. To calculate the distance travelled in the first 8 s we have:

$$u = 3 \text{ m/s}$$
$$v = 27 \text{ m/s}$$
$$a = ?$$
$$t = 8 \text{ s}$$
$$s = ?$$

using $s = \left(\frac{u+v}{2}\right)t$ we have $s = \left(\frac{3+27}{2}\right)8 = 120$ m

Now if we measure the area under the graph (which is a trapezium) we find it also gives 120.

AREA UNDER A VELOCITY-TIME GRAPH = DISTANCE TRAVELLED

WORKED EXAMPLE 7

An object accelerates uniformly from rest to a velocity of 12 m/s in 4 s. It then maintains this velocity for a further 5 s before decelerating to rest in another 3 s.

(*a*) Sketch the velocity-time graph.

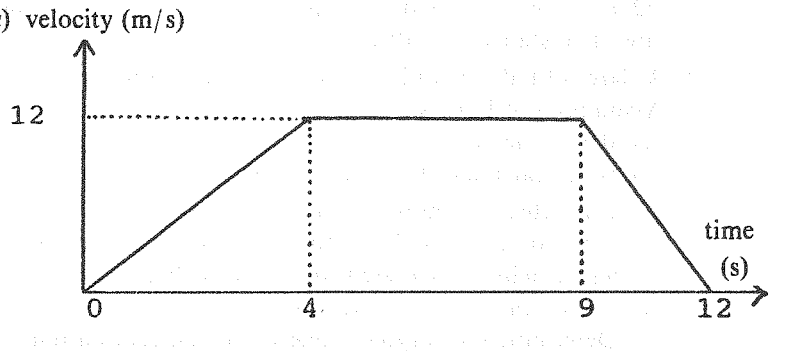
(*b*) For each stage of the motion calculate

(i) the object's acceleration.

(ii) the distance travelled.

(*c*) Find the object's average velocity over the full period of its motion.

(*a*)



Note carefully the values on the time scale.

(*b*) (i) During the first 4 s acceleration = gradient
$= 12/4 \text{ m/s}^2$
$= 3 \text{ m/s}^2$

For the next 5 s the velocity is uniform, thus the acceleration is 0.

During the last 3 s acceleration = gradient
$= -12/3 \text{ m/s}$
$= -4 \text{ m/s}^2$

(ii) For each stage of the motion, distance travelled is given by the area under that portion of graph.

During the first 4 s area $= \frac{1}{2} \times 4 \times 12$
$= 24 \text{ m} =$ distance

During the next 5 s area $= 5 \times 12$
$= 60 \text{ m} =$ distance

During the last 3 s area $= \frac{1}{2} \times 3 \times 12$
$= 18 \text{ m} =$ distance

$$\text{Average velocity} = \frac{\text{total distance travelled}}{\text{total time taken}}$$
$$= \frac{(24 + 60 + 18)}{12}$$
$$= 8{\cdot}5 \text{ m/s}$$

EXERCISE C

1. A vessel accelerates from a velocity of 1 m/s at a rate of 0·2 m/s² for 30 seconds. Using a velocity-time graph calculate the distance travelled.
2. A flare is fired vertically upwards with a velocity of 120 m/s. Assuming $g = 10 \text{ m/s}^2$
   (*a*) sketch the velocity-time graph,
   (*b*) hence calculate the maximum height achieved.
3. In being lifted by a hoist a cargo first accelerates from rest to a velocity of 2 m/s in 3 s. It then maintains this velocity for 4 s before being decelerated to rest in further 1 s.
   (*a*) Sketch the velocity-time graph,
   (*b*) Determine the cargo's acceleration and retardation.
   (*c*) Find the total distance through which the cargo is lifted.

9.7 RELATIVE VELOCITY

This term is used to denote the velocity of one body relative to another.

When the distance between two points A and B is altering, either in magnitude or direction, each is said to have a velocity "relative to the other".



For example, two motor cars are travelling along a straight road as shown, it is clear that "the velocity of A relative to B" is 5 km/h. How was this obtained? By subtraction, one might say.

But an alternative answer might be, "by applying B's velocity reversed to both". This would bring B to rest, and would reduce A's velocity to 5 km/h.

This argument is quite general, viz.

To find the velocity of A relative to B, apply B's velocity reversed to both.

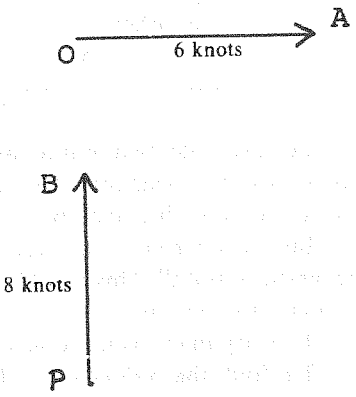
This has the effect of bringing B to rest (mathematically speaking). In deciding which body to stop, it is helpful to regard B in this example as the "observer". It is B who wants to know A's relative velocity — therefore stop B. Conversely, if we want B's velocity relative to A, then stop A, by applying A's velocity reversed, to both.

The two relative velocities, are of course, equal in magnitude but opposite in direction.

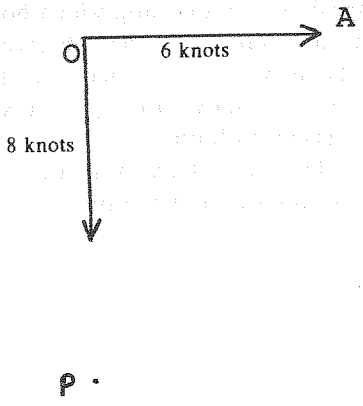
WORKED EXAMPLE 8

At a particular time a vessel *A* is due north of a vessel *B*. Vessel *A* has a speed of 6 knots, course 090 T whilst vessel *B* has a speed of 8 knots, course 000 T. Calculate the velocity of *A* relative to *B*.

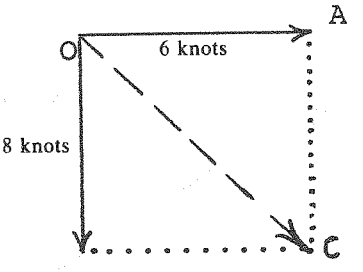
The situation is as shown:



We wish to stop *B*. Therefore the diagram becomes:



Thus to find the velocity of A relative to B simply becomes a matter of vector addition as dealt with in Chapter 3. We have

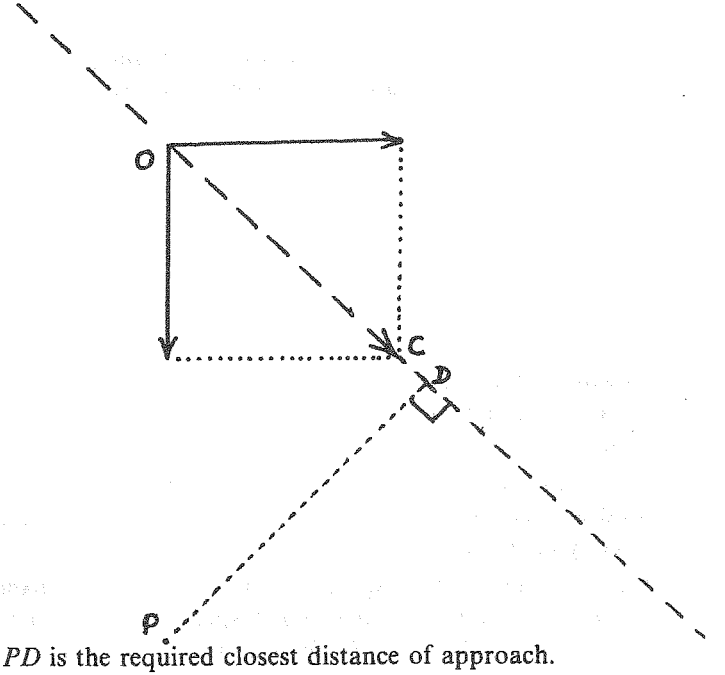


We can now regard the situation as being that vessel B is stopped and vessel A is proceeding along the direction OC.

OC is the required velocity.

OC can be calculated to be 10 knots.

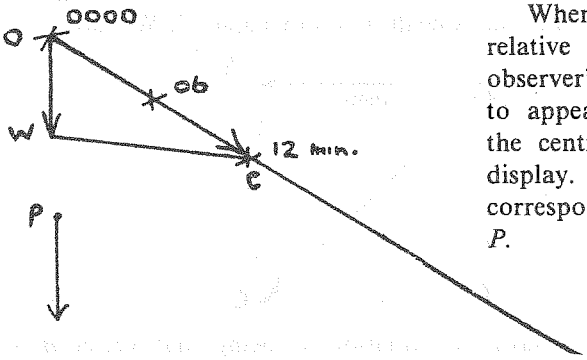
If we now wished to find the closest distance of approach we would proceed as follows



PD is the required closest distance of approach.

The above, of course, is an important factor in radar plotting.

9.8 APPLICATION TO RADAR PLOTTING



When keeping a relative plot, the observer's ship is made to appear stationary at the centre of the radar display. Suppose this corresponds to position *P*.

The motion observed in the echo *O* is its motion relative to *P*, that is the resultant obtained by compounding its own (true) motion with *P*'s motion reversed.

Hence, we have to reconstruct the parallelogram used in the previous example, working from the relative motion *OC* to get back to the true motion of *O*.

Using the notation commonly employed in radar plotting, we therefore draw $OW = P$'s motion reversed (that is course, and distance travelled in 12 min.).

Join *WC*. This gives us the true motion of the echo.

EXERCISE D

Use scale drawings to solve the following:

1. A vessel is travelling 030°T at 8 knots. The wind is actually blowing FROM 170°T at 12 knots. Determine the wind's apparent velocity and direction to someone on board the vessel.
2. To an observer in a fast launch travelling 000°T at 22 m/s the wind appears to blow from 090°T at 4 m/s. Find the true velocity of the wind.
3. Ship *A*, course 100°T speed 12 knots, is 47 nautical miles N. 40° W. of ship *B*, course 000°T speed 8 knots. Find the velocity of *A* relative to *B*. What is the distance of nearest approach, and at what time would this occur?

9.9 FORCE

Dynamics is that portion of mechanics which deals with things in motion, **Statics** with things at rest.

We shall refer to these terms again shortly.

Definition

Force is that which causes, or tends to cause, motion VI

The above statement sums up what is really **Newton's First Law of Motion,** namely:— **Every body continues in its state of rest or of uniform motion in a straight line unless acted upon by an impressed force.**

So only a force can either change the state of a body which is at rest or change its state of motion if it is in motion.

Hence there are two definitions in one here — let us consider them separately.

DYNAMICAL NOTION OF FORCE

Force is that which causes change of motion — to which we might add, "and is measured by the change of motion (i.e., of velocity) it produces per second".

Now a change of velocity per second is an acceleration.

So force can be measured in terms of the acceleration it produces.

9.10 UNIT OF FORCE

The **newton.**

Definition: In SI, the unit of force is the newton (N) which has already been defined as the force which when applied to a mass of 1 kg, imparts to it an acceleration of 1 m/s².

"From this definition if we want to give a body an acceleration of 2 m/s² instead of 1 m/s², we must have twice as much force; 5 m/s², five times as much force, and so on.

Or if there were 2 kg instead of 1 kg, we should need twice as much force, and so on.

And finally, if there were 2 kg and we wanted to give it 5 m/s², we should need 2 × 5, i.e., 10 times as much force, i.e., 10 newtons".

In short,

Force (N) = mass (kg) × accel. (m/s²).

$$F = ma \quad \text{..... VII}$$

WORKED EXAMPLE 9

A train of mass 210 t starts from rest with constant acceleration and after 88 s is travelling at 40 km/h. Find the force applied by the engine.

$F = ma$ and so we first have to calculate the acceleration a.

$u = 0$

$$v = 40 \text{ km/h} = 40 \times 1000 \text{ m/h} = \frac{40 \times 1000}{3600} \text{ m/s} = 11.11 \text{ m/s}$$

$a = ?$

$t = 88$ s

$s = ?$

$$v = u + at$$
$$11{\cdot}11 = 0 + a.88$$
$$a = (11{\cdot}11)/88 = 0{\cdot}1263 \text{ m/s}^2$$

$$F = ma$$
$$F = 210 \times 1000 \times 0{\cdot}1263 \text{ N}$$
$$F = 26520 \text{ N } (26{\cdot}52 \text{ kN})$$

EXERCISE E

1. The same train as in the worked example above, when travelling at 40 km/h, has its brakes uniformly applied and comes to rest after 1 minute 40 seconds. Find the average retarding force of the brakes.
2. A vessel of mass 600 t accelerates from 1 m/s to 9 m/s in 32 s. Calculate the force required.

9.11 STATICAL OR GRAVITATIONAL NOTION OF FORCE

Now return to the definition (VI) and consider the other half of it, "force is that which **tends** to cause motion".

If a 5 kg mass rests on a table, it exerts a force on the table top. Nothing is moving, yet there is a force acting. This force arises from the attraction of the earth on the mass.

Moreover, if the mass could suddenly be released, it would fall with an accel. g. and therefore the force on the mass, pulling it towards the earth, would be

$$\text{mass} \times \text{accel} = 5\,g\ \text{N}$$

This force is called the **weight** of the body, and, as we saw in Chapter 1, it is sometimes quoted in "kilogramme force" (kgf) instead of newton to avoid having to multiply by g. If there is no gravitational attraction then the body has no weight but of course it still has its mass. Herein lies the distinction between mass and weight. A body's weight depends upon the gravitational force of attraction whereas its mass does not. A body has the same mass on the Earth as on the moon or in free space. However because the force of gravity is different in these places so is the body's weight. (In free space where there is no gravity the body would be weightless).

9.12 MOMENTUM

This term is used a good deal in everyday speech and most of us have some concept of its meaning. If a body is in motion, the force necessary to change its motion depends upon its velocity as well as its mass. A large rugby forward running towards one has more momentum than a small child in similar circumstances.

In mathematics however, it is necessary to be more precise than this, and by definition:

$$\textbf{Momentum} = \textbf{mass} \times \textbf{velocity}$$

$$\textbf{i.e., Momentum} = \textbf{mv} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad \textbf{VIII}$$

Units: In using this formula, the mass must be in kilogrammes and the velocity in metres/second.

Hence, the unit of momentum in SI units is the "kilogramme-metre per second (kg m/s).

Again:

$$\text{momentum} = mv \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

We already have $F = ma$, i.e., $m = \dfrac{F}{a}$

and also $v = at$, so substituting in (1) we have

$$\text{Mom.} = \frac{F}{a} \times at = Ft$$

$$\textbf{i.e., Momentum} = \textbf{Force} \times \textbf{time}$$

Thus an alternative S.I. unit is Ns.

Also from above

$$\textbf{Force (N)} = \frac{\textbf{Momentum (kg m/s)}}{\textbf{Time (s)}} \quad \ldots\ldots\ldots \quad \textbf{IX}$$

This is a particularly important formula, which was originally expressed in **Newton's Second Law**, viz.

The rate of change of momentum (i.e., change of momentum per second) is proportional to (i.e., with correct units, is equal to) the impressed force, and takes place in the direction of the force.

In one line, this means:

$$\textbf{Average force} = \textbf{change in momentum per second} \quad \ldots \quad \textbf{X}$$

Let us work Worked Example 9 above using this principle.

$40 \text{ km/h} = 11{\cdot}11 \text{ m/s}$

Momentum of train $= mv$

$= 210 \times 1000 \times 11{\cdot}11 \text{ Ns}$

$= 2\,333\,100 \text{ Ns}$

$$\text{Rate of change of momentum} = \frac{\text{momentum change}}{\text{time}}$$

$$= \frac{2333100 - 0}{88} \quad \frac{Ns}{s}$$

$= 26512{\cdot}5 \text{ N}$ (approx. $26{\cdot}51 \text{ kN}$)

which is what we obtained approximately, in our previous solution.

EXERCISE F

1. A train of mass 96 tonnes starts from rest and after 2 min is travelling at 20 km/h. Find the whole tractive force if there is a steady force of resistance of 0·96 tf.
2. Find the momentum of a truck, mass 12 t, travelling at 15 km/h. If the brakes are applied and speed is changed to 12 km/h in 4 seconds, find the average force exerted by the brakes.
3. A shot of mass 120 kg and travelling with a velocity of 1500 m/s enters a bank of earth and is brought to rest in 3·03 seconds. Find the average force exerted by the bank.

9.13 NEWTON'S THIRD LAW

Since reference has already been made to Newton's first and second laws of motion, this may be an appropriate place to mention his *third law*, which states:

To every action there is always an equal and contrary reaction, or as it is sometimes expressed, action and reaction are equal and opposite.

This law is within common experience, and many instances will occur to the reader.

9.14 CONSERVATION OF MOMENTUM

This important theorem states that if two or more bodies collide, then **the total momentum of the system remains unchanged by their mutual reactions.**

In particular, the algebraic sum of the momenta along any chosen direction remains unchanged by their mutual reactions.

This is usually summed up in the words:

momentum before impact = momentum after impact ... **XI**

(in any chosen direction) (in the same chosen direction)

WORKED EXAMPLE 10

A man weighing 84 kg runs at 20 km/h directly after a 5 tonne bus, travelling at 12 km/h and jumps on board. Find the velocity of the bus (with man) immediately after boarding.

20 km/h = 5·556 m/s and 12 km/h = 3·333 m/s

man → bus →

Before impact. Momentum in direction →

(man) (bus)

$= (84 \times 5{\cdot}556) + (5000 \times 3{\cdot}333)$ *Ns* ..... (1)

$= 17132$ *Ns*

After impact. Let their combined velocity be V.

Then momentum $= (84 + 5.10^3)\,V$ ..... (2)

Now from the principle of the Conservation of Momentum

momentum before impact = momentum after impact

(1) = (2)

$17132 = 5084\,V$

$V = 3{\cdot}37$ m/s

WORKED EXAMPLE 11

A pile driver, mass 200 kg, falls from a height of 16 m on to a pile, mass 1 tonne. Find the velocity of the pile immediately after impact.

$$u = 0$$
$$v = ?$$
$$a = 9{\cdot}81 \text{ m/s}^2$$
$$t = ?$$
$$s = 16 \text{ m}$$
$$v^2 = u^2 + 2as$$
$$v^2 = 0 + (2 \times 9{\cdot}81 \times 16)$$
$$v = 17{\cdot}72 \text{ m/s}$$

Therefore momentum before impact $= 200 \times 17{\cdot}72$ *Ns*

$= 3544$ *Ns* ..... (1)

After impact, (pile and driver move as one) at velocity V, so

momentum $= (200 + 1000)\,V$ ..... (2)

(1) = (2) so $V = \dfrac{3544}{1200} = 2{\cdot}95$ m/s approx.

EXERCISE G

1. An object of mass 3 t is travelling at 8 m/s when it collides with a stationary body of mass 9 t. Calculate the velocity with which the two bodies move off together after the collision.
2. A mass of 5 kg travelling with a velocity of 12 m/s collides with a mass of 3 kg travelling at 10 m/s in the same direction. Calculate their combined velocity after collision.
3. Rework question 2 but this time the two masses are initially moving in opposite directions. They still move off together after the collision.

TEST PAPER 9

Motion, Relative Velocity, Force, Momentum

1. A train starts from rest with uniform acceleration. After 70s it is running at 30km/h. It continues at this speed for 20 minutes and is then uniformly retarded to rest in 220m. Find the total distance travelled in kilometres.
2. A test vehicle on a straight track accelerates uniformly from rest to a speed of 40m/s in 8s. It maintains this speed for 12s and then decelerates uniformly to rest in 16s.

   (*a*) Draw a velocity-time diagram to illustrate the motion of the vehicle.

   (*b*) From your diagram find:

   (i) the initial acceleration:

   (ii) the total distance travelled:

   (iii) the distance travelled in the last 8s.

   (SCOTVEC Dec 1989)
3. A shackle is dropped from a point 8·5m above a ship's deck. It falls straight down an open hatch and hits the tank top which is 11·5m below the ship's deck. Ignoring air resistance, calculate:

   (*a*) the speed with which the shackle hits the tank top:

   (*b*) the time taken for the shackle to fall from deck level to the tank top.

   (SCOTVEC Mar 1990)
4. A ship accelerates at a uniform rate. It starts from rest and reaches a speed of 8m/s after 12 minutes. Calculate:

   (*a*) the acceleration of the ship:

   (*b*) the distance travelled:

   (*c*) (i) the further time interval required to increase speed to 10m/s.

   (ii) the additional distance travelled to increase speed to 10m/s.

   (SCOTVEC July 1991)

5. (*a*) A body starts at 3m from datum at time zero and moves with constant velocity to a position 8m from the datum after 10s. Sketch the displacement-time graph and from your sketch find the constant velocity.
(*b*) A body accelerates uniformly from rest to a speed of 45m/s in 10s. It maintains this speed for 20s and then decelerates uniformly to rest.
(i) Sketch a graph of speed against time to illustrate the motion of the body.
(ii) If the total distance travelled is 2025m. Calculate:
(1) the time spent decelerating
(2) the magnitude of the deceleration.
(SCOTVEC Dec 1991)

6. A test vehicle moves along a straight track. The vehicle starts from rest and accelerates uniformly to a speed of 40m/s in a distance of 80m,
(*a*) Calculate the acceleration of the vehicle.
(*b*) Calculate the time taken.
(*c*) If the vehicle continues to accelerate at the same rate, calculate the further time it will take to reach a speed of 180m/s and the further distance it will travel in doing so.
(SCOTVEC Mar 1992)

7. (*a*) Define each of the following:
(i) velocity (ii) acceleration
(*b*) A sports car starting from rest accelerates at $4m/s^2$ for a period of 8s. The speed attained by the end of the acceleration period is maintained for 3 minutes and then the car is brought to rest, with a uniform retardation, in 12s.
(i) Calculate the maximum speed attained.
(ii) Sketch the velocity-time graph to represent the motion of the car.
(iii) Calculate the total distance travelled.
(SCOTVEC July 1993)

8. A ship is sailing North at 16m/s through a current setting 225°T at 4m/s. A passenger throws a quoit with a velocity of 20m/s right across the deck from port to starboard. Find graphically the velocity of the quoit:
(*a*) relative to the sea.
(*b*) relative to the earth.

9. A vessel is steering 045°(T) at 12kn. The position at 1000 hours is 20° N, 30° W. The following ranges and bearings of another ship were observed on the radar display:

Time	*Range*	*True Brg.*
1000	8 miles	075°
1010	5½ miles	072°
1020	3 miles	065°

Show on a plot to a scale of 1 inch to 1 mile the position of both ships at 1020, and state the course and speed of the other vessel.

10. A certain force acting on a mass of 2kg gives it an acceleration of 3m/s². What acceleration would six times this force give to a mass of 400g?
11. A certain force acting on a truck of mass 10 tonnes for 6 seconds gives it a velocity of 10m/s. Find the force.
12. A train of mass 210 tonnes starts from rest with constant acceleration and after 88 seconds is travelling at 64km/h. Find the resultant tractive force applied by the engine.
13. A man on top of a tower 12m high throws a stone vertically upwards with an initial velocity of 30m/s. After what time will it hit the ground and with what velocity?
14. A ship on launching (from ways assumed nearly horizontal) entered the water at a speed of 2·8m/s and was stopped by the drag chains in 35 seconds. Find the average breaking force exerted by the drags if the launching weight of the ship was 9520 tonnes.
15. A mass of 8kg travelling at 16m/s collides end-on with a mass of 5kg travelling at 12m/s in the opposite direction. If the masses coalesce, find their common velocity after impact.
16. Find the momentum of a launch, mass 22 tonnes travelling at 36 knots. The engines are stopped and speed falls uniformly to 18 knots in 44 seconds. Find the average retarding force. (Take 1 knot = 0·515m/s.).

CHAPTER 10

DYNAMICS

Energy, Work, Power, Machines

In the previous chapter we have seen how to define velocity and acceleration, which in turn led us to the definition of **Force**.

We are now going to consider what happens when forces are in motion, that is, are doing **Work**.

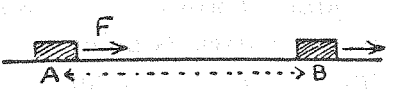
10.1 WORK

In mechanics, work is done when a force moves its point of application and the amount of work done is proportional to the distance moved in the direction of the force. That is,

Work done =

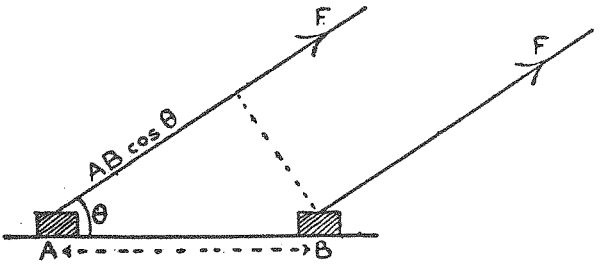
Force × distance moved in the direction of the force . I

For example. (*a*) If a force F moves its point of application a distance AB, then work done $= F \times AB$.

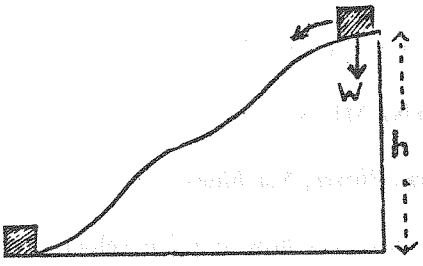


Or again,

(*b*) If a force F moves the mass as shown, then: work done $= F \times AB \cos \theta$, since $AB \cos \theta$ is the distance moved *in the direction of the force.*



(*b*)



(*c*)

Or again,

(*c*) Suppose a mass M slides as shown. The only force acting on it is that due to its own weight, which acts vertically downwards, and is $M \times g$ absolute units of force. Hence regardless of the path followed, the work done is $M \times g \times h$ absolute units of work.

This is true even if the mass falls vertically. The work done on the mass is Mgh.

Conversely, if a mass M is raised through a height h, then the work done is Mgh absolute units.

10.2 UNITS OF WORK

In S.I., the unit of work is the **joule** (J).

Definition:

1 joule of work is done when a force of 1 newton moves its point of application 1 metre in the direction of the force ..... **II**

WORKED EXAMPLE 1

How much work is done in raising 4 tonne of coal from a mine 200 m deep?

$$\begin{aligned}\text{Work done} &= \text{Force} \times \text{distance} \\ &= 4 \times 10^3 \times 9{\cdot}81 \times 200 \text{ J} \\ &= 7{\cdot}848 \text{ MJ}\end{aligned}$$

WORKED EXAMPLE 2

A chain weighing 12 kg/m and 200 m long hangs vertically with a load of 2 t on the end. Find the work done in winding up chain and load.

$$\text{Mass of chain} = 12 \times 200 = 2400 \text{ kg}.$$

The C.G. of this chain is at its mid-point, so in winding it up, the C.G. is raised 100 m.

$$\begin{aligned}\text{Work done on chain} &= \text{Force} \times \text{distance} \\ &= 2400 \times 9{\cdot}81 \times 100 \text{ J} \\ &= 2{\cdot}3544 \text{ MJ} \\ \text{Work done on load} &= \text{Force} \times \text{distance} \\ &= 2 \times 10^3 \times 9{\cdot}81 \times 200 \text{ J} \\ &= 3{\cdot}924 \text{ MJ} \\ \text{Total work done} &= 2{\cdot}3544 \text{ MJ} + 3{\cdot}924 \text{ MJ} \\ &= 6{\cdot}2784 \text{ MJ}\end{aligned}$$

EXERCISE A

1. Find the work done in drawing a load of 2 tonne up a smooth slope 200 m long, rising 1 m for every 20 m along the slope.
2. Find the total work done in heaving up a cable of mass 220 kg/m and length 46 m which is hanging vertically with a 3 tonne load on the end.
3. Find the work done in raising the materials into the form of a vertical stone tower 40 m high, the tower being of uniform density 2000 kg per metre of height.
4. A man weighing 80 kg puts a load of 20 kg on his back and climbs to the top of a ladder 6 m long, inclined at 30° to the vertical. Find the work done.

10.3 ENERGY

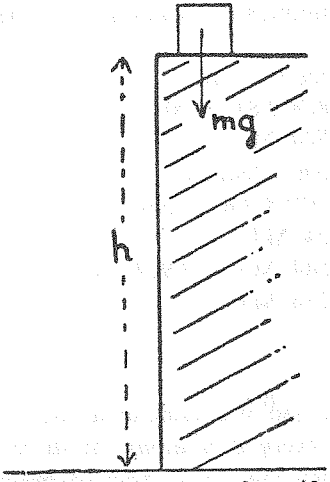
Energy is capacity for doing work. There are many sources of energy, chemical, electrical and so on, but we shall only consider two forms here, namely, **potential energy**, and **kinetic energy**.

One of the most fundamental concepts in mechanics is that of the interchangeability of work and energy. If energy is "capacity for doing work", it follows conversely that if work is done on a body,

then that body will possess energy. Since work and energy are measured in precisely the same units, it follows that

work done = change in energy produced ..... NB

(*a*) **Potential energy:**



This is the energy a body possesses **in virtue of its position.** We have already seen that if a body of mass m is raised to a height h then the work done on it is mgh.

Conversely, therefore, if a body of mass m kg is at a height h m it possesses a potential energy (P.E.) of mgh joule, and if it is released it will yield up this quantity of energy, i.e., do this quantity of work, in falling through a height h.

(*b*) **Kinetic energy**

This is the energy a body possesses **in virtue of its motion.**

If a force F acts on a body of mass m (at rest) which is free to move in a straight line, then the body will be put into a state of motion and its K.E. at any instant will be equal to the work done on it.

If the force F moves through a distance s then

$$\text{work done} = F \times s$$

Now $F = ma$

and $v^2 = u^2 + 2as$

but $u = 0$ giving

$$v^2 = 2as$$

$$\text{i.e., } s = \frac{v^2}{2a}$$

so substituting,

$$\text{work done} = ma \times \frac{v^2}{2a} = \tfrac{1}{2}mv^2$$

$$\textbf{i.e., K.E.} = \tfrac{1}{2}\textbf{m v}^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots \textbf{III}$$

Note that if m in kg, v in m/s, K.E. is in joule.

Note also that the formula for potential energy contains a DISTANCE whilst that for kinetic energy contains a VELOCITY. Thus it may be necessary in a problem to find these quantities first by using the Equations of Motion of section 9.4.

WORKED EXAMPLE 3

An object of mass 2 kg is fired vertically upwards from sea-level. At the instant it reaches a height of 60 m it is travelling at a velocity of 30 m/s. At this instant calculate the object's

(*a*) potential energy.

(*b*) kinetic energy.

(Assume $g = 9{\cdot}81$ m/s^2)

(*a*) P.E. $= mgh$
$= 2 \times 9{\cdot}81 \times 60$ J
$= 1177{\cdot}2$ J ($1{\cdot}1772$ kJ)

(*b*) K.E. $= \tfrac{1}{2}\,mv^2$
$= \tfrac{1}{2} \times 2 \times 30^2$ J
$= 900$ J

WORKED EXAMPLE 4

For the object in Worked Example 3 calculate

(*a*) the maximum height it reaches.

(*b*) the potential energy at maximum height.

(*a*) At maximum height the velocity is zero. Thus using the instant when the object has reached a height of 60 m as an initial reference point we have

$u = 30$ m/s

$v = 0$

$a = -9{\cdot}81$ m/s^2

$t = ?$

$s = ?$

$$v^2 = u^2 + 2as$$
$$0 = 30^2 + 2(-9{\cdot}81)s$$
$$0 = 900 - 19{\cdot}62\,s$$
$$19{\cdot}62\,s = 900$$
$$s = 45{\cdot}87 \text{ m}$$

Since the object has already travelled 60 m the maximum height reached = 60 m + 45·87 m = 105·87 m.

(*b*) P.E. = mgh

$= 2 \times 9{\cdot}81 \times 105{\cdot}87$ J

$= 2077{\cdot}2$ J

Note that the potential energy at maximum height (2077·2 m) is the sum of the potential and kinetic energies calculated in Worked Example 3 (1177·2 J + 900 J).

We shall consider why this is so in the next section.

EXERCISE B

Assume $g = 9{\cdot}81\ \text{m/s}^2$ where necessary.

1. A ship of displacement 6000 tonne has a velocity of 20 km/h. Calculate its kinetic energy.
2. Calculate the P.E. possessed by the water which fills a vertical cylindrical tower 100 m high and 10 m in diameter. (Assume the R.D. of the water is 1, the volume of a cylinder is given by its cross-sectional area, i.e. the area of a circle, $\times$ its height).
3. A ball of mass 0·3 kg is thrown vertically upwards with an initial velocity of 24·53 m/s. Calculate its potential and kinetic energies at the following heights:
   (*a*) zero height.
   (*b*) maximum height.
   (*c*) half way to its maximum height.

10.4 CONSERVATION OF ENERGY

Energy cannot be created, nor can it be destroyed — it can only be converted into some other form of energy, the sum total of energy in the universe remaining constant.

This is a very splendid theory, but in practice energy can be "dissipated", that is, lost as far as our purpose is concerned. For example, Man is using up the natural stores of energy in the form of fuel at a great rate — more have been used in the past 200 years than in all the previous history of the earth. Much of the heat energy thus released is converted into mechanical energy, or electrical energy, but much is lost to the atmosphere. It still remains in the universe somewhere, but is said to be dissipated, or wasted.

Within mechanics Worked Example 3 and 4 give an illustration. All the potential energy and kinetic energy possessed by the object at a height of 60 m has become potential energy at its maximum height. This is true ONLY if we ignore air resistance. In practice there will be air resistance causing some dissipation of energy. Thus at maximum height the potential energy still possessed by the object will actually be less than the work done in getting it to that height. All moving bodies lose energy by doing work against frictional forces such as air resistance.

If an impact between two bodies occurs, energy is dissipated in the form of heat, sound, crushing of materials and so on. Hence, although we can say (Section 9.14)

momentum before = momentum after

we can not carry an energy equation through an impact,

i.e. we can NOT say

energy before = energy after

unless we also include the energy lost in the forms mentioned above.

WORKED EXAMPLE 5

An object of mass 5 kg moving with a velocity of 12 m/s collides with a stationary object of mass 7 kg. Calculate:

(1) the velocity with which they move off together.
(2) the kinetic energy before impact.
(3) the kinetic energy after impact.

(1) momentum before = momentum after

$$5 \times 12 = (5+7)\,v$$
$$60 = 12\,v$$
$$v = 5 \text{ m/s}$$

(2) k.e. before $= \frac{1}{2} mv^2$

$$= \tfrac{1}{2} \times 5 \times 12^2 \text{ J}$$
$$= 360 \text{ J}$$

(3) k.e. after $= \frac{1}{2} mv^2$

$$= \tfrac{1}{2} \times (5+7) \times 5^2 \text{ J}$$
$$= \tfrac{1}{2} \times 12 \times 25 \text{ J}$$
$$= 150 \text{ J}$$

Thus we can see that 210 J of kinetic energy have been transferred to other forms of energy in the collision.

Finally, returning to the example of an object placed some height above the earth's surface, if the object is at rest then all its energy is POTENTIAL. If it is allowed to fall then, IGNORING AIR RESISTANCE, all its potential energy is gradually converted to kinetic energy as it falls. At the instant before hitting the ground all the potential energy will now be converted to kinetic energy whereas halfway down the object's energy is half potential and half kinetic.

10.5 POWER

So far we have talked about the "amount of work done" without considering the time taken to do it. Now, it is obvious that if one machine does a certain amount of work in 10 minutes, whilst another machine does exactly the same amount of work in 5 minutes, then the second machine is doing work **at twice the rate of the first.**

We might say in fact, that the second machine had twice as much **Power** as the first.

Definition:

Power is the rate of doing work, i.e., power is the amount of work done per second IV

$$\text{Power} = \frac{\text{Work done}}{\text{Time taken}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{V}$$

10.6 UNITS OF POWER

In S.I. units:

The unit of power is the watt (W). Larger units may be the kW (kilowatt) and MW (megawatt) and smaller, the mW (milliwatt) and μW (microwatt).

Definition:

1 watt is a rate of working of 1 joule per second.

E.g. a 3-watt motor does 3 joules of work every second; a 2 kW heater takes 2000 joules of electrical energy out of the circuit every second.

WORKED EXAMPLE 6

How long would it take a winch developing 15 kW to raise a load of 3 tonne through 23 m at uniform speed? Assume $g = 9{\cdot}81$ m/s^2.

$$\text{Total work to be done} = mgh$$
$$= 3000 \times 9{\cdot}81 \times 23 \text{ J}$$
$$= 676890 \text{ J}$$
$$\text{power} = \frac{\text{work}}{\text{time}}$$
$$15000 = \frac{676890}{t}$$
$$t = \frac{676890}{15000} \text{ s}$$
$$= 45{\cdot}13 \text{ s}$$

WORKED EXAMPLE 7

Find the power of a locomotive which can draw a train at 60 km/h along the level against a steady resistance of 450 kgf.

$$60 \text{ km/h} = \frac{60 \times 10^3}{60 \times 60} = \frac{100}{6} = \frac{50}{3} \text{ m/s}$$

Resistance $= 450 \times 9{\cdot}81$ N and this is the force which is being overcome by the engine.

$$\text{work done per s} = \text{force} \times \text{distance moved in 1 s}$$
$$\text{so work done per s} = \frac{450 \times 9{\cdot}81 \times 50}{3}$$
$$= 73575 \text{ J}$$

(i.e., this is the power of the loco. in watts)

i.e., power $= 73{\cdot}6$ kW.

EXERCISE C

1. A man weighing 65 kg puts a load of 16 kg on his back and climbs a ladder 12 m long inclined at 70° to the horizontal. How many joules of work are done? If it takes him 55 seconds, at what power is he working?
2. How long would it take a motor developing 60 kW to raise 4 t through 150 m?
3. Find the power of a locomotive that can draw a train at 70 km/h along the level against a resistance of 520 kgf?
4. A winding engine can raise 4 t of coal from a mine 220 m deep in 2·5 minutes. Find the power of the engine?
5. A steam engine cylinder is 32 cm diameter. If the average pressure during a stroke is 62×10^4 N/m^2, and the length of stroke is 48 cm, find the work done in one stroke and the power if it does 88 strokes per minute?
6. A 15 kW pump is delivering water into a tank at an average height of 12·5 m. How many tonne per hour will it deliver?
7. Find the power of a winch that can hoist a 5 t load at a steady speed through 8 m in 9 seconds?
8. A 900 kW locomotive is pulling a 400 tonne train along the level. Resistance is 6 kgf per tonne mass of train. Find the maximum speed it can attain km/h?
9. A man is working a hand windlass to hoist 2 t of coal, 100 kg at a time, out of a bunker 9 m deep. If he works at the rate of 150W, how long will it take him, assuming he lowers down twice as quickly as he hoists up and no time is lost otherwise?
10. A winch is being used to haul a load of 2 t up a smooth slope which rises 1 m for every 10 m along the slope. If the load is hauled up a slope, a distance of 78 m, in 3 mins, find the power of the winch?
11. A chain weighing 15 kg per meter and 84 m long hangs vertically from a winding engine, and carries a load of 0·8 t on the end. If the whole is wound up in 72 seconds, find the power of the engine?

10.7 MACHINES

In mechanics, a machine is any appliance that enables a small force to apply a greater force. The lever, hand windlass (wheel and axle), capstan, tackle are all simple machines.

The force applied is called the **Effort.**

The force overcome is called the **Load**, and for *all* machines.

$$\textbf{Mechanical Advantage (MA)} = \frac{\textbf{Load}}{\textbf{Effort}} \quad \ldots\ldots\ldots \textbf{VI}$$

This formula is quite independent of whatever arrangement of wheels and levers go to make up the machine. However simple, however complicated, if an effort of 10 kgf at one end enables a load of 120 kg, to be moved at the other end, then the mechanical advantage of that machine is 120/10. i.e., 12, at that particular loading. For a "perfect" machine as defined below, this ratio would be the same at all loadings and the effort would always be $^1/_{12}$ of the load.

Of course the load will not move so far, nor so fast, as the effort itself moves.

In a 3-fold purchase for instance, with six parts at the moving block, if we pull off 6 m on the hauling part, the load will only move 1 m. Other examples will occur to the reader.

This brings us to another important definition.

10.8 VELOCITY RATIO

For *all* machines,

$$\textbf{Velocity Ratio (V.R.)} = \frac{\textbf{Distance travelled by effort}}{\textbf{Corresponding distance travelled by load}} \quad \ldots\ldots \textbf{VII}$$

This formula also applies to every machine, however simple or complex.

10.9 EFFICIENCY

Efficiency of a machine is defined as follows

$$\text{Efficiency} = \frac{\text{work out}}{\text{work in}} \times 100\%$$

$$\text{Now work out} = \text{load} \times \text{distance travelled by load}$$

$$\text{and work in} = \text{effort} \times \text{distance travelled by effort}$$

$$\text{Therefore Efficiency} = \frac{\text{load} \times \text{distance travelled by load}}{\text{effort} \times \text{distance travelled by effort}} \times 100\%$$

$$= \text{M.A.} \times \frac{1}{\text{V.R.}} \times 100\%$$

$$\textbf{Efficiency} = \frac{\textbf{M.A}}{\textbf{V.R.}} \times \textbf{100\%} \quad \ldots\ldots\ldots\ldots \textbf{VIII}$$

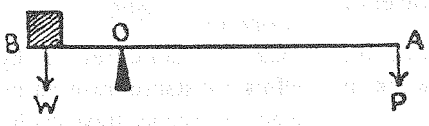
For a perfect machine, i.e. one which is absolutely frictionless, the efficiency would be 100%. This means **M.A.** = **V.R.** for a perfect machine. In practice of course no machine is 100% efficient.

EXERCISE D

1. A certain machine moves a load of 300 N through a distance of 0·2 m by using an effort of 15 N. If the effort moves through a distance of 5 m. Calculate
   (a) the **M.A.**
   (b) the **V.R.**
   (c) the machine's efficiency.
2. A 60% efficient machine causes a load of 144 N to move through a distance of 0·4 m when the effort moves 2·4 m. Calculate
   (a) the **V.R.**
   (b) the **M.A.**
   (c) the effort.
3. A 75% efficient machine has a **M.A.** of 8. An effort of 420 N moving through 0·8 m is required to overcome a certain load. Calculate
   (a) the **V.R.**
   (b) the load distance.
   (c) the load.

10.10 TYPES OF MACHINES

1. Lever. This is about the simplest form of machine. Levers can be used in various ways, but the principle is the same.



Simple lever.

In the example shown, a load W is being moved by an effort P. Taking moments about O and using the principle of moments,

$$W \times BO = P \times AO$$

$$\frac{W}{P} = \frac{AO}{BO}$$

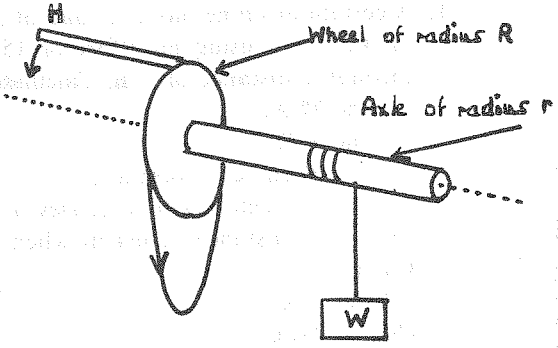
$$\textbf{M.A.} = \frac{\textbf{Arm of effort}}{\textbf{Arm of load}} \quad \ldots\ldots\ldots\ldots\ldots\ldots \textbf{IX}$$

However, most problems on levers are best solved by using the principles of moments, and not by formula XI.

2. Wheel and Axle. This is also sometimes called a hand windlass or a hand winch.

It might be worked by a rope or chain round the wheel, or by a handle H, as shown.

The load is raised by a cord round the axle.



Wheel and axle.

Find the V.R.

Dist. travelled by effort (1 rev) $= 2\pi R$

Dist. travelled by load (1 rev) $= 2\pi r$

$$\textbf{Hence, V.R.} = \frac{2\pi \mathbf{R}}{2\pi \mathbf{r}} = \frac{\mathbf{R}}{\mathbf{r}} \quad \ldots\ldots\ldots\ldots\ldots\ldots \textbf{X}$$

Note. If the thickness of the rope is given then R and r respectively must be taken to the **centre** of the rope.

WORKED EXAMPLE 8

In a hand windlass, the axle is 12 cm diam and the rope round the axle is 2·4 cm diam. The effort is supplied by means of a handle distant 50 cm from the axis. Find the effort needed to raise a load of 120 kg. *(a)* disregarding the rope, *(b)* allowing for the rope.

Refer to the diagram above, which illustrates our problem.

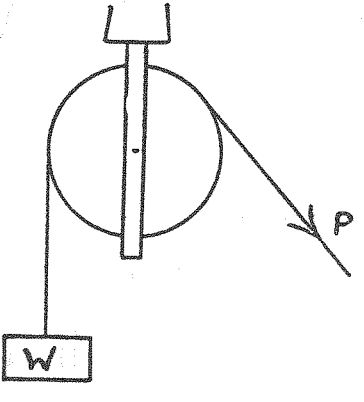
(a) Here, R is 50 cm, r is 6·0 cm, and the V.R. (and also the M.A. if the machine is 100% efficient).

$$= \frac{R}{r} = \frac{50}{6} = \frac{25}{3}$$

So the effort to raise 120 kg $= \dfrac{120 \times 3}{25} = 14{\cdot}4$ kgf.

(b) This time, $R = 50$ cm, $r = 6 + 1{\cdot}2 = 7{\cdot}2$, and M.A. $= \dfrac{50}{7{\cdot}2}$

Thus, effort required $= \dfrac{120 \times 7{\cdot}2}{50} = 17{\cdot}3$ kgf



Single fixed pulley.

3. Pulley. The diagram opposite shows a single fixed pulley. Here the distance travelled by the effort is equal to the distance travelled by the load, i.e., the M.A. = 1, again assuming the system is 100% efficient.

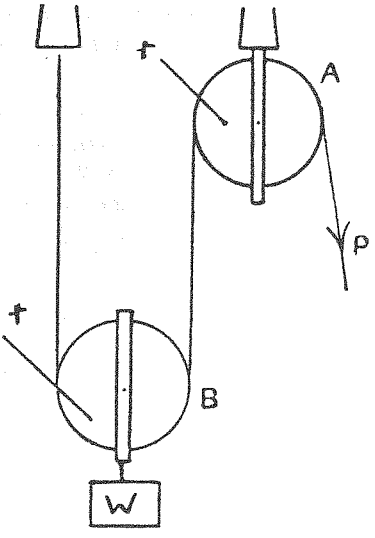
This is only another way of saying that there is *no* mechanical gain. The effort applied has to be equal to the load, and the only purpose of such a pulley is to enable the effort to be applied in a more convenient way.

The diagram opposite shows a better arrangement. Distance travelled by effort (1 rev) $= 2\pi r$.

Distance travelled by load $= 2\pi r/2 = \pi r$, because of course, the length pulled off at A has to be divided between the two cords at B.

Hence, V.R. (and also M.A. for 100% efficiency)

$$= \frac{2\pi r}{\pi r} = 2.$$



Single movable pulley.

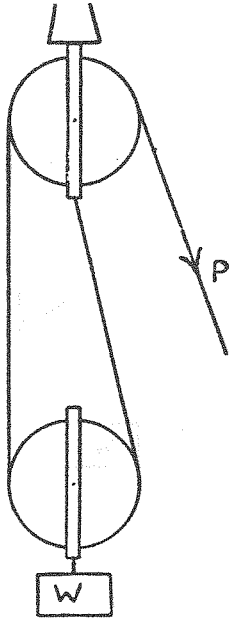
A little consideration will show that the diagram opposite, the gun tackle, is identical with the pulleys shown in the previous diagram.

So the V.R. for a gun tackle used as shown is 2, that is, the number of parts at the moving block. This can be extended to cover any arrangement of pulleys, viz:—the V.R. is equal to the **number of parts at the moving block.**

If all the pulleys are smooth, then the M.A. = the V.R.

On board ship as we know, tackles are anything but smooth, so that the M.A. is, in fact, much less.

However we shall deal with friction later. For the present, we shall assume that all our pulleys are smooth.



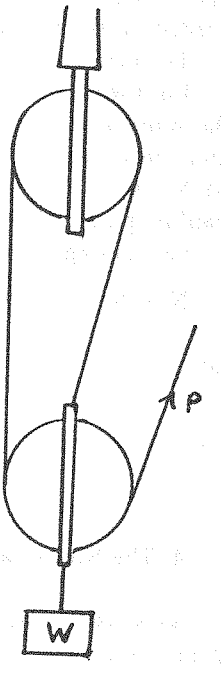
Gun tackle.

10.11 IMPORTANT NOTE

Tackle used to "Advantage" or "Disadvantage".

For the gun tackle in the previous diagram the number of parts at the moving block is 2, and so the V.R. is 2.

In the following diagram we are using the tackle a different way. Here the number of parts at the moving block is 3, and the V.R. is 3. This is called using the tackle "to advantage". whilst previously we are using it "to disadvantage".



Gun tackle used to advantage.

WORKED EXAMPLE 9

A luff tackle is being used to disadvantage to raise a 600 kg lift the hauling part leading to a hand winch, with axle 16 cm diam operated by a handle distant 56 cm from the axis. Find the force needed on the handle, neglecting friction.

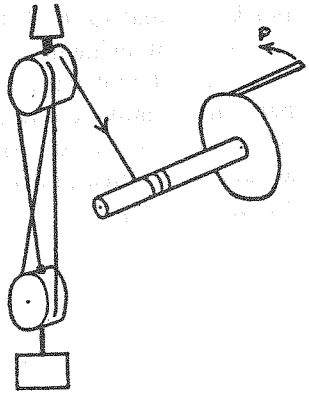
The arrangement is as shown.

For the tackle, there are 3 parts at the moving block, so the V.R. is 3, and since there is no friction, the M.A. is 3 also. Hence, load on hauling part is ⅓ of 600 kg,

i.e., 200 kg.

Now for the winch, V.R. $= \frac{56}{8} = 7$

and therefore effort $P = \frac{200}{7} = 28{\cdot}6$ kgf



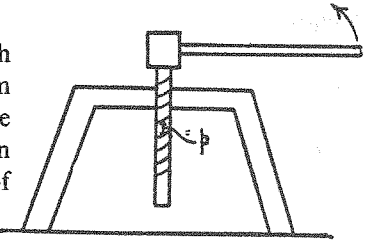
4. The Screw Jack

This useful little machine is in general use everywhere. The principle is illustrated in the figure following.

By the **pitch** of a screw we mean the distance it will travel forward for one revolution of the screw. In a mean single-threaded screw, it is the distance between threads (p in the fig.).

WORKED EXAMPLE 10

Using a screw jack of pitch 0·4 cm, length of lever 42 cm everything smooth, find the load that can be raised by an effort of 10 kgf at the end of the handle.



Find the V.R.

Dist. travelled by effort (1 rev) $= 2\pi \times 42 = 84\pi$ cm

Dist. travelled by load (1 rev) $= 0{\cdot}4$ cm

$$\text{V.R.} = \frac{\text{Dist. travelled by effort}}{\text{Dist. travelled by load}} = \frac{84}{0{\cdot}4} \times \frac{22}{7} = 660$$

Hence, load moved $= 10 \times 660 = 6{\cdot}6$ tonne.

5. Capstan

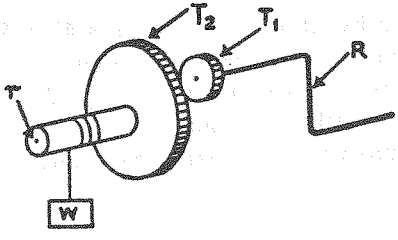
This may also be classed as a simple machine. Like the lever however, it is best dealt with using the principle of moments, as explained in chapter 4, and we will not go into it again.

6. Simple Winch (sometimes known as a "crab").

The priniciple is illustrated in the sketch, and may be found in a simple wire reel, for instance.

To find the V.R.

In one revolution of the handle, distance travelled by the effort $= 2\pi R$.

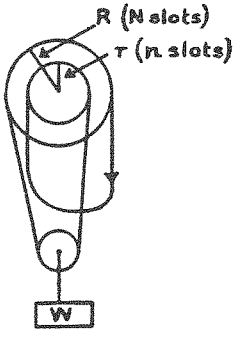


The pinion will also make a complete revolution, and so T_1 teeth will engage on the wheel which will therefore complete T_1/T_2 of a revolution, i.e., the shaft will also complete T_1/T_2 of a revolution, and so the load will travel through a distance T_1/T_2 of $2\pi r = 2\pi r T_1/T_2$.

$$\text{Hence } V.R. = \frac{\text{dist. travelled by effort}}{\text{dist. travelled by load}} = \frac{2\pi R}{2\pi r}\ \frac{T_2}{T_1} = \frac{R}{r} \times \frac{T_2}{T_1} \quad \ldots\ldots\ldots\ldots\ldots \text{N.B.}$$

7. Weston Differential Purchase

This is in fairly common use on board ship. The endless chain engages in slots on the wheels. Note particularly how the chain is rove. Since all slots are the same size, we may use the "slot" as if it were a unit of length.



To find the V.R.

In one revolution of the big wheel, the distance travelled by the effort equals the circumference $= 2\pi R$ *(or* N slots).

At the same time, one of the parts leading to the weight will haul up a distance $2\pi R$ (or N slots), whilst the other part will lower a distance equal to $2\pi r$ (or n slots), and since this difference is shared between two parts, we have,

$$\text{distance moved by load} = \frac{2\pi R - 2\pi r}{2} \text{ or } \frac{N-n}{2} \text{ slots}$$

Hence: (i) V.R., i.e., $\dfrac{\text{dist. travelled by effort}}{\text{dist. travelled by load}}$

$$= \frac{2\pi R \times 2}{2\pi R - 2\pi r} = \frac{2R}{R-r} \quad \ldots\ldots\ldots\ldots\ldots\ldots \textbf{N.B.}$$

$$\text{or} \quad \text{(ii)} = \frac{N \times 2}{N-n} = \frac{2N}{N-n} \quad \ldots\ldots\ldots\ldots\ldots\ldots \textbf{N.B.}$$

Note in all these machines, 1 to 7, do not memorise the velocity ratios. Note how to derive them.

EXERCISE E

(Neglect friction throughout).

1. In a wheel and axle machine, the wheel is 48 cm diam and axle 8 cm diam. Find the effort needed to support a load of 86 kgf. What load would an effort of 22 kgf raise?
2. A man of mass 72 kg is being raised up a shaft at uniform speed by a hand windlass, wheel 64 cm diam, axle 10 cm diam, rope on the axle 2·4 cm diam. Find the force needed on the handle, which is attached to the rim of the wheel?
3. A weightless lever AB is 2·8 m long, has the fulcrum 0·6 from A, a load of 1·5 t at A, a man of mass 80 kg sitting on it 2·4 m from A. What additional force will be needed at B so as just to raise the load?
4. A drum of mass 30 kg is being raised by a hand winch. The circle described by the handle is 84 cm circumference, and while it makes five revolutions the drum rises 1·2 m. What is the smallest force that can be employed?
5. A pair of nutcrackers is 14·4 cm long and when a nut is placed 2·4 cm from the hinge, a force of 15 N will just crack the nut. What mass in kg, if simply placed on top of the nut, would crack it?
6. A hand winch is operated by a handle of radius 36 cm, the shaft being of radius 12 cm. The pinion has 16 teeth and the wheel 56 teeth. Find the load that could be raised by an effort of 12 kgf on the handle?
7. In a Weston purchase, the larger pulley has 12 slots and the smaller 8 slots. What load could be lifted by an effort of 9 kgf?

TEST PAPER 10
Work, Power, Machines

Note: Neglect friction throughout.

1. Find the power of a winch that can haul in 68 m of cable of mass 3·2 kg per metre together with a load of 3 t on the end of the cable, the whole hanging vertically, in 45 seconds.
2. A load of 3 t is raised by a winch through 12 m in 15 s at constant speed. What is the power of the winch?
3. Find the power of a pump that will deliver 44 t of water per hour from a depth of 18 m.
4. A man of mass 72 kg carries a load of 18 kg to the top of a 6 m ladder inclined at 30° to the vertical. Find the total work done. If it takes him 30 s, at what power is he working?
5. A load of 175 kg is being raised by a gun tackle used to disadvantage, rope 2·6 cm diam, the hauling part being led to a hand winch, axle 16 cm diam, wheel 80 cm diam. Find the force necessary on the handle, which is attached to the rim of the wheel, in order to raise the load at a steady speed.
6. A 200 kW locomotive is drawing a train along the level, the total mass, train and locomotive, being 215 t. If the force of resistance is 7 kgf per tonne mass of train, find the maximum speed attainable. (*Note*: the only work done is against the resistance).
7. A capstan is being used to haul a load of 1·5 t up a smooth inclined plane which rises 1 m for every 10 m along the slope. If the load is hauled up the full length of the slope, 34 m, in 4 mins, find the work done and the power developed by the capstan.
8. A truck of mass 4 t arrives at the foot of an incline of 1 in 81 along the slope at 40 km/h. How far will it run up the slope? How long will it take?
9. A launch of mass 3 t is proceeding at 22 km/h when the engine is stopped and she comes to rest in 260 seconds. What is the average force of resistance of the water? What work is done on the launch?

CHAPTER 11

DYNAMICS

Friction and Efficiency, Machines

11.1 FRICTION

In general, the nature of friction is well known. Whenever a body moves over another body its motion is opposed by a resistance due to the roughness of the two surfaces. This resisting force is called the **force of friction**, or often just **friction**.

However smooth the surfaces may be, such as two highly polished steel surfaces, there is always some friction between them. Thus, although hitherto we have dealt with machines that were assumed to be "smooth", in real life there is no such thing as a perfectly smooth machine. There is always some force of friction to be overcome, and this force of friction is overcome by the the effort driving the machine.

Friction thus represents just so-much **wasted effort**, and whether the motive power is electrical, steam, or simply manpower, some of the effort has to be used in overcoming the friction in the machine itself.

The greater this wasted effort is, the less efficient the machine is said to be. In a very efficient machine, the friction (wasted effort) is small.

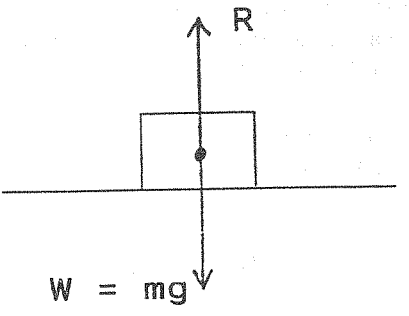
In practice, these somewhat generalised notions have to be subjected to a more rigorous mathematical treatment.

11.2 COEFFICIENT OF STATIC FRICTION

Consider a mass m at rest on a table top. It is in equilibrium under the action of TWO forces:-

(1) Its weight $W = mg$ acting vertically downwards.

(2) a reaction R vertically upwards by the table top on the mass. R is called the NORMAL REACTION.



Since the mass is in equilibrium $R = W = mg$.

Suppose we now attempt to push the mass across the table top using a small force P. The mass may not move. This is because there is an equal and opposite force $f = P$ due to friction. As we gradually increase P so will f but eventually the mass will begin to move. Thus there is a limit to the frictional force f which can oppose the motion. This is called LIMITING FRICTION and it is found that this value of f, call it F, obeys the relationship:

F is directly proportional to R

$F \alpha R$

$F = \text{constant} \times R$

The constant is given the name COEFFICIENT OF STATIC FRICTION and is denoted by μ (pronounced "mu").

$$\mathbf{F = \mu R} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad \mathbf{I}$$

The value of μ depends on the nature of the two surfaces in contact. It takes on values between 0 (two completely smooth surfaces) and 1. For example for wood on wood it has a value of 0·3 to 0·8.

11.3 COEFFICIENT OF DYNAMIC FRICTION

Suppose the mass above is subjected to a force which overcomes the limiting frictional force and causes the mass to continue in motion. As long as the mass is moving there will be an opposing frictional force. However its value is less than the limiting frictional force. Thus, for example, if to make the mass start to move we have to overcome a limiting frictional force of 25 N then the frictional present whilst it is moving will be less than 25 N, say 20 N.

The relationship $F = \mu R$ still holds but F, and therefore μ, will have different values to the static case. In this case μ is the COEFFICIENT OF DYNAMIC FRICTION.

WORKED EXAMPLE 1

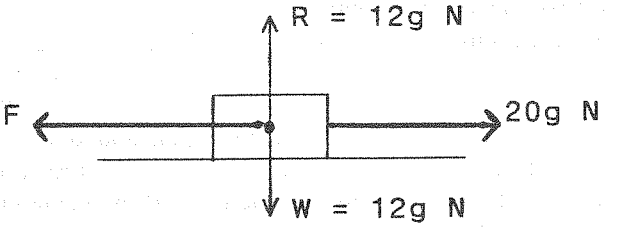
A body of mass 3 kg rests on a surface with which it has a coefficient of static friction of 0·4. Calculate the force needed to just cause the body to move.

$$\text{Weight of body} = 3 \times 9{\cdot}81 \text{ N}$$
$$= 29{\cdot}43 \text{ N}$$
$$\text{Therefore normal reaction} = 29{\cdot}43 \text{ N}$$
$$F = \mu R$$
$$= 0{\cdot}4 \times 29{\cdot}43 \text{ N}$$
$$= 11{\cdot}772 \text{ N}$$

Thus an equal and opposite force to this limiting friction will be needed to just cause the body to move i.e. a force of 11·772 N is required.

WORKED EXAMPLE 2

A 12 kg mass rests on a rough plane of coefficient of dynamic friction 0·4 and is pulled by a steady horizontal force of 20 kgf. Calculate the acceleration of the mass.



$$F = \mu R$$
$$= 0{\cdot}4 \times 12 \times 9{\cdot}81 \text{ N}$$
$$= 47{\cdot}088 \text{ N}$$

Hence resultant force causing motion

$$= (20 \times 9{\cdot}81) - 47{\cdot}088 \text{ N}$$
$$= 149{\cdot}112 \text{ N}$$
$$F = ma$$
$$149{\cdot}112 = 12a$$
$$a = 12{\cdot}426 \text{ m/s}^2$$

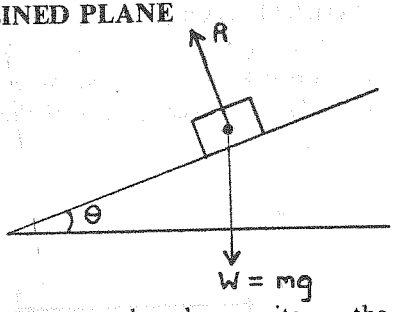
EXERCISE A

1. A horizontal force of 60 N is just sufficient to overcome the limiting frictional force between a crate of mass 30 kg and a deck. Calculate the coefficient of static friction.
2. An object of weight 160 N is pulled along a rough surface by a force of 250 N. If the coefficient of dynamic friction is 0·5 calculate the object's acceleration.

11.4 FRICTION ON AN INCLINED PLANE

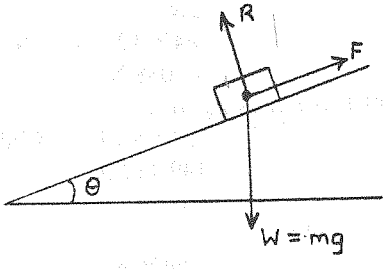
Consider a mass m at rest on an inclined plane but on the point of slipping down the plane.

At first glance it appears that there are only two forces acting.

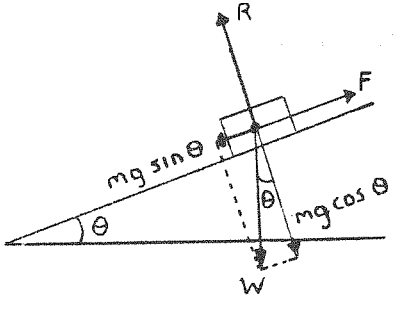


However these two forces are not equal and opposite — the mass cannot be in equilibrium due to these two forces alone.

There is, of course, a force of friction UP the plane preventing the mass from slipping down the plane.



For equilibrium the resultant of these three forces must be zero. To analyse the situation further let us first resolve the weight of the mass into two components at right angles to each other:— one parallel to the plane and one at right angles to the plane. Study carefully the following diagram, especially how the geometry of right-angled triangles is used to obtain the second θ.



It is the component down the plane which would cause slipping and on the point of slipping is exactly balanced by F.

From the diagram above we can see that:

$$\mathbf{R = mg.cos\ \theta} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \mathbf{II}$$

$$\mathbf{F = mg.sin\ \theta} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \mathbf{III}$$

Now $F = \mu R$ and therefore substituting into this equation from II and III we have

$$mg.\sin\theta = \mu.mg.\cos\theta$$

$$\frac{mg.\sin\theta}{mg.\cos\theta} = \mu$$

$$\mu = \frac{\sin\theta}{\cos\theta}$$

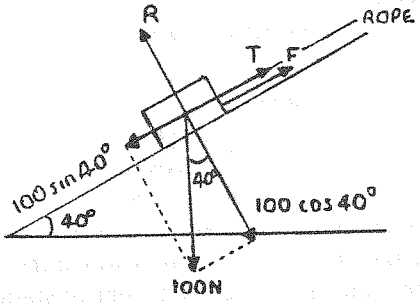
$$\mu = \tan\theta$$

i.e. if a plane is gradually inclined from a horizontal position the tangent of the angle to the horizontal at which the mass just begins to slip down the plane gives the coefficient of static friction.

A similar analysis to the above is used to solve inclined plane problems such as those below.

WORKED EXAMPLE 3

A crate of weight 100 N is placed on a plane inclined at 40° to the horizontal. On its own the crate would slip down the plane. Therefore a rope is used as shown on the diagram below to hold the crate in place. Given that the coefficient of friction is 0·6 calculate the tension in the rope.



T is the tension in the rope.

Now $\mu = \tan\theta$ does NOT apply in this case because the plane is inclined at an angle greater than that at which would just cause slipping.

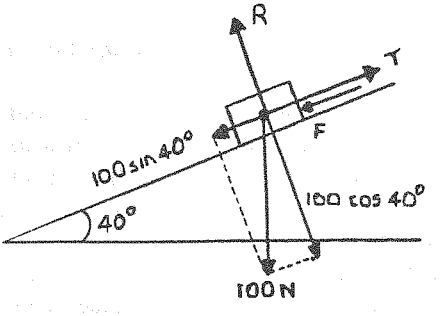
However we can say that because the crate is in equilibrium forces down the plane must equal forces up the plane.

$$T + F = 100.\sin 40°$$
$$T + \mu R = 64{\cdot}2788$$
$$T + 0{\cdot}6.100.\cos 40° = 64{\cdot}2788$$
$$T + 45{\cdot}9627 = 64{\cdot}2788$$
$$T = 18{\cdot}32\ \text{N}$$

WORKED EXAMPLE 4

In worked example 3 above calculate the tension in the rope if it is being used to pull the crate up the plane with constant velocity. In this case assume $\mu = 0{\cdot}5$.

In this case, since the frictional force will oppose the motion, F will act down the plane.



Since the crate has no acceleration we can say:

Forces up the plane = Forces down the plane

i.e. $T = F + 100.\sin 40°$

$= \mu R + 64{\cdot}2788$

$= 0{\cdot}5 \times 100.\cos 40° + 64{\cdot}2788$

$= 38{\cdot}3022 + 64{\cdot}2788$

$= 102{\cdot}6$ N

EXERCISE B

1. A wooden box of mass 120 kg rests on a horizontal deck, the coefficient of static friction between the box and deck being 0·3.
   (a) To what angle to the horizontal can the deck tilt before the box begins to slide?
   (b) If the deck tilts to 20° to the horizontal what force parallel to the deck would be required to prevent the box sliding?
2. A lifeboat is held on an inclined ramp by a wire parallel to the ramp. The coefficient of dynamic friction between the boat and ramp is 0·2. If the ramp is inclined at 15° to the horizontal calculate the tension in the wire when
   (a) the boat slips down the ramp at constant velocity.
   (b) the boat is hauled up the ramp at constant velocity.

11.5 MECHANICAL ADVANTAGE

It was stated in section 10.7 that for all machines.

$$\text{M.A.} = \frac{\text{Load}}{\text{Effort}} \text{ and}$$

it is only necessary to confirm here that *this applies whether there is friction or not.*

Whatever the nature of the machine, if we are told, for instance, that "a pull of 20 N on the hauling part moves a load of 360 N", then the actual Mechanical Advantage of that machine is 18.

This means that under those conditions of loading, the actual effort required will be $^1/_{18}$th of the load.

The theoretical value of the mechanical advantage of any machine is always higher than the actual value. This is because of friction. So we now have yet another point of view. We have already spoken of friction as "so much wasted effort". We can also regard it as so much **lost load.**

It is the ratio between these quantities at any given loading which enables us to define the efficiency of a machine at that loading, and we will now consider this a little more carefully.

11.6 EFFICIENCY OF A MACHINE

As we have seen in section 10.9, for a *perfect* machine the work done by the effort is equal to the work done on the load, i.e.,

work got out = work put in (for a perfect machine).

However, machines are not perfect. There is always a loss due to friction.

Again in section 10.9 we saw that

$$\text{Efficiency} = \frac{\text{M.A.}}{\text{V.R.}} \times 100\% \quad \ldots\ldots\ldots\ldots\ldots \text{IV}$$

An alternative definition of efficiency is

$$\text{Efficiency} = \frac{\text{actual M.A.}}{\text{theoretical M.A.}} \times 100\% \quad \ldots\ldots \text{V}$$

11.7 FURTHER NOTE ON POWER

We have already defined power as "the rate of doing work", or the amount of work done per second, and a 1 kW machine as one that does 1000 joules of work in 1 second.

However, from what we have said above it is clear that some of the power developed (say) in the cylinders of an engine has to be used in overcoming the force of friction. Two definitions of power have therefore to be understood.

Indicated Power

This is the actual power developed by the engine, by the steam in the case of a steam engine, by the combustion of petrol in an internal combustion engine, and so on. It is a theoretical value arrived at by calculations based on the design of the engine, and it is the total power available.

When an engine is in service, the total power available may be checked by measuring the mean effective pressure within the cylinder(s). This is done by an "indicator card", leading naturally to the term "indicated power".

All the indicated power is not available for useful purposes, however. Some of it has to be used to overcome friction in the engine itself.

Effective Power

The quantity

Indicated Power — Power lost in overcoming friction

is sometimes called the Effective Power, or Brake Power, or Shaft Power, of the engine.

The term "brake power" is usually reserved for reciprocating engines, and is derived from the brake or dynamometer used in laboratory or workshop to determine the actual power output, or Brake Power.

The term "shaft power" is usually reserved for turbines. It is not possible to measure the indicated power on a turbine and the power output is measured by a torsionmeter mounted on the shaft of the installed turbines — the Shaft Power.

The brake power is the actual power output of the engine, and the ratio of these two quantities, brake, and indicated, powers gives us yet another way of defining efficiency, thus:

$$\textbf{Efficiency} = \frac{\textbf{Brake power}}{\textbf{Indicated power}} \times \textbf{100\%} \quad \ldots\ldots \textbf{VI}$$

WORKED EXAMPLE 5

A pump delivers 2 t of fresh water to a height of 22 m every minute. If its efficiency is 80% what power would be required to drive it?

Weight of water $= 2 \times 1000 \times 9{\cdot}81$ N $= 19620$ N

Work done by pump $= 19620 \times 22$ J $= 431640$ J

Power used (Brake power) $= \frac{431640 \text{ W}}{60} = 7194$ W

$$\text{Efficiency} = \frac{\text{Brake power}}{\text{Indicated power}} \times 100\%$$

$$80 = \frac{7194}{x} \times 100$$

$$x = \frac{7194}{80} \times 100 = 8992{\cdot}5 \text{ W}$$

WORKED EXAMPLE 6

A dynamo has an output of 15 kW but the engine driving it has a brake power of 25 kW. Find the efficiency of the dynamo.

The power input to the dynamo is the power output from the engine i.e. the dynamo's INDICATED power is the engine's BRAKE power.

For the dynamo

$$\text{Efficiency} = \frac{\text{Brake power}}{\text{Indicated power}} \times 100\%$$

$$= \frac{15}{25} \times 100$$

$$= 60\%$$

WORKED EXAMPLE 7

A winch of 28 kW (indicated) raises a load of 3 t at a steady speed to a height of 11 m in 20 s. Find the efficiency of the winch.

Weight of load $= 3 \times 1000 \times 9{\cdot}81 = 29430$ N

Work done on load $= 29430 \times 11$ J $= 323730$ J

Power required (Brake power) $= \frac{323730 \text{ W}}{20} = 16186{\cdot}5$ W

$$\text{Efficiency} = \frac{\text{Brake power}}{\text{Indicated power}} \times 100\%$$

$$= \frac{16186{\cdot}5}{28000} \times 100\% = 57{\cdot}81\%$$

EXERCISE C

1. What is the effective power of a winch that can raise 7 t at a steady speed through 6 m in 22 seconds? If the winch has an efficiency of 65%, what is its indicated power?
2. A dynamo is found to have an effective power of 7 kW and is driven by an engine of indicated power 16 kW and efficiency 60%. Find the efficiency of the dynamo.

11.8 MECHANICAL ADVANTAGE OF TACKLES

The tackles in use on board ship are not smooth, and allowance must be made for friction. It will still be true of course, that the **V.R. is equal to the number of parts at the moving block** (section 10.10) but the actual mechanical advantage will be much less than the V.R.

Practical Method

The practical method which is in general use is as follows. Although not mathematically accurate, it has been found by experience to give good results for practical work on board ship.

(*a*) **Increase the load by $^1/_{10}$th of itself for every sheave the rope runs over. Treat this as the total load to be overcome.**

(*b*) **Divide this total load by the V.R., i.e. by the number of parts at the moving block. The result will be the effort required.**

WORKED EXAMPLE 8

Using a 3-fold purchase to disadvantage, find the effort needed to raise a load of 4 tonne.

There are 6 sheaves, so,

$$\text{Total load to be overcome} = 4 + \frac{6}{10} \text{ of } 4 = 4 + 2{\cdot}4$$
$$= 6{\cdot}4 \text{ t}$$

Also, there are 6 parts at the moving block, so

$$\text{effort} = \frac{6{\cdot}4}{6} = 1{\cdot}07 \text{ tf}$$

or, effort in the hauling part $= 1070 \times 9{\cdot}81$ N
$= 10496{\cdot}7$ N

Theoretical Method

The formula:

$$\textbf{Efficiency} = \frac{\textbf{M.A.}}{\textbf{V.R.}} \times \textbf{100\%}$$

can be applied to a tackle just as to any other machine. The only drawback in practice is that the necessary information is seldom available. However, its use may be required from time to time, and one or two examples should help to fix ideas.

WORKED EXAMPLE 9

Using a 2-fold purchase to disadvantage, an effort of 40 kgf was necessary in order to move a load of 112 kgf. Find the efficiency of the tackle.

Remember that regardless of type of machine or any other factors, if 40 moves 112, then the M.A. $= 112/40$, also the V.R. of this tackle = no. of parts at the moving block, i.e. V.R. $= 4$.

$$\text{Efficiency} = \frac{\text{M.A.}}{\text{V.R.}} \times 100\%$$
$$= \frac{112}{40 \times 4} \times 100\%$$
$$= 70\%$$

EXERCISE D

1. Using a luff tackle to advantage what effort is needed to raise a load of 2·6 t?
2. Using a 2-fold purchase to disadvantage, find the effort needed in the hauling part to raise 3·2 t.
3. Find the pull in the hauling part of a 3-fold purchase used to disadvantage to raise 8 t.
4. Using a certain tackle, of V.R. 7, an effort of 416 kgf was necessary in order to move a load of 2·24 t. Find the efficiency of the tackle.
5. A ship's propeller weighing 8 t is being lifted by a 4-fold purchase used to disadvantage. What is the tension in the hauling part?
6. If a luff tackle used to advantage is attached to the hauling part in the previous problem, what effort is needed in the hauling part of the luff tackle to raise the propeller?
7. A load of 4 t is being raised with a 2-fold purchase used to advantage, the effort necessary being 1 tf. Find the efficiency of the tackle.

11.9 STRENGTH OF ROPE, WIRE, CHAIN

In addition to finding the actual pull or tension needed in the hauling part it is sometimes necessary to calculate the size of rope, wire or chain that would be necessary to stand that tension with safety.

The breaking strengths depend very much on the quality, and also on the construction. Manufacturers will tell us that they can make a wire rope of a given size to lift 5 tonne, or 10 tonne — it all depends on what is wanted.

Hence it is not our intention here to go into the strength of ropes, a subject more appropriate to a manual on seamanship.

However it should be remembered that a FACTOR OF SAFETY of at least 2, and preferably 3, should be allowed even for an occasional lift. A factor of safety of something like 6 should be used for continuous working. Bearing this in mind we can arrive at a Safe Working Load (S.W.L) for the given rope.

TEST PAPER 11
Friction, Machines

1. A horizontal force of 6kgf is just sufficient to move a mass of 30kg along a horizontal table. What is the coefficient of friction? To what angle of inclination must the table be tilted so that the mass will just slide?

2. A block of teak, mass, 14kg, rests on a teak plank 3·2m long. If $\mu = 0{\cdot}45$, to what height must one end of the plank be raised so that slipping is just about to commence. With the plank in this position, calculate the work done in pushing the block the full length of the plank up the incline.

3. A body of mass 200kg rests on a plane surface for which the coefficient of friction is 0·2.
   (*a*) If the surface is steadily tilted, calculate to what angle to the horizontal it can be raised before the body begins to slide.
   (*b*) If the surface is tilted to an angle of 30° to the horizontal, calculate the minimum force parallel to the slope which must be applied to stop the body sliding down the slope.
   (SCOTVEC Dec 1989)

4. A boat in a launch cradle rests on a ramp at an angle of 25° to the horizontal. The combined mass of the boat and cradle is 4·2 tonnes. The coefficient of friction between the cradle and the ramp is 0·15. A wire from the cradle leads parallel to the same ramp in the upward direction. Calculate the tension in the wire when:
   (*a*) lowering the boat and the cradle down the ramp at constant speed.
   (*b*) hauling the boat and cradle up the ramp at constant speed.
   (SCOTVEC Dec 1990)

5. (*a*) Define EACH of the following:
   (i) the coefficient of static friction.
   (ii) the coefficient of dynamic friction.
   (*b*) A ship of mass 7000 tonnes lies on a slipway inclined at an angle of 10° to the horizontal. The coefficient of friction for the stationary ship and slipway is 0·25, which falls to 0·2 on motion.

(i) Calculate the minimum force P required to start the ship moving down the slipway.

(ii) Assume the force P in Q.5 (*b*) (i) is maintained and the ship accelerates uniformly, calculate the force causing the acceleration.

(iii) Calculate the acceleration of the ship.

(SCOTVEC Mar 1993)

6. A load of 400kg is being raised by a pair of double blocks used to disadvantage. Making the usual allowance for friction, what is the tension in the hauling part.

7. A mass of 28kg rests on a rough table, $\mu = 0{\cdot}6$, and is pulled by a steady force of 380N. Find how far the mass will move from rest in 10 seconds.

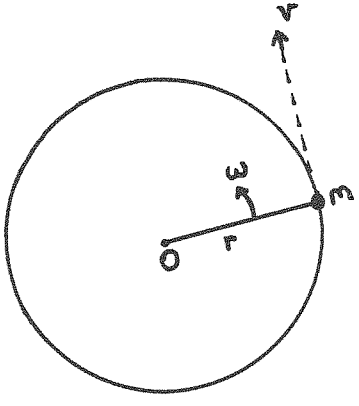
CHAPTER 12

DYNAMICS

Motion in a circle. Simple harmonic motion

This is a very wide subject and it will only be possible to deal with it briefly in this chapter

12.1 ANGULAR VELOCITY



Motion in a horizontal circle.

Consider the particle of mass m which is moving in a circular horizontal path with **constant tangential velocity** v. It is clear that the radius vector joining the particle to the centre O is also sweeping out angles at a constant rate. This leads to an alternative means of defining the rotation of the particle in terms of **angular velocity**, which is the **speed of rotation in radians per second.** (Symbol, Greek letter omega, ω).

Hence, we can think of the particle as having

either a tangential velocity of v metres per second (m/s)

or an angular velocity of ω radians per second (rad/s)

Elementary trigonometry reminds us that

length of arc of a circle = radius × radian measure of angle

i.e., arc length $= r\theta$ (in general)

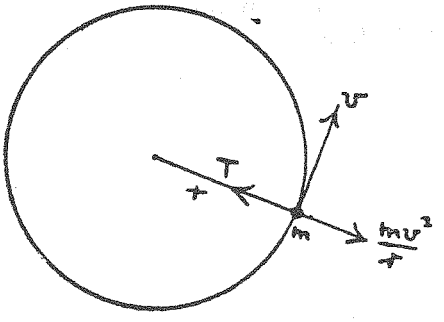
So, in our case, if the particle sweeps out ω radians in 1 second, then the equivalent length of arc traversed in 1 s $= r \times \omega$.

But, of course, "length of arc traversed in 1 s" is just v. Hence,

$$v = \omega r \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{I}$$

12.2 HORIZONTAL CIRCLE

It is within common experience that when a particle travels in a circular path a force is required to constrain it in that path. Examples will occur to the reader, such as a small mass m whirled round at the end of a piece of cord. This force is provided by T, the tension in the cord.



The particle is trying to fly **outwards**, and this outward force is called the **centrifugal force**. A force equal and opposite and acting **inwards** is called the **centripetal force**.

It can be shown in more advanced works that the force acting on such a particle in order to constrain it in a circular path is **always directed towards the centre** and is given by

$$\mathbf{F\ (newtons) = \frac{mv^2}{r}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \mathbf{II}$$

where m, v, r are in SI units.

Of course, the outward, or centrifugal force is equal in magnitude but opposite in direction. This force, mv^2/r is shown.

Also, since $v = \omega r$, we may write, by substituting in II

$$\mathbf{F = m\omega^2 r} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \mathbf{IIa}$$

Note that T is a force (newtons) and force = mass $\times$ accel. Hence, in the expression on the R.H.S. of equation II, the acceleration on the particle is v^2/r or $\omega^2 r$.

WORKED EXAMPLE 1

A sphere of mass 2 kg rests on a smooth horizontal table and is connected by a chord through a hole in the table to a mass of 5 kg hanging vertically. Find the speed at which the sphere on the table must travel in a circle of 1 m radius to preserve equilibrium.

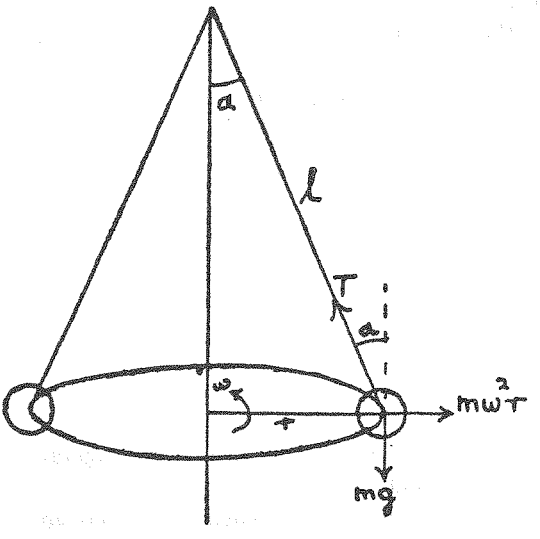
The tension in the chord due to the 5 kg mass hanging vertically is $5g$ N and the centrifugal force provided by rotation of the 2 kg mass must, therefore, equal $5g$ N. Hence

$$F = \frac{mv^2}{r}$$

$$5g = \frac{2 \times v^2}{1}, \qquad v^2 = \frac{5 \times 9{\cdot}81}{2}$$

and $v = 4{\cdot}95$ m/s.

12.3 APPLICATION TO GOVERNORS



Conical governor.

This device is illustrated. Its purpose is to "govern" or regulate, the speed of machinery, acting as a kind of conical flywheel. The position of the spheres can also be made to operate a slide or valve, thus controlling the power supply to the machine. The heavy spheres of mass m are secured to a central rotating shaft by rigid rods of length l. As the shaft rotates, the spheres fly out. At any instant, considering one sphere only it is in dynamical equilibrium, that is, it is in **motion under three balanced forces**, namely the tension T in the rod, its weight mg and the centrifugal force $m\omega^2 r$, as shown in the diagram.

Hence, resolving vertically,

$$T \cos \alpha = mg \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

and resolving horizontally

$$T \sin \alpha = m\omega^2 r \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

and also, $r = l \sin \alpha$, so in (2)

$$T = m\omega^2 l \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

Since m and l are constant for any given machine, the above three equations enable us to deal with any problem.

WORKED EXAMPLE 2

A pair of conical governor balls, each of mass 25 kg, is rotating at 42 revs per min. Find the tension in each tie rod, which are 3 m long.

Since 1 complete revolution $= 2\pi$ radians, we have

$$42 \text{ revs/min} = 42 \times 2\pi \text{ radians/min}$$

$$\therefore \quad \omega = \frac{42 \times 2\pi}{60} \text{ rads/s}$$

$$= 4{\cdot}398 \text{ rads/s}$$

$$T = m\omega^2 l$$

$$= 25 \times 4{\cdot}398^2 \times 3 \text{ N}$$

$$= 1451 \text{ N}$$

EXERCISE A

1. A wheel revolves 180 times per minute. Find its angular velocity in radians per second.
2. Find the revolutions per minute of a flywheel which has an angular velocity of 30 radians per second.
3. A railway truck weighs 2000 kg and travels at 60 km/h round a bend whose radius of curvature is 1200 m. Find the centrifugal force on the truck.
4. A heavy particle of mass 45 g attached to a chord 20 cm in length travels round the circumference of a horizontal circle with a uniform velocity of 120 cm/s. Find the tension in the chord.
5. Deduce the fundamental equations for a pair of heavy spheres arranged as a conical governor.

   If each sphere weighs 18 kg, the length of the rod is 1·5 m, and the governor is doing 70 revs per min, find the tension in the rod and the angle made by the rod with the vertical.
6. A horizontal flywheel 12 cm in radius has a mass of 1 g attached to the rim. Find the centrifugal force in Newton on the mass when the wheel is doing 1200 r.p.m.

12.4 SIMPLE HARMONIC MOTION (S.H.M.)

Definition: A particle is said to perform S.H.M. about a fixed point, called the origin, when the acceleration of the particle is always proportional to the distance of the particle from the origin, and always acts towards the origin.

Like many other definitions, this needs careful study.



Simple harmonic motion.

Firstly, note that S.H.M. has to do with oscillations — the oscillation of some particle P about some fixed point, or origin, O.

The distance of P from O at any instant will be called its **displacement** (x).

Let us imagine that P is moving to the right. The acceleration acting on it is *towards the origin* — i.e., it is slowing P down. Moreover, the further P moves over to the right, the greater the acceleration becomes (*it is proportional to the distance from the origin*). The limiting point is reached when P becomes *stationary*, at A.

At that instant,

its displacement is a maximum,
its acceleration is a maximum,
its velocity is zero.

It now begins to move under its acceleration towards O, with increasing velocity, for although the acceleration is diminishing (because the displacement is diminishing) it still continues to act towards O right up to the very instant when the particle is at O. At that instant,

its displacement is zero,
its acceleration is zero,
its velocity is a maximum.

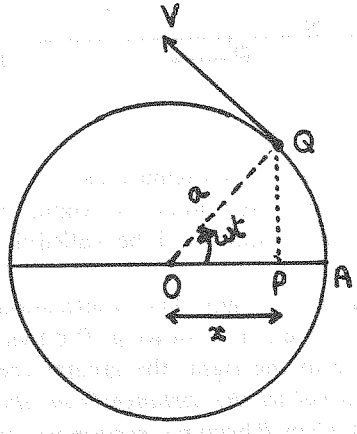
It continues moving over to the left, slowing down under increasing retardation, until finally, at B, the situation is exactly as at A.

The whole operation is now repeated, back to A, out to B, back to A, often many times a second. A tuning fork for example, for middle C, performs 256 oscillations per second.

The time taken for one complete oscillation from A back to A is called the **period**, symbol T (seconds).

The maximum displacement either side, OA or OB (being equal) is called the **amplitude** of the motion, symbol a.

12.5 EQUATIONS OF S.H.M.



S.H.M. of point P.

It is convenient to illustrate S.H.M. by the sketch as shown opposite.

If Q is a particle moving round the circumference of the circle with *constant angular velocity* ω (or constant tangential velocity V), **then P, the foot of the perpendicular, performs S.H.M. about O.**

Let the circle be of radius a i.e. the maximum displacement of P will be a. Also let the displacement of P at any instant t be x.

Note that the circle is simply to help us. It is P that is performing the S.H.M.

Now if Q starts from A with angular velocity ω radians/seconds, then after any time t second the angle swept out will be ωt, as shown in the diagram above, and with the notation already given, the displacement at that instant is given by

$$\mathbf{x = a \cos \omega t} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \textbf{III}$$

This gives us the displacement after any time t.

(Note that ωt is just an angle, in radians).

To find the velocity of P (v).

Resolving V horizontally,

$v = V \sin \omega t$, and $V = \omega a$,

$$\mathbf{so\ v = a\omega \sin \omega t} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \textbf{IV}$$

This gives the velocity of P after any time t.

To find the acceleration of P (f) — using f for acceleration to avoid confusion with a for amplitude.

The accel. on Q, acting towards the centre, is, from IIa, $a\omega^2$, and resolving horizontally, accel. on P is given by

$$\mathbf{f = a\omega^2 \cos \omega t} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \mathbf{V}$$

This gives the accel. of P after any time t.

Finally, note that the time of one complete oscillation is given by

$$\mathbf{T} = \frac{2\pi \text{ (radians in a circle)}}{\omega \text{ (radians/second)}} \quad \ldots\ldots\ldots\ldots\ldots \mathbf{VI}$$

These are the fundamental equations of S.H.M.

In using them note that displacement to the right should be positive, to the left negative. Also, the correct signs for sin, cos, etc., of angles up to 360° must be used.

12.6 ALTERNATIVE DERIVATION

They may also be derived quite easily by the use of calculus.

Let us start with the basic assumption that the distance OP, i.e., x, from the origin after any time t, is given from the triangle OPQ in which,

$$\frac{OP}{OQ} = \cos \omega t, \text{ i.e.,}$$

$$x = a \cos \omega t \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

an equation in which, remember, a and ω are constants.

Since $v = dx/dt$, we have, differentiating

$$v = a. - \sin \omega t. \omega$$

$$v = -a\omega \sin \omega t \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

(the minus sign just means that v is in the reverse sense to x, i.e., we measured x to the right, and v is in the direction towards the left).

And again, since acceleration $f = dv/dt$, we have from (2)

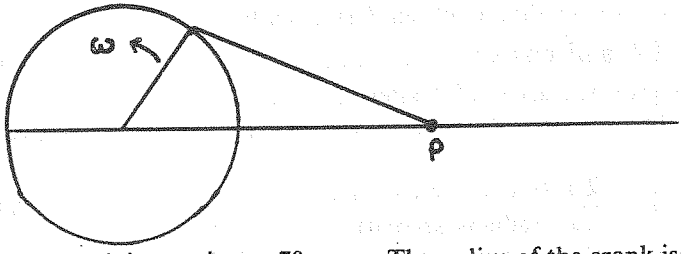
$$f = \frac{dv}{dt} = -a\omega. \cos \omega t. \omega$$

$$f = -a\omega^2 \cos \omega t \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

(Again the acceleration as we know, is acting towards the centre, i.e., towards the left).

If the correct signs are used for cos ωt and sin ωt according to the quadrant in which they may fall as a result of the value of the angle ωt, then these equations will always give not only the correct position of P (i.e., to left or right of the origin) but also the correct direction of the motion.

WORKED EXAMPLE 3



A crank is rotating at 70 r.p.m. The radius of the crank is 1·5 m. Find the maximum acceleration on the crank pin, P, given that the motion of P is simple harmonic.

Maximum acceleration is at either end of the stroke.

Since: $f = a\omega^2 \cos \omega t$, f will be a maximum when $\cos \omega t = 1$,

i.e., $\omega t = 0°$ (or 180°)

$f = a\omega^2$

find ω:

$$70 \text{ r.p.m.} = \frac{70 \times 2\pi}{60} \text{ radians/second}$$

$$\text{from which } \omega = \frac{7\pi}{3} \text{ radians/second}$$

$$\text{Thus } f = a\omega^2$$

$$= 1{\cdot}5 \times \left(\frac{7\pi}{3}\right)^2 \text{m/s}^2$$

$$= 80{\cdot}6 \text{ m/s}^2$$

WORKED EXAMPLE 4

A tuning fork loaded with a small mass of 0·05 g does 512 oscillations per second, the observed amplitude being 1 mm. Find the maximum force on the mass.

Maximum force = mass × maximum acceleration and the maximum acceleration will be at either end of the motion.

$$f = a\omega^2$$

$$= 0{\cdot}001 \times (512 \times 2\pi)^2 \text{ m/s}^2$$

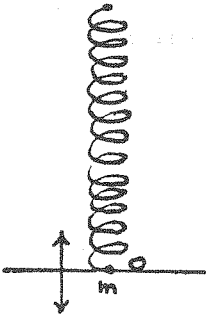
$$= 10349 \text{ m/s}^2$$

$$F = mf$$

$$= \frac{0{\cdot}05}{1000} \times 10349 \text{ N}$$

$$= 0{\cdot}5175 \text{ N}$$

12.7 MOTION OF A HELICAL SPRING



Let a mass m be suspended from an elastic cord or helical spring and let O be the position in which m would remain at rest if placed there.

If m is now slightly displaced, then it will perform **simple harmonic oscillations about O as origin.**

To deduce the equations of motion for m.

From equation V in section 12.5 we have

$$f = a\omega^2 \cos \omega t$$

Also from III,

$$x = a \cos \omega t$$

and substituting,

$$f = \omega^2 x,$$

i.e., accel. = disp't $\times \omega^2$ ..... (1)

Now, for the particular case when the displacement is unity (1 m) we have

$$f = \omega^2$$

that is, $mf = m\omega^2$, in which mf is the force (in Newtons) to cause unit displacement, or

Force to cause unit displacement $F = m\omega^2$ ..... (2)

We already have

$$T = \frac{2\pi}{\omega} \text{ so } \omega = \frac{2\pi}{T}$$

and in (2)

$$F = m \cdot \frac{4\pi^2}{T^2}$$

$$T^2 = \frac{4\pi^2 m}{F}$$ and taking square root,

$$\mathbf{T = 2\pi \sqrt{\frac{m}{F}}} \quad \text{..... VII}$$

This particularly important formula may be used for all linear oscillations which are simple harmonic.

WORKED EXAMPLE 5

A 7 kg mass attached to the end of a helical spring produces 40 oscillations in 10 s when disturbed from rest. What extension of the spring would be caused by a mass of 11 kg?

40 in 10 s gives T, the time for 1 osc'n = ¼ s.

From VII above we have $T = 2\pi\sqrt{\frac{m}{F}}$

$$\text{i.e. } {}^1/_4 = 2\pi\sqrt{\frac{7}{F}}$$

$${}^1/_{16} = 4\pi^2 \times \frac{7}{F}$$

from which $F = 4\pi^2 \times 7 \times 16 \text{ N} = 4421{\cdot}58 \text{ N}$

11 kg has a weight of $11 \times 9{\cdot}81 \text{ N} = 107{\cdot}91 \text{ N}$

4421·58 N causes an extension of 1 metre

107·91 N will cause an extension of $\frac{1}{4421{\cdot}58} \times 107{\cdot}91 \text{ m}$

$= 0{\cdot}0244 \text{ m}$

WORKED EXAMPLE 6

A mass of 10 g is suspended from a helical spring and is at rest when the length of the spring is 16 cm, its unstretched length being 14 cm. If the mass is now displaced slightly, find the period of the resulting oscillation.

We must first find the force in Newtons to cause unit (1 m) displacement.

10 g causes an extension of 0·02 m

Therefore $\frac{10 \text{ g}}{0{\cdot}02}$ causes an extension of 1 m i.e. $500 \text{ g} = 0{\cdot}5 \text{ kg}$

Force to cause 1 m extension $= 0{\cdot}5 \times 9{\cdot}81 \text{ N} = 4{\cdot}905 \text{ N}$

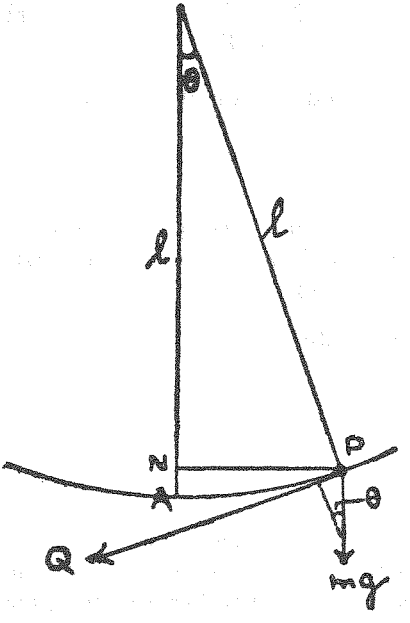
In the formula,

$$T = 2\pi\sqrt{\frac{10 \times 10^{-3}}{4{\cdot}905}}$$

$T = 0{\cdot}28$ s.

12.8 SIMPLE PENDULUM

A simple pendulum is one in which the bob can be regarded as a heavy particle and the string weightless, and which is allowed to oscillate under gravity through a very small arc.



Simple pendulum.

The diagram illustrates the motion. Note that since the oscillations are small, if θ is in radians then $\sin\theta = \theta$ (very nearly). Also PN may be assumed $= PA$. Now in the displaced position, the only force acting on the bob is that due to its own weight, mg, and is the component of mg resolved tangentially along PQ.

i.e.: Force causing motion $= mg \sin\theta$

$$= mg \times \frac{PN}{l} = \frac{mg}{l} \times PA \quad \ldots\ldots \quad (1)$$

In this expression, m, g, l are constants, and so the force acting on m **is proportional to PA, its displacement from A, the neutral position, and also acts towards A.**

Hence the motion, by definition, is simple harmonic.

The period is therefore given by

$$T = 2\pi\sqrt{\frac{m}{F}}$$

Now the force causing a displacement PA is, from (1) $\frac{mg}{l} \times PA$. The force is to cause *unit* displacement will be $\frac{1}{PA}$ th of this, i.e., $\frac{mg}{l}$

Hence, $\mathbf{T = 2\pi\sqrt{(m)\left|\left(\frac{mg}{l}\right)\right.} = 2\pi\sqrt{\frac{l}{g}}}$ ..... **VIII**

This is the formula giving the period of a simple pendulum.

WORKED EXAMPLE 7

Find the length of a "seconds pendulum" taking $g = 9{\cdot}81$ m/s^2.

We will take a "seconds pendulum" to mean one that "beats seconds", i.e., it has a period of 2 seconds.

Hence, $2 = 2\pi\sqrt{\frac{l}{9{\cdot}81}}$ and reducing this gives $l = 0{\cdot}994$ m.

EXERCISE B

1. A helical spring is extended 2·4 cm when a mass of 1 kg is suspended. The 1 kg is replaced by a 3 kg mass. Find the period of oscillation if this mass is slightly displaced from the equilibrium position.
2. A particle is describing S.H.M. of amplitude 0·05 cm and period 0·05 s. Find its acceleration and velocity when its displacement is respectively (*a*) 0·05 cm (*b*) 0·025 cm (*c*) zero cm from the origin.
3. A particle is moving in a straight line with S.H.M. When it is 3 m from the origin its velocity at the same instant is 4 m/s. If the amplitude of the motion is 5 m, calculate the angular velocity ω and hence the period of the motion.

TEST PAPER 12
Circular Motion, Simple Harmonic Motion

1. Assuming that the earth rotates once in 24 hours and that its equatorial radius is $6{\cdot}5 \times 10^6$ m, find the centrifugal force acting on a man of mass 80 kg when on the equator.
2. A particle is describing S.H.M. of period 0·3 s and amplitude 10 cm. Find its maximum velocity and maximum acceleration.
3. A simple pendulum beats quarter seconds at a place where $g = 9{\cdot}80$ m/s^2. Find its length.
4. Find the height of the cone of revolution in a conical governor doing 40 r.p.m., length of arm 1 m.
5. In order to control a steam valve inlet it is desired to arrange the cone of revolution in a governor to be 20 cm. in height. Taking $g = 9{\cdot}81$ m/s^2, find the speed in r.p.m. to achieve this.
6. A simple pendulum 6 m long swings to and fro 10 cm each side of the mean position. Find its period, its velocity at its lowest point and acceleration at its highest point. (Take $g = 9{\cdot}81$ m/s^2).
7. A mass of 5 kg is suspended on the end of a spring which stretches 5 cm for each 10 kg of load. How many oscillations per minute will it make when disturbed from rest?
8. The reciprocating parts of an engine of 0·6 m stroke weigh 120 kg. Assuming S.H.M., find the accelerating force at the beginning of the stroke, engine running at 240 revs per minute.
9. A particle of mass 0·04 kg is moving in a straight line with S.H.M. When it is 1 m from the origin its velocity is 2 m/s. If the angular velocity ω is $\pi/6$ rad/s, calculate the amplitude of the motion.
10. A point describes S.H.M. of amplitude 6 cm. Its velocity when passing through the mean position of the motion is 10 cm/s. Find the period of the motion.

CHAPTER 13

HEAT

General Introduction. Thermometry

13.1 HEAT AND TEMPERATURE

When a substance is heated it is said to gain or absorb heat. When it cools it is said to lose or give out heat.

So the question is, what is heat? Evidently it is not material, for the weight of a substance does not change as it absorbs or gives out heat.

Heat is a form of energy and when a substance gains heat it gains energy. This energy is given to the molecules of any given substance. These molecules (of which all matter is composed, though we need not go into that here), are vibrating to and fro. As a body is heated, the vibration increases, i.e. the molecules gain kinetic energy. Hence the body has absorbed "heat energy". When it cools, the vibration diminishes and the surplus heat energy is given out as the molecules lose kinetic energy.

Heat is thus energy—molecular energy. *It must in no sense be confused with "hotness"*, which we shall refer to again. All substances contain some heat. A block of ice contains heat, and if put in contact with something even colder still heat will flow from the ice into the colder body.

Heat is a form of molecular energy, then, and broadly speaking, all substances contain some of it.

Heat, a Measurable Quantity. A measurable quantity is one that can be expressed in terms of a fixed unit of its own kind.

Since all substances contain some heat, and they can absorb or give out heat, it looks as though heat is something which could be measured in its own units.

Since heat is energy however, it is usual to measure heat as a quantity of energy, i.e., in "joules of thermal energy". We shall also

refer to this again later. For the present, let us note that there is such a concept as a "quantity of heat".

Temperature. We have already referred to "hotness", and degrees of hotness are recognisable even to a comparatively inaccurate instrument like the hand. When a body absorbs heat it becomes hotter. We might say it acquired a higher "degree of hotness", but to speak of degrees of hotness would be clumsy and inconvenient. We therefore use the term **Temperature** instead, and when a body has acquired any particular degree of hotness it is said to be at a certain temperature.

To sum up.

Heat is a measurable quantity, a quantity of molecular energy that a body possesses. It is the sum total of the kinetic energies of all the molecules of the body. The molecules in a large block of ice may each have a small amount of kinetic energy but the total kinetic energy of them all could well be more than that of the molecules in, for example, a small cup of boiling water. In short the block of ice could contain more heat than the cup of boiling water.

Temperature is a degree of hotness and in fact it depends on the extent of the molecular activity already referred to. Temperature is a measure of the AVERAGE kinetic energy of the molecules in a body. In a cup of boiling water the average kinetic energy of the molecules is higher than that of the molecules in a block of ice. Boiling water is at a higher temperature than ice.

Difference of Temperature. Two bodies are at different temperatures if, when brought into proximity, heat flows from one to the other. In fact, difference of temperature is an essential condition for a transfer of heat to take place. Heat always passes from the body at a higher temperature to the body which is at a lower temperature.

Conversely, if two bodies are at the same temperature no transfer of heat will take place.

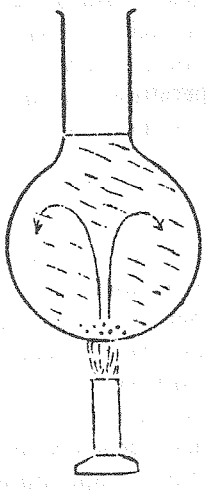
13.2 TRANSMISSION OF HEAT

Remember always to think of heat as something that can be transferred from place to place in measurable quantities.

There are three ways in which this can occur, namely, by **Convection,** by **Conduction** or by **Radiation.**

Convection. The sketch shows how convection currents may be demonstrated.

The flask contains water. In the bottom are placed a few crystals of colouring matter. When the flask is heated, coloured streams will be seen moving upwards and then downwards as in the sketch.



Convection.

These are called convection currents. We will refer to convection again later.

Definition. Convection is the motion of the hot body itself carrying its heat with it.

Conduction. If a poker is placed in the fire the handle becomes warm. This is because heat is passing along the poker from the end that is in the fire. This method of transmission is called conduction.

Definition. Conduction is the flow of heat through an unequally heated body from places of higher to places of lower temperature.

Some substances are better conductors than others. All metals are good conductors, silver being the best. This explains why a silver teaspoon gets very hot. Substances like wood and stone are bad conductors.

Examples of good and bad conductors

On a *cold* day (i.e., at temperatures much below that of the hand) a good conductor feels colder to the touch than a bad conductor. This is because the good conductor conducts heat away more rapidly from the hand.

A piece of iron feels colder than a piece of wood, even though both may be at the same temperature. At normal cold temperatures this makes the handling of steel tools an uncomfortable job. In Arctic temperatures however, so much heat may be drawn from the hand that it is liable to freeze to the steel.

On a *hot day* (i.e., at temperatures much above that of the hand) a good conductor feels hotter to the touch than a bad conductor.

This is because the good conductor allows heat to flow more rapidly into the hand.

An iron deck and a wooden deck in the sun are at the same temperature. We know which is easier to walk on in bare feet, however. The iron provides a continuous supply of heat flowing rapidly into the foot. In the case of the wood, whilst the foot quickly takes up the small quantity of heat from where it stands, there is no corresponding rush from the rest of the deck to make up the deficiency.

Uses of good and bad conductors

Boilers are made of good conductors such as copper, aluminium or steel. This increases the rate at which heat is taken from the furnace. The efficiency of a boiler is greatly reduced by a layer of scale on the inner surface. The scale is a bad conductor and consequently a layer of scale only 3 mm thick will cause a big reduction in the rate at which heat is taken from the furnace.

Air is a bad conductor. Clothes and the coats of animals owe their warmth chiefly to the air they contain. Fur and wool contain much air and so are bad conductors. Thus the heat of the body cannot escape. This is an important point. Wool and fur are not warm in themselves—it is only the *heat of the body* they keep in. Hence, it is not much use wrapping a cold man in cold blankets, or putting thick woollen socks on to feet that are numb with cold. It is essential to get some warmth there first—then wrap up.

Clothes are just insulators. Other materials are used to insulate, for example, cold storage chambers or for lagging hot pipes.

Radiaton. Heat reaches the earth from the sun. Since its journey is mainly through a vacuum there can be no question here of conduction or convection. This method of transferring heat is known as radiation.

Definition. In radiation, the hotter body loses heat and the colder body gains heat by transmission through some intervening medium which does not itself become hot.

Notice that—"the intervening medium does not become hot". The Earth's surface is warmed by radiation from the sun. The air is not appreciably warmed by the sun's ray passing through it. It is warmed by contact with the Earth's surface. This is referred to again.

Heat radiation is known to have much in common with the transmission of light, that is, it obeys some of the same laws as light rays. Radiant energy travels in straight lines like light waves. Radiant energy is not *heat*—it is only converted into heat on striking matter. Radiant energy waves can be focused to a point. Parabolic reflectors on electric fires utilise this fact. A convex lens ("burning glass") can cause such a concentration of rays that inflammable material will be ignited.

Examples of good and bad radiators

Brightly polished bodies radiate badly. A silver teapot is a case in point; it keeps the tea hot. Hot-water radiators on the other hand should always be painted with a dull paint to increase their radiating power, and they are always constructed so as to expose as much surface as possible.

Black bodies radiate heat more rapidly than polished or light coloured bodies. They also absorb heat more rapidly than light coloured bodies. For this reason dark clothing is unsuitable for tropical wear.

The thermos flask is another example. Most people know that the glass envelope is a vacuum. But why does it appear silver? It is silvered on the interior of the glass walls to reduce loss of heat by radiation.

13.3 GENERAL EFFECTS OF HEAT

When a body is heated or cooled various effects are produced. The most important are as follows:

(a) Change in physical properties.
(b) Promotion of chemical action.
(c) Change of temperature.
(d) Change of state.
(e) Expansion or contraction.

consider these in turn.

(a) Change in Physical Properties. Heating and cooling modify in many ways the properties of substances. Iron heated to redness loses its magnetism. Steel may be softened or tempered by heating and cooling. Many substances becomes brittle when cold. The passage of electricity through a conductor is influenced by the

temperature of the conductor. Sound travels through air or water at slightly differing speeds according to the temperature of the air or water.

Though not spectacular in themselves, these changes can be very important in some connections.

To sum up:—nearly all the physical properties of matter change with change of temperature.

(b) Chemical Change. The most common change under this heading is that caused when combustion occurs. Coal, wood, oil for example, when heated sufficiently combine vigorously with the oxygen in the air and change chemically into other substances such as carbon and gas, giving out a large amount of heat as they do so. This process is called **combustion** or **burning.**

Other chemical changes can be promoted by heat, but we will not consider these here.

(c) Change of Temperature. This is almost fundamental. We have already said that *difference* of temperature causes heat to flow.

Conversely, if heat is put into a body, that body must rise in temperature. If heat is taken out of it, its temperature falls.

The only exception to this law is when a body is "changing its state".

(d) Change of State. This is much more important from our point of view.

Nearly all substances may exist in three states, solid, liquid, gas, and a substance is said to be "changing its state" when it is changing from solid into liquid or from liquid into gas, or the reverse.

A familiar example is ice-water-steam. Heat has to be put into a block of ice to turn it into water, but while the change of state is taking place, *no rise in temperature occurs.* All the energy of the heat is absorbed in working the conversion, and only when the last bit of ice is melted will the temperature finally start to rise. It will go on rising till the water boils (as long as we continue to put heat into it, of course) and then the rise will cease again. The boiling water will remain at a constant temperature and the steam leaving it will be at the same temperature. When all the water is converted into steam (though not in a marine boiler we hope), then the temperature of the steam would start to rise again.

It then becomes "superheated" steam, which exerts a greater pressure. This fact is utilised in marine engines (superheating the steam) though not quite on the lines of the above discussion.

To sum up:—While a substance is changing its state, no change in temperature occurs. The heat is absorbed in the process of changing state and cannot therefore go into stimulating the molecular energy, which is necessary for a rise in temperature.

The heat used in changing state is not lost however—it is said to be **Latent** (hidden) and it is given up again when the process is reversed. We shall discuss this in more detail later.

(e) Expansion and Contraction. Almost all substances expand when heated and contract when cooled.

The only notable exception to this rule is water which contracts on cooling down to 4° C, but then expands again as the temperature falls to freezing point at 0° C. Thus a given quantity of water occupies its smallest bulk at 4° C.

From what we have said already about density (under hydrostatics—Chapter 1), if a given mass of water occupies its smallest bulk, this is only another way of saying its density is greatest at that moment.

Water is at its greatest density at 4° C.

This fact has important results. For instance, it explains why water always freezes on the surface first (because the less dense water at 0° C rises to the top over the denser water at 4° C). It also partly explains why ice floats.

With the exception of water, however, (and a few other little known substances), all substances expand when heated and contract when cooled.

13.4 EXPANSION OF SOLIDS

Of all the common substances, metals expand most when heated. Exceptions are platinum and invar (a nickel steel), which expand much less than steel, copper, brass, silver, and lead, for example. Also stone, glass and most woods expand comparatively little. However, *all* solids expand when heated, contract when cooled, even though the amount of the expansion varies a good deal from one substance to another.

Force of Expansion and its Application. In some respects this expansion is a disadvantage. In the Golden Gate Bridge at San Francisco, provision is made by sliding panels at each end for an expansion of from 1·2 m to 1·8 m in the length of the bridge

roadway. Owing to expansion in the bridge suspension cables, the roadway may be as much as 3 m nearer to the water surface in hot weather. In all steel structures similar precautions have to be taken, since the force exerted by a metal rod, if prevented from expanding or contracting, is very great.

On board ship, steam pipes are provided with a big looped bend. When setting up funnel guys on a cold funnel, the guys should be left slack. The balance wheel of a chronometer is constructed with its rim being a bimetallic strip so as to allow for expansion and contraction and so to enable the chronometer to keep correct time. Other examples will occur to the reader.

On the other hand, sometimes the force of expansion or contraction is turned to good account. A successful riveted joint depends amongst other things on the contraction of the rivets on cooling, which thus make a tight joint. Heavy castings, such as a crank, or the rudder quadrant, are "shrunk" on to the crank shaft or rudder stock. That is, they are hammered on to the shaft while still hot and on cooling grip the shaft tightly.

Bi-Metallic Thermometers. Two strips of different metal are riveted or soldered together in the form of a spiral. When this arrangement is subjected to changes in temperature, it alternatively coils itself together, or unwinds itself. The spiral is held rigid at one end, and the other end carries a pen which records a trace on a revolving drum. Thus we have self-recording thermometer, or **Thermograph.** Of course, there is rather more to a thermograph than just this, but at least this is the principle, and one more useful application of the expansion and contraction of metals.

13·5 EXPANSION OF GASES

Gases when heated expand much more rapidly than solids. Also whereas each solid substance has its own particular rate of expansion, **all gases expand equally when heated** (provided that the outside pressure remains constant). Air is a gas, so we shall confine our discussion to air. Anything we say about air would apply to any other gas in similar circumstances.

Convection. Air is a fluid, and a fluid may be defined as a substance incapable of resisting any force, however small, tending to charge its shape or position. Now, if we heat a solid, it still,

broadly speaking, remains in the same position. This is not the case with air, which can flow freely in any direction.

Consequently, if a given sample of air (say a portion of a large air mass) is warmed by contact with a warm surface, it will expand. In doing so, it follows that it becomes **less dense** than the air just above it which has not felt the warming influence.

In other words, it is exactly like a cork held under water—it is in an environment heavier than itself, and being free to move, it does so—upwards. This is happening to all the samples of air, all over the heating surface, so that in such an air mass we have innumerable trickles of air ascending, mixing in with the air above, more air taking its place at the surface and being warmed and rising in turn.

This phenomenon, this sort of system of circulation, is called **Convection.**

Gradually, of course, the whole air mass tends to get warmer—at least, as far as the upper limit of the convectional currents. In fact, an air mass absorbs very little heat from the sun's rays passing through it. It is chiefly warmed by convection.

Convection is also found in liquids. We shall refer to this later.

13.6 CAUSE OF WIND

Wind is air in motion in a horizontal direction. But what starts it moving?

All motion of the air owes its origin to differences in temperature.

We have seen how heating from below causes convectional currents. Thus, when air is in contact with a warm portion of the earth's surface this will in time result in a large air mass becoming considerably warmer than the other air masses round about it.

Warm air weighs less than cold air, and so, where a warm air mass was located we would have a region of **low barometric pressure.** Adjacent to it, where the colder air is, we would have a region of heavier air, that is, **high barometric pressure.**

The final step is easily seen. Air flows from the high pressure towards the low pressure, though not *directly* towards it.

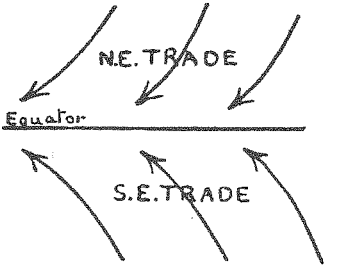
To sum up:— Wind is caused by differences in pressure between one locality and another. These differences in pressure are themselves caused by differences in temperature.

Direction of Air Flow. For reasons which we cannot go into here, the air does *not* flow directly from the high pressure towards the low.

Owing to the rotation of the earth, the air stream tends to be deflected, to the **Right** in the **Northern Hemisphere**, to the **Left in the Southern Hemisphere** (right and left, that is, if we face the direction in which the stream is moving).

The **Trade Winds** provide a good example of this.

The heating of the Equitorial regions causes a permanent low pressure area at the Equator. Thus, air from north and south flow in towards the Equator, and being deflected, as we have stated, they become the N.E. and S.E. Trades.



Trade Winds.

13.7 LAND AND SEA BREEZES

Land absorbs heat, and loses it again, more rapidly than water.

During the day, the land becomes heated, pressure falls and air flows in from the sea, thus causing a **Sea Breeze**.

At night, the land cools rapidly, pressure rises and air flows towards the sea, causing a **Land Breeze**.

These are just local winds set up by temperature differences on a fairly small scale. There is no time for deflection of the air stream to become apparent, and broadly speaking, air flows in from the sea by day, off the land by night.

In many parts of the world these breezes are quite strong. Along the North African Coast for instance, the sea breeze by late afternoon may be quite a strong boisterous wind, dying away soon after sunset.

13.8 CONVECTION IN LIQUIDS

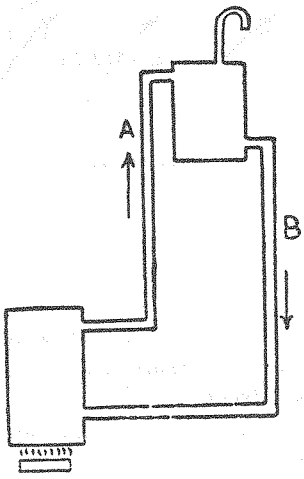
When a liquid in a vessel is heated from below, the convection currents caused by the heating establish a system of circulation in the liquid.

Under this system the liquid circulates round and round, every part of it in turn being brought near the surface of heat until finally it boils.

All liquids are heated by convection, just as all air masses are. It is for this reason that boilers are heated from below.

Central Heating. The system of heating buildings by hot water pipes is a case of convectional currents in liquid.

The sketch illustrates the principle.



Central Heating.

Hot water in the boiler (hottest at the *top* of the boiler remember) is pushed up the branch *A* by the cooler water in branch *B*. The branch *B* is the one that passes through all radiators. So even when the whole system is quite hot, branch *B* will always be cooler than *A* and so the circulation will be kept up.

For a really successful circulation, the boiler should be at the lowest possible point, and the outflow *A*, should go by the most direct route to the highest possible point in the system.

All the radiators are on branch *B* and the coldest one will be the last one before the water re-enters the boiler.

The tank at the top is just a "storage tank", to ensure there is always plenty of water in the system. It is fitted with an expansion pipe, in case the water boils.

13.9 EXPANSION OF LIQUIDS

Liquids expand when heated, contract when cooled. This has to be allowed for in tankers, for example.

Of course, a liquid has to be contained in a vessel of some sort, and the vessel itself expands. Thus, the **apparent expansion** of a liquid in a vessel is the combined result of the **real expansion** of the liquid itself and the expansion of the vessel.

The apparent expansion is always less than the real expansion.

Like solids, each liquid has its own rate of expansion (or **Coefficient of expansion**, as it is called).

One of the most important applications of the expansion of liquids is to measure temperature. If a liquid is contained in a glass vessel, then the liquid will expand or contract as the temperature of its surroundings varies. This is the principle of all thermometers, and we will now consider these in some detail.

13.10 THE MERCURY-IN-GLASS THERMOMETER

The graduation of an ordinary mercury thermometer is based upon the apparent expansion of mercury in glass. Since mercury expands about seven times as much as glass, any change in temperature causes a noticeable increase in the volume of the mercury. This expansion of the mercury is read off on a graduated stem.

The degree divisions of the stem are all equal. Thus, the assumption is made that both the mercury and the glass *expand uniformly* throughout the whole range of temperature. This is not strictly correct, but is very nearly so, and any errors are further reduced by making the bulb large by comparison with the volume of the stem.

Construction of a Mercury-in-Glass Thermometer. The general appearance of a thermometer is well known. It consists of a glass tube with a very fine bore having a fairly large bulb on one end (large that is, by comparison with the bore of the stem).

The bore must be fine so that a small expansion of the mercury will be easily seen. The bulb is comparatively large, *(a)* so that an appreciable amount of mercury will be forced up the stem and *(b)*

so that errors which arise from the expansion of the glass itself will be minimised.

The bulb and stem are first completely filled with mercury. This is a tedious job, and it is done by alternatively heating the bulb (so as to drive a little air out) and cooling it again (so as to suck a little mercury in). At long last it is filled completely.

The mercury is then boiled so as to make sure that no bubbles of air remain. As it cools, the open end is kept immersed in mercury, so that it draws this in as the mercury in the thermometer contracts. Finally, the whole thing is cool and completely filled with mercury in which no bubbles of air remain.



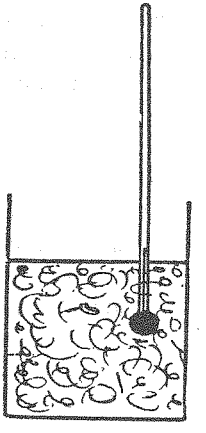
Constructing a thermometer.

Bulb and stem are now heated again to a temperature somewhat higher than the instrument will ever have to register. This causes mercury to overflow of course, and in this condition the open end of the tube is sealed off. As the mercury cools once more, a perfect vacuum remains in the stem above the column of mercury.

Determination of the Fixed Points. In order to furnish a thermometer with a scale, two points are first marked on the stem which correspond to two definite temperatures. They are: the "lower fixed point" that is, the melting point of ice, which is the same as the freezing point of pure water, the "higher fixed point" that is, the boiling point of water.

The Freezing Point. The freezing point is determined by placing the thermometer in a vessel containing a mixture of pure ice and distilled water. As we have seen already (section 13.3 (d)) ice and water together will remain at the temperature of freezing water. The stem must be immersed as far as possible consistent with the upper end of the mercury thread being just visible.

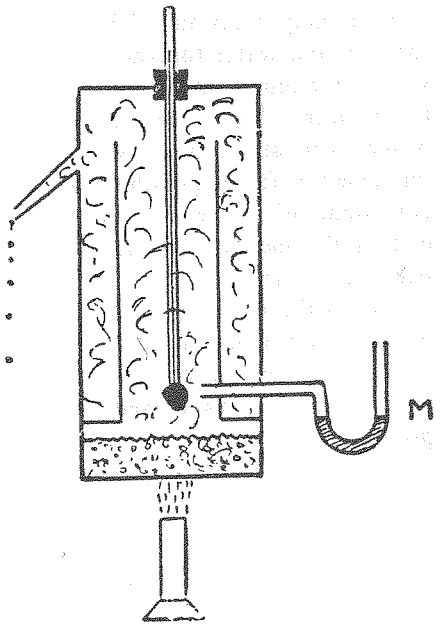
When the mercury level becomes stationary, a mark is carefully traced on the stem. This is the **lower fixed point.**



Determining the lower fixed point.

The Boiling Point. To determine the upper fixed point, the thermometer is not placed *in* the boiling water, since the presence of even minute impurities in water affects the boiling point slightly.

Using an instrument called a hypsometer, the thermometer is placed in the steam from boiling water. The steam is always at the correct temperature, providing the pressure inside the hypsometer does not vary from standard atmospheric pressure. This is taken to be 759·587 mm and a variation in pressure affects the temperature of steam. (To take a rather extreme case, tea cannot be made successfully on top of Mont Blanc because water boils at too low a temperature).



Determining the higher fixed point.

To ensure that pressure does not rise inside the hypsometer, a water manometer *(M)* is fitted.

When the mercury level ceases to rise, a mark is carefully traced on the stem. This is the **upper fixed point.**

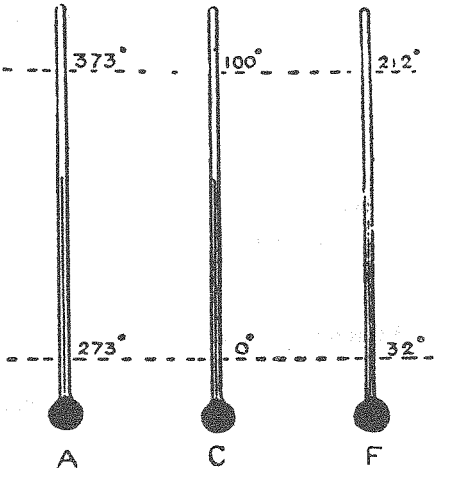
Graduation of the Thermometer. The interval between the freezing and boiling points might be divided into any desired number of equal parts, called "degrees". In practice, only three such scales need be considered.

Celsius Scale.* The freezing point is marked 0°, the boiling point is 100°, and the interval between divided into 100 equal degrees.

Fahrenheit Scale. The freezing point is marked 32° and the boiling point 212°, the interval between being divided into 180 equal degrees.

In both cases, the degrees scale is extended above and below the fixed points.

In both scales, temperatures *below zero* are shown by a *minus sign.*



Graduation of thermometers.

Absolute or **Kelvin Scale.**† The freezing point is marked 273 and the boiling point 373. Thus, there are 100 divisions, so degrees on the Kelvin scale are the same as on the Celsius. Note the symbol for Kelvin is NOT °K but simply K e.g. 273 K.

The Kelvin scale is much used in Scientific work, and will be dealt with in greater detail later. It is the correct S.I. unit for temperature.

13.11 CONVERSION BETWEEN TEMPERATURE SCALES

(1) Celsius — Kelvin

These conversions are relatively straightforward since the size of a degree is the same on both scales.

To convert Celsius into Kelvin simply ADD 273.

For example: 10 °C = 10 + 273 K = 283 K.

To convert Kelvin into Celsius simply SUBTRACT 273.

For example: 300 K = 300 − 273° = 27 °C.

**Formerly the Centigrade scale. Renamed in honour of its Swedish inventor, Anders Celsius* (1701-1744).

†*Formerly known only as the Absolute scale. Renamed in honour of its British inventor, William Thomson, later Lord Kelvin, who proposed it in* 1848. *The term "absolute zero" is still used.*

(2) Celsius — Fahrenheit

This is not as straightforward because the size of 1 °C is not the same as the size of 1 °F.

Between the fixed points we have 100 °C or 180 °F,

i.e. $100\ °C = 180\ °F$

therefore $1\ °C = \frac{180}{100}\ °F$ which cancels down to give

$$1\ °C = \frac{9}{5}\ °F$$

We must also take into account the fact that these scales allocate different values to freezing point.

WORKED EXAMPLE 1

Convert 35 °C into F.

35 °C is also 35 °C above freezing point.

$$35\ °C = 35 \times \frac{9}{5}\ °F$$

$$= 63\ °F \text{ above freezing point.}$$

On the Fahrenheit scale freezing point is 32 °F.

Therefore $35\ °C = 63 + 32\ °F$

$$= 95\ °F.$$

Summarising to convert from °C into °F use the formula

$$F = \left(C \times \frac{9}{5}\right) + 32 \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{ I}$$

To convert 95 °F back to °C we reverse the procedure, i.e. we use the formula

$$C = (F - 32) \times \frac{5}{9} \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{ II}$$

Hence for 95 °F we have

$$C = (95 - 32) \times \frac{5}{9}\ °C$$

$$C = 63 \times \frac{5}{9}\ °C$$

$$= 35\ °C.$$

(3) Kelvin — Fahrenheit

We will not convert directly between these two scales. First we will convert the degrees Fahrenheit or Kelvin into degrees Celsius and then into the required scale.

WORKED EXAMPLE 2

Convert 14 °F into Kelvin.

First convert to °C

$$C = (F - 32) \times \frac{5}{9}$$

$$= (14 - 32) \times \frac{5}{9}$$

$$= -18 \times \frac{5}{9}$$

$$= -10\ °C$$

Now convert to Kelvin i.e. $-10 + 273 = 263$ K.

EXERCISE A

1. Convert the following into K
   (a) 20 °C *(b)* 135 °C *(c)* −80 °C
2. Convert the following into °C
   (a) 298 K *(b)* 127 K *(c)* 401 K
3. Convert the following into °C
   (a) 77 °F *(b)* −4 °F *(c)* 241·6 °F
4. Convert the following into °F
   (a) 25 °C *(b)* −20 °C *(c)* 112 °C
5. Convert the following into K
   (a) 68 °F *(b)* 23 °F *(c)* 124 °F
6. Convert the following into °F
   (a) 308 K *(b)* 238 K *(c)* 280 K

13.12 ADVANTAGES OF MERCURY

Some of the advantages of mercury as a thermometer liquid have already been mentioned. To sum up:—

(a) High coefficient of expansion by comparison with glass.

(b) Expands very nearly uniformly throughout the whole range; hence, degree divisions are all the same length.

(c) Good conductor of heat and so it rapidly attains the temperature it is required to measure.

(d) Does not wet the glass nor evaporate into the vacuum above the end of the thread.

(e) Does not boil until 357° C. This is high enough for most ordinary purposes, though not when it comes to measuring the temperature of furnaces, for example. For this, other means have to be employed.

(f) freezes at −39° C. This, although quite low, is not low enough for all purposes. Temperatures well below that are obtained in some of the colder parts of the earth. For thermometers intended for use in very cold climates, *alcohol* must be used instead of mercury.

Spirit Thermometers. The spirit used is pure alcohol (sometimes called "spirits of wine"). This becomes a thick liquid at −90° C and solidifies into a white mass at −130° C.

Such a thermometer is useless at high temperatures however, as alcohol boils at 78° C.

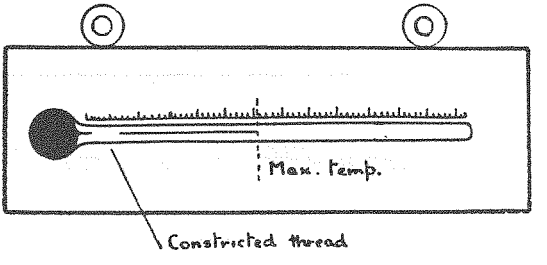
13.13 MAXIMUM AND MINIMUM THERMOMETERS

These are thermometers designed to register the highest or lowest temperatures attaining during any interval, usually 24 hours.

The maximum and minimum temperatures attained during a day are of considerable interest to meteorologists, and for this purpose the thermometers are kept out of doors, in a screen specially constructed for the purpose, and called a Stevenson screen, from its designer. This consists of a box-like cupboard, with louvred sides, so as to allow free access of air to the thermometers, without however, any direct sunlight reaching them.

Another common use for the maximum thermometer is in the ordinary clinical thermometer used by doctors. This retains a fixed record of the patient's temperature so that the doctor has time to read it. He then resets the thermometer by shaking the mercury down into the bulb again.

Maximum Thermometer. The instrument commonly used is an ordinary mercury thermometer, with the bore of the tube much constricted close to the bulb. As a consequence, the mercury expanding in the bulk forces its way past the constriction into the stem of the thermometer. On the other hand, when temperature falls, the thread of mercury in the stem fails to make its way back into the bulb.



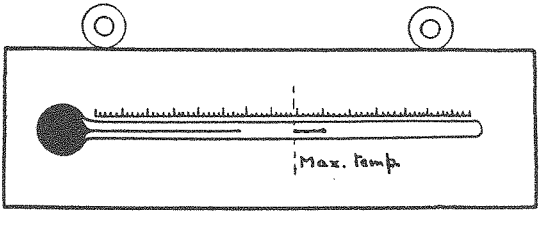
Maximum Thermometer.

The extremity of the thread thus records the maximum temperature reached.

The thermometer is usually mounted horizontally and is reset by gently swinging the thermometer, bulb downwards, so as to persuade the mercury back into the bulb.

As already mentioned, the clinical thermometer is of this type.

Another type of maximum thermometer makes use of a metal index.

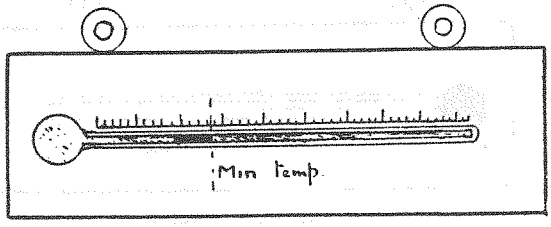


Maximum Thermometer.

As the mercury expands it pushes the index before it in the stem, and when the temperature falls, the index is left behind.

The inner end of the index thus records the maximum temperature and the instrument is re-set by holding it in a sloping position and gently allowing the index to slide back into contact with the mercury.

Minimum Thermometer. This is a spirit thermometer, mounted horizontally and is furnished with a light dumb-bell-shaped enamel or glass index which is in the spirit. When expansion occurs, the spirit flows past the index along the bore, but when temperature falls, the contracting spirit draws the index back with it. The outer end of the index thus records the minimum temperature reached.



Minimum Thermometer.

The instrument is reset by gently allowing the index to slide along the tube until it again makes contact with the inner surface of the spirit.

Note: *Maximum and minimum thermometers are often constructed with the bulbs turned downwards at right angles to the stem.*

TEST PAPER 13
Temperature Scales, Thermometers

1. *(a)* Convert the following temperatures:
   (i) from 20 °C to Fahrenheit:
   (ii) from − 100 °C to Kelvin:
   (iii) from 302 °F to Kelvin.
   (b) Sketch a mercury-in-glass thermometer, clearly labelling all the parts.
   (c) State the reason why this thermometer is **NOT** reliable when navigating in polar areas.
   (SCOTVEC July 1992).
2. An ungraded thermometer is attached to a cm scale and is found to read 6·4 cm in melting ice and 22·8 cm in steam at boiling point. What would be the temperature in *(a)* °F, *(b)* °C when the mercury stood at 10·5 cm.
3. On a certain day the readings on a Celsius and Fahrenheit thermometer were the same. What was the temperature?
4. *(a)* Distinguish between "temperature" and "heat".
   (b) "Three forms of heat transfer occur in the production of land and sea breezes".
   With the aid of sketches, discuss this statement, identifying the forms of heat transfer involved at each stage.
   (SCOTVEC July 1990).
5. Complete the following table by carrying out the necessary conversion:

	°F	°C	K
(a)	50		
(b)		65	
(c)			328

CHAPTER 14

HEAT

Units, Specific Heat Capacity, Thermodynamics

14.1 HEAT AS ENERGY — SI UNITS

We have already spoken (section 13.1) of heat as a form of energy. Just as we have been accustomed to distinguish between different forms of energy in such terms as "kinetic energy", "potential energy", "electrical energy" and so on, in the same way it is customary to speak of heat as "thermal energy". The reader should appreciate this point. The two terms are synonomous; heat is thermal energy, thermal energy is heat. And when heat is transferred from one body to another, what has happened is simply a transfer of energy.

When thermal energy is transferred from one hot body to another at a lower temperature, the energy so transferred goes into stimulating the action of the molecules of the body which gains heat, whilst the molecular activity is reduced in the body which loses heat. The increase in molecular energy shows itself by a rise in temperature, whilst a reduction in molecular energy causes a fall in temperature.

So what is transferred is simply energy — thermal energy, to be precise — and it follows that, like all other forms of energy, it is measured in joules (J) i.e., joules of thermal energy, or joules of heat, whichever is preferred; they mean the same.

14.2 SPECIFIC HEAT CAPACITY

In general terms, the **heat capacity** of any given mass of a substance is the heat required to raise the temperature of that mass by 1°C. Whilst this concept has certain (limited) usefulness, it will be appreciated that without knowing exactly what the mass amounts to, it is not a very helpful generalisation.

In practice, we limit ourselves to unit mass, in which case it is called the **Specific Heat Capacity**. Hence:

Definition:

The Specific Heat Capacity (abbreviation c) of a substance is the quantity of heat required to raise the temperature of 1 kg of that substance through 1°C.

For example 4180J of heat energy is required to raise the temperature of 1 kg of water by 1°C. Thus we say that the specific heat capacity of water is 4180J/kg°C. Note the units for specific heat capacity: 4180J/kg°C means 4180J for every kg for every °C rise in temperature. Strictly speaking we should say 4180J/kg°K since Kelvin is the correct S.I. unit for temperature. However in the previous chapter we saw that the size of 1°C and 1K were the same. Thus a 1°C rise in temperature is the same as a 1K rise and we can quote the specific heat capacity using either of these.

Examples of other approximate values for c are given in Table 1.

Table 1

Approximate specific heat capacities in J/kg°C

Iron	450	Brass	380	Fresh water	4200
Copper	400	Lead	140	Ice	2100
		Glass	660		

It will be seen that the specific heat capacity of water is very high — in fact by far the highest of any common substance — a fact which is of far reaching importance and has consequences which affect many aspects of our daily lives. We shall refer to this again later. First let us consider the following.

From Table 1 above we can see that, for example, 400 J of heat energy are required to raise the temperature of 1 kg of copper by 1°C.

Thus to raise the temperature of, say, 2 kg of copper by 50°C would require $2 \times 400 \times 50$ J $= 40\,000$ J.

More generally if we wish to raise the temperature of m kg of a substance of specific heat capacity c J/kg°C by T°C then the quantity of heat required, Q, is given by

$$\mathbf{Q = mcT} \quad \ldots\ldots\ldots\ldots\ldots \quad \mathbf{I}$$

Note that if we wish to cool the substance by T°C then the above formula gives the amount of heat energy to be given out.

WORKED EXAMPLE 1

Calculate the amount of heat required to raise the temperature of 3 kg of iron by 85°C given that its specific heat capacity is 450 J/kg°C.

$$Q = mcT$$
$$= 3 \times 450 \times 85 \text{ J}$$
$$= 114\,750 \text{ J } (114{\cdot}75 \text{ kJ})$$

WORKED EXAMPLE 2

1·26 MJ of heat energy are extracted from 5 kg of water at an initial temperature of 80°C. Calculate the final temperature of the water. (c for water = 4200 J/kg°C).

$$Q = mcT$$
$$1{\cdot}26 \times 10^6 = 5 \times 4200 \times T$$
$$\frac{1{\cdot}26 \times 10^6}{5 \times 4200} = T$$
$$T = 60°\text{C}$$

This is the temperature CHANGE. Thus the final temperature of the water is 80 − 60 = 20°C.

EXERCISE A

1. Calculate the amount of heat energy required to raise the temperature of 5 kg of brass by 120°C. (c for brass = 380 J/kg°C).
2. Some water increases its temperature by 80°C when 13·44 MJ of heat energy are delivered to it. Calculate the mass of the water. (Assume c for water = 4200 J/kg°C).
3. 4·5 kg of lead require 63 kJ of heat to raise its temperature by 100°C. Calculate its specific heat capacity.

14.3 MIXING HOT WITH COLD

If we mix hot water with cold water then, of course, the resulting mixture will be at some temperature between that of the hot water and that of the cold water.

In such cases we can say that the heat lost by the hot water is equal to the heat gained by the cold water. This assumes no heat is lost to the surroundings or to the container.

WORKED EXAMPLE 3

44 kg of water at 92° C is mixed with some cold water which is at a temperature of 20° C. The final temperature of the mixture is 69·5° C. Calculate the amount of cold water present.

Assume c for water = 4200 J/kg° C.

For the hot water heat given out = mcT

$= 44 \times 4200 \times (92 - 69{\cdot}5)$ J

$= 4\,158\,000$ J

For the cold water heat received = mcT

$= m \times 4200 \times (69{\cdot}5 - 20)$ J

$= 207\,900 \times m$

Heat gained by cold water = Heat lost by hot water

$207\,900 \times m = 4\,158\,000$

$m = 20$ kg

The same sort of calculation can be made when mixing two different substances which are at different temperatures.

WORKED EXAMPLE 4

A piece of iron of mass 5 kg and specific heat capacity 450 J/kg° C is at a temperature of 100° C. It is quickly immersed in 6 kg of water at 25° C. Find the resulting temperature of the mixture. Assume c = 4200 J/kg° C for water.

Let t be the final temperature of the mixture.

For the hot iron: heat lost = mcT

$= 5 \times 450 \times (100 - t)$ J

For the cold water: heat gained $= 6 \times 4200 \times (t - 25)$ J

Note the temperature change in each case.

Heat lost by iron = Heat gained by water

$5 \times 450 \times (100 - t) = 6 \times 4200 \times (t - 25)$

$2250 \times (100 - t) = 25200 \times (t - 25)$

$225000 - 2250t = 25200t - 630000$

$225000 + 630000 = 25200t + 2250t$

$855000 = 27450t$

$t = 31{\cdot}15$° C

In practice in both the above examples some of the heat lost by the hot substance is used in raising the temperature of the container.

Suppose that in the last example the container has a mass of 0·75 kg and a specific heat capacity of 424 J/kg°C. To take account of this we would amend the calculation as follows:

$$\text{Heat gained by container} = 0{\cdot}75 \times 424 \times (t - 25)\ \text{J}$$
$$= 318 \times (t - 25)\ \text{J}$$

Note we have assumed that the container was initially at the same temperature as the water it contains.

This time we say

Heat lost by iron = Heat gained by water + Heat gained by container

$$2250\,(100 - t) = 25200\,(t - 25) + 318\,(t - 25)$$
$$225000 - 2250t = 25200t - 630000 + 318t - 7950$$
$$225000 + 630000 + 7950 = 25200t + 318t + 2250t$$
$$862950 = 27768t$$
$$t = 31{\cdot}08°\text{C}$$

Thus we see that including the container in our calculations made very little difference to the final answer. This is because of its low specific heat capacity and its small mass compared to that of the iron and the water.

EXERCISE B

In the following calculations assume the specific heat capacity of water is 4200 J/kg°C.

1. 60 kg of water at 80°C are mixed with 12 kg of water at 14°C, contained in a vessel of mass 10 kg and c = 1260 J/kg°C. Find the resulting temperature.
2. 30 kg of water at 90°C are poured into a vessel of mass 10 kg and specific heat capacity 1344 J/kg°C containing a block of iron of mass 7·8 kg and specific heat capacity 450 J/kg°C. If the initial temperature of the iron and vessel was 22°C calculate the final temperature of the mixture.

14.4 THERMODYNAMICS.

The study of the movement of heat forms a branch of physics known as thermodynamics (a word which readily suggests "heat", and "motion" as we have seen for example, in earlier chapters). Interest in heat transfer was stimulated during the early part of the nineteenth century, largely as a result of the development of the steam engine by James Watt, with its dependence on heat flow. The term is nowadays used, however, in its wider implications, to cover all aspects of the transfer of thermal energy from one place to another.

We have frequently used the words "assuming no losses" — and of course, all consideration of heat flow must assume, to begin with, that heat (which remember, is energy) cannot vanish into nothing. This is the law of the conservation of energy. As far as our immediate purpose is concerned, it may be "wasted" (by moving parts of the appliance itself, for example) — it may be "dissipated", that is, lost altogether into some unavailable form — but it cannot vanish into nothing. It still exists, somewhere — and usually by warming some part of its surroundings and so creating internal molecular energy in those surroundings.

So important is this concept in thermodynamics that it gave rise to what is usually termed the **First Law of Thermodynamics**.

In general terms, this states:—

The total energy of a closed system remains constant.

This pre-supposes that there is such a thing as a closed system — nothing getting out, nothing coming in. The production of mechanical energy (as in the steam engine) by the combustion of fuel, for example, is anything but a closed system. Wastage is enormous.

Conversely however, the production of heat by mechanical means can approximate fairly closely to a closed system, if suitable precautions are taken. Joule showed this very convincingly, and demonstrated that there was a very close correlation between the mechanical work done and the thermal energy generated. The amount of energy delivered to the receiving body was very nearly equal to the total energy created by mechanical means.

We ourselves have used this principle in our problems on mixing hot substances with cold — we assume that all the energy is

transferred across. In reality, however, there must always be some loss of available energy.

The **First Law** is sometimes expressed in rather more formal terms, as follows:—

Heat and mechanical work are mutually convertible and in any operation involving such conversion of one form of energy to the other, within a closed system, the total energy present remains constant.

This may seem very basic — and indeed it is so. But it must be remembered it was formulated at a time when even the very nature of energy was hardly understood.

The first law of thermodynamics (the law of conservation of energy) makes it clear that in no circumstances can one extract more energy from a closed system than the total energy present.

14.5 SECOND LAW OF THERMODYNAMICS

The first law merely states that the total energy of a closed system is constant — it does not tell us anything about how the energy may move from place to place within the system.

This is left for enunciation by the **Second Law**, which states:—

In any closed system heat will flow from a hot region to a cold region.

Early thought along these lines was directed, around 1824, by Carnot, who analysed in formal terms the working of a steam engine and gave his name to Carnot's cycle. In outline, he showed that the heat flow was from the hot region (the boiler) to the cold region (the condenser) and it was upon the difference in temperature between these two that the availability of energy depended. In no circumstances could a spontaneous heat flow take place in the reverse direction.

Moreover, the availability of energy required that the temperature difference be maintained. In a closed system, the temperature difference would ultimately diminish to zero. In a steam engine, an injection of fresh fuel keeps the boiler temperature up, and cold water keeps the condenser temperature down. There are examples in nature — within the atmosphere for example — where a fresh injection of "fuel" into a region can make more energy available and give rise to dynamical results.

However, the laws of thermodynamics only apply to closed systems, and if the first law confirms that one cannot possibly get more out of a system than the total energy present, the second imposes and even tighter limit. From the second law, it is plainly impossible to extract more energy from a system than the quantity of *available* energy present. Moreover, the *available energy is always less than the total energy.*

Hopeful inventors of perpetual motion machines over many decades have consistently ignored both laws.

WORKED EXAMPLE 5

A mass of 10 kilos in descending a distance of 15 metres is made to stir 2 kilos of fresh water by means of paddles. If all the energy lost by the mass goes into heating the water, calculate the rise in temperature of the water.

$$\text{Work done by mass} = mgh = 10 \times 9{\cdot}81 \times 15 \text{ J} = 1471{\cdot}5 \text{ J}$$
$$\text{Heat given to water} = mcT = 2 \times 4200 \times T = 8400 \text{ J}$$
$$\text{Work done by mass} = \text{Heat given to water}$$
$$1471{\cdot}5 = 8400\,T$$
$$T = 0{\cdot}18^\circ\text{C}$$

EXERCISE C

1. A mass of 2 kilos descends freely and in so doing operates a paddle which stirs 200 g of oil, $c = 1900$ J/kg°C. Assuming that all energy goes into heating the oil, how far would the mass have to fall to raise the temperature of the oil by 2°C?
2. Two men working in relays apply a steady force of 7 kgf to the handle of a wheel, radius 0·4 m, which stirs 11 kg of fresh water by means of paddles. Assuming that all work done goes into heating the water, find the rise in temperature after 150 turns of the wheel.

14.6 NATURAL SOURCES OF HEAT

Heat, being a form of energy, cannot be created — it can only be obtained from the natural reserves of heat in the earth, or by methods which themselves depend upon the earth's reserves of heat.

The principal commercial source of heat is by the combustion of fuel. A fuel is any substance capable of chemical combination with the oxygen in the atmosphere and may be solid, liquid or gaseous.

The efficiency of a fuel is usually defined by its **calorific value**, that is, the quantity of heat given out by the complete combustion of unit quantity of the fuel.

Some examples of calorific values are as follows. They do of course, vary somewhat according to the "grade" of any particular fuel.

Table 2

Calorific value

Wood	18×10^6 J/kg
Soft Coal	26×10^6 J/kg
Anthracite	33×10^6 J/kg
Fuel Oil	44×10^6 J/kg

14.7 PETROLEUM FUELS

In many parts of the world, and at varying depths, there are accumulations of hydrocarbons formed long ago by the decomposition of animal and vegetable remains. In its widest sense, the term **petroleum** embraces all hydrocarbon deposits occurring naturally in the earth, and petroleum itself may be liquid, gaseous or solid (such as bitumen or asphalt). In the commercial sense, however, the term is usually restricted to crude oil, and its gaseous form is known as **natural gas**.

Crude oil is refined by distillation into four main categories of fuel, gasoline, kerosene, gas oil (which includes diesel oils) and fuel oil. These fuels are burnt under widely varying conditions and each type of fuel must have combustion characteristics appropriate to its use. Of these, the most important are:—

(*a*) **Volatility**, or ability to give off vapour. Gasoline is a liquid that vaporizes readily, while kerosene and fuel oils require higher temperatures to induce vaporization.

(*b*) **Boiling Point.** Volatility is associated with boiling point. A liquid with a low boiling point is more volatile than one with a

higher boiling point. The temperature at which a pure liquid boils is its boiling point and this temperature remains constant until all the liquid has evaporated.

Most oil fuels are mixtures and hence each ingredient has its own characteristic boiling point, so that the "boiling point" of the fuel may extend over a considerable range of temperature.

(*c*) **Flash point.** This is the lowest temperature at which a combustible material will give off enough vapour to form an inflammable mixture with air. Thus the flash point assesses both the volatility and the inflammability of a fuel. Gasoline has a flash point well below normal atmospheric temperatures.

If an oil is heated above the flash point until it will burn continuously as an open flame, the temperature at which this occurs is called the **fire point.**

14.8 CRUDE OIL — ITS DISTILLATES AND THEIR USES

The four common fuels have already been referred to above, and it is now only necessary to note briefly their properties.

Gasoline (or petrol) is a very volatile fuel, used in spark-ignition internal combustion engines. It is highly inflammable and boils at temperatures varying from 30°-200°C. The combustion occurs in the combustion space in the cylinder, by means of a spark which ignites the mixture of fuel vapour or droplets and air. The flame passes rapidly across the combustion space causing a rapid rise in pressure and so provides a thrust to the piston. In this way, heat energy is converted into mechanical energy.

Kerosene is used primarily as an illuminant. It is less volatile and less inflammable than gasoline, boiling between 140°-300°C.

Diesel Fuel. For the most part these are grades, or blends, of all those distillates which are heavier than kerosene and are commonly known as **gas oil**, mostly boiling between 180°-370°C.

Various qualities are marketed to suit the type of engine. For the relatively slow moving marine diesel engine the oil may be quite thick, or viscous.

Diesel oil is the fuel used in compression-ignition internal combustion engines, usually referred to after their inventor as Diesel engines.

In these, air is drawn into the cylinder and is then compressed by the piston, thus raising its temperature. The liquid fuel is then

sprayed into the hot air in the form of fine droplets which are immediately ignited, rapid combustion occurs and the resulting high pressure provides thrust to the piston.

Fuel oils. These consist almost entirely of the residue from the distillation of the heavier crude oils. Fuel oil is burnt to raise steam in conventional boilers. They are viscous and are hard to vaporize and must first be heated to reduce viscosity and then atomised in special burners before they will burn efficiently.

The steam so raised may be used to provide thrust to pistons, as in the conventional steam reciprocating engine, or it may be used to provide rotary motion directly, as in the steam turbine, in which the steam passes through jets and causes rotation of a large number of blades arranged around a central axis. A fuller discussion on all types of engines mentioned above (reciprocating, internal combustion, turbine) will be found in any textbook on elementary engineering knowledge.

14.9 GASEOUS FUELS

The principal gaseous fuel has been coal gas. Natural gas obtained by boring into the earth's crust has been increasingly employed as a result of the continuing need to find and exploit new sources of energy. Important deposits of natural gas have been found, for example, in Europe and North Africa.

14.10 NUCLEAR FUEL

Finally, use is now being made of the heat energy released by the nuclear fission in the enriched uranium fuel elements in an atomic reactor. The use of this source of energy involves a degree of scientific knowledge and control which preclude it from use other than in a controlled environment and so far, developments have mainly been in atomic power stations. Here, the heat energy resulting from the nuclear fission is converted through some intermediate agent such as steam, into mechanical energy in the generators and thence into electrical energy. In this form, the energy is transmitted through overhead power lines to the consumers.

TEST PAPER 14

Specific Heat Capacity, Thermodynamics

(Use Table 1 or Table 2 as requisite)

1. How much thermal energy would be given out by a 7 kg ingot of copper in cooling from 400° K to 80° K? By how much would this raise the temperature of 50 kg of water?
2. If 10 kg of anthracite coal are fully consumed and only 5% of the heat generated is converted into mechanical energy, find the mechanical energy produced. If suitable machinery can be provided, 50% efficient, find the mass of water that may be raised to a height of 30 metres.
3. A boiler weighs 5 tonnes and is made of steel, $c = 430$ J/kg° C. It contains 4 tonnes of fresh water. Find the quantity of heat required to raise the temperature of the whole from 15° C to 85° C. If the fuel oil has a calorific value of 44×10^6 J/kg, find the mass of oil consumed, if only 20% of the heat from combustion goes into heating the boiler.
4. A piece of metal weighing 65 kg is removed from boiling water into a tank, of mass 89·25 kg and $c = 400$J/kg° C. The final temperature is observed to be 16·5° C. find the specific heat capacity of the metal.
5. Water falls freely from a chute 60 m high and as a result, the rise in temperature of the water is observed to be 0·12° C. Calculate the value of the specific heat capacity of water as given by this experiment.
6. A brass cube of mass 50 kg was removed quickly from boiling water and immersed in a vessel of heat capacity 80 J/° C containing 75 kg of oil at 14° C. The final temperature of vessel and contents was 22° C. Ignoring losses, calculate the specific heat capacity of the oil.
7. In a test in a closed vessel, 1·2 g of fuel oil was burned. The vessel contained 3 kg of water and the heat capacity of the vessel was 360 J/° C. After combustion was completed, the rise in temperature was 3·6° C. Find the calorific value of the fuel oil in J/kg.
8. A paddle driven by a 75 W motor is used to stir 1·24 kg of water in a vessel of heat capacity 112 J/° C, the initial temperature being 15° C. Assuming an overall loss of 60% of the energy produced, calculate the temperature of the water after 10 minutes.

CHAPTER 15

HEAT

Thermal Expansion

15.1 EXPANSION OF SOLIDS

We have already noted (section 13.3 (*e*)) that in general, all substances expand when heated, contract when cooled. Numerous everyday examples can be cited. It is convenient to express the extent of this expansion in terms of coefficients.

Coefficient of linear expansion

This may be defined as the increase in length which a bar of unit length undergoes when its temperature is raised through 1K or 1°C (strictly speaking from 0°C to 1°C).

The usual symbol for this coefficient is α.

Also note that α is independent of the units of length employed. The statement "the coefficient of linear expansion of brass is 0·000019" can be interpreted as

1 metre of brass wire expands by 0·000019 metre per 1°C rise.

1 cm of brass wire expands by 0·000019 cm per 1°C rise.

and so on.

Strictly speaking the unit for α is /°C i.e. α for brass is 0·000019/°C.

Now, if α is the amount by which a unit length expands for 1° rise in temperature then if there are L_0 units of length at 0°C, clearly the expansion will be $L_0 \times \alpha$, and if the temperature is raised, not by 1°, but by t degrees, then the *expansion* will be $L_0 \alpha t$.

Thus, the new length will be $L_0 + L_0 \alpha t$, and if we call this new length L_t, we have

$$L_t = L_o + L_o \alpha t$$

$$\textbf{i.e., } \mathbf{L_t = L_o(1 + \alpha t)} \text{ exactly} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad \mathbf{I}$$

This formula is quite easy to remember and should be learned.

The coefficient of expansion is not constant over all ranges of temperature, but for many practical purposes may be assumed to be so.

Hence, if the temperature rises from t_1 to t_2 one may write (with a fair degree of accuracy).

$$\mathbf{L_2 = L_1[1 + \alpha(t_2 - t_1)]} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad \mathbf{II}$$

Note we could again write this formula as

$$L_2 = L_1(1 + \alpha t)$$

but here $t = t_2 - t_1$

WORKED EXAMPLE 1

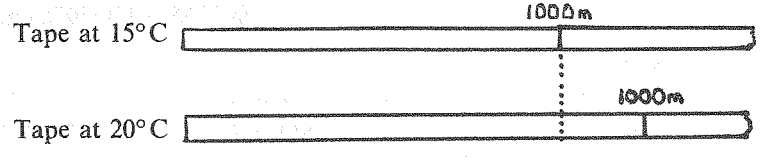
A straight length of iron pipe is found to measure 10 metres at a temp. 45° C. Find the increase in length when steam at 105° C flows through the pipe. (Assume that the pipe reaches 105° C and take $\alpha = 0{\cdot}000012/°\,C$).

$$L_2 = L_1(1 + \alpha t)$$
$$L_2 = 10\,(1 + 0{\cdot}000012 \times 60)$$
$$L_2 = 10{\cdot}0072 \text{ m}$$

Therefore the increase in length is 0·0072 m i.e. 7·2 mm.

WORKED EXAMPLE 2

A steel tape used for measuring distances is correct at 20° C. If the tape is used when the temperature is 15° C and the measured distance is 1000 metres, what is a better estimate of the correct length? (Take $\alpha = 0{\cdot}000012/°\,C$).



$$L_2 = L_1(1 + \alpha t)$$
$$1000 = L_1(1 + 0{\cdot}000012 \times 5)$$
$$\frac{1000}{1{\cdot}00006} = L_1$$
$$L_1 = 999{\cdot}94 \text{ m}$$

We can see from the above examples that the coefficient of linear expansion for metals is quite a small number.

For example for steel we took $\alpha = 0{\cdot}000012/°\,C$.

An alternative way of quoting this value would be $12 \times 10^{-6}/°\,C$.

Also the coefficient is often referred to as LINEAR EXPANSIVITY.

Coefficient of superficial expansion β

This may be defined as the increase in area which a surface or plate of unit area undergoes when its temperature rises through 1°C (strictly, from 0°C to 1°C).

It can be shown to be very nearly equal to twice the linear coefficient, that is,

$$\text{coeff. superficial expansion} = 2\alpha \quad \ldots\ldots\ldots \quad \text{III}$$

$$\beta = 2\alpha$$

Coefficient of cubical expansion γ

Again, this is the increase in volume which a unit volume undergoes when its temperature is raised from 0°C to 1°C.

It can be shown to be very nearly equal to three times the linear coefficient, that is,

$$\text{coeff. cubical expansion} = 3\alpha \quad \ldots\ldots\ldots\ldots \quad \text{IV}$$

$$\gamma = 3\alpha$$

WORKED EXAMPLE 3

A hollow brass vessel has a volume of 120 cm³ at 20°C. Find its volume at 65°C (α for brass = 0·000019/°C).

$$V_{65} = V_{20}\,(1 + 45 \times 3\alpha)$$
$$= 120\,(1 + 45 \times 0{\cdot}000057)$$
$$= 120 \times 1{\cdot}002565 = 120{\cdot}3078 \text{ cm}^3$$

EXERCISE A

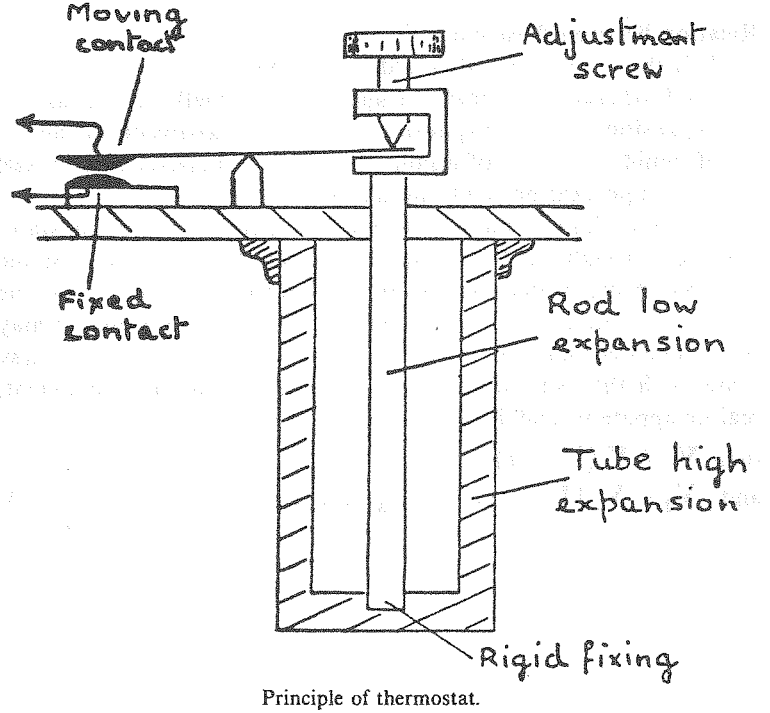
1. A metal bar has a length of 8 m at a temperature of 4°C. Calculate its length at a temperature of 94°C if the linear expansivity of the metal is 19×10^{-6}/°C.
2. When a steel rod is at a temperature of 11°C it has a length of 2·7 m. Determine the change in its length if it is heated to a temperature of 143°C. Assume the linear expansivity of steel is 12×10^{-6}/°C.
3. When heated through a temperature change of 120°C a rod changes length from 5·625 m to 5·670 m. Calculate the rod's linear expansivity.
4. A rectangular sheet of metal measures 1·2 m by 2·3 m at a temperature of 17°C. Calculate the area of the sheet at a temperature of 78°C given that the coefficient of linear expansion of the metal is 0·000017/°C.

5. A solid cube has side 9 cm at a temperature of 0°C. Calculate its sides' length at a temperature of 100°C given that its linear expansivity is $12 \times 10^{-6}/°C$.

Practical application — the thermostat

This useful device has many applications in modern life, being used to control the temperature at which some specific apparatus is required to operate.

The principle of a simple type of thermostat is illustrated in the sketch, and calls for little explanation. If the temperature of the substance surrounding the tube rises, the tube expands more than the rod, until finally contact is broken and the electric supply cut off.



Principle of thermostat.

As temperature falls again, the reverse happens, contact is made and the electric supply resumed.

15.2 EXPANSION OF LIQUIDS

Liquids have no shape and therefore no fixed dimensions. Thus, we are concerned only with their cubical expansions.

Moreover, they have to be contained in some vessel. Owing to the expansion of this vessel, the apparent expansion of the liquid is less than the true value (see also section 13.9).

Two coefficients of expansion are used:—

The coefficient of **apparent** expansion is the apparent increase of unit volume for 1° rise in temperature.

As with solids, the coefficient is usually quoted for 1°C rise from 0°C to 1°C.

The coefficient of **absolute or real expansion** is the true increase in unit volume for 1° rise in temperature. That is, allowance is made for the expansion of the vessel.

Relation between these coefficients

It is shown in more advanced works that

coeff. of real expansion of liquid = **coeff. of app. expansion of liquid** + **coeff. of cubical expansion of the material of the vessel**

This important principle should be remembered.

For most liquids the coefficient of expansion is not constant, varying throughout the temperature range, the most notable example being water (see section 13.3(*e*)). In other cases the variation is not great, however, and as in the case of solids, it may be assumed constant for many practical purposes, so that we may write, with the usual notation, if γ is the appropriate coefficient, real or apparent, and V_0 is the volume at 0°C.

then $\mathbf{V_t = V_0(1 + \gamma t)}$ **exactly**

and $\mathbf{V_2 = V_1\,[1 + \gamma\,(t_2 - t_1)]}$ very nearly V

15.3 APPLICATION TO LIQUID CARGOES

The expansion and contraction of liquid cargo is of great importance and must be taken into account, the commonest case of course, being that of bulk oil carried in tankers. *(A fuller treatment of this subject will be found in any standard work on tankers).*

One factor that must be appreciated is that the density of a given mass of (say) oil will change if the volume changes since

$$\text{Density} = \frac{\text{mass}}{\text{volume}}$$

So if the volume increases as a result of a rise in temperature, the density decreases, and conversely. This factor must be taken into account when computing the weight of oil in a tank, and tank tonnage tables are available for the purpose.

A common method is to measure the **ullage** — that is, the vertical depth of the empty space contained between the underside of the deck and the surface of the oil in the tank. From this, the volume of oil in the tank may be extracted from ship's *Calibration Tables*, and by using the density appropriate to the oil at its temperature, from the tables, the weight of oil in the tank may be calculated. This type of exercise, it will be appreciated, has as its object the determination of the exact weight of cargo on board.

Alternatively, and quite apart from the density changes, the changes of volume itself must be allowed for whenever the oil, during the voyage, is liable to reach a temperature higher than that at the loading port. In this case, if the tank is filled too full at a low temperature, the cargo will be forcing its way out through the tank valves on deck when the temperature rises. This is particularly important in the case of light-grade spirit cargoes, which must be loaded high in the tank to ensure that the ship will take its proper deadweight. When carrying the heavier crude oils, there is usually sufficient ullage to ensure that it will not overflow in any circumstances.

The coefficient of expansion, or **volume coefficient** as it is sometimes called, is tabulated for various grades of oil, and throughout the range of temperatures normally experienced. The volume coefficient may be used for computing the loading ullages so that a full cargo may be carried without any of the tanks overflowing when temperature rises during the voyage.

WORKED EXAMPLE 4

A tank is loaded at a temperature of 12°C, the ullage giving a volume for the liquid in the tank as 803 m^3. The discharging temperature is estimated to be 31°C. At this temperature the tank has a capacity of 815 m^3. Determine if the tank has been overloaded.

(Assume the volume coefficient of the liquid is $720 \times 10^{-6}/°C$)

$$V_2 = V_1 (1 + \gamma t)$$
$$= 803 (1 + 720 \times 10^{-6} \times 19)\ m^3$$
$$= 813{\cdot}99\ m^3$$

Thus the tank has not been overloaded.

This rather theoretical example serves to illustrate the principle. In practice, the whole operation would be done by the use of tables. The essential object is to allow sufficient ullage when loading to ensure that the tank will not overflow at the higher temperature.

15.4 GASES — THE GAS LAWS

One characteristic property which distinguishes a gas from a solid or a liquid is that a gas always fills any enclosure in which it is placed, its pressure and temperature adjusting themselves to the volume available.

We have referred elsewhere to the molecular structure of matter. A gas consists of molecules which are in continuous motion, and the "pressure" exerted by a gas is the result of the enumerable encounters between moving molecules and between molecules and the walls of the vessel. Note that we have used the word "encounters" deliberately — the molecules do not actually collide, since their mutual repulsion keeps them apart. In all these encounters, any kinetic energy lost by one molecule is transferred to another, so that the total internal kinetic energy of the gas remains constant. This is known as the **Kinetic Theory of Gases**. There are other provisions made by this theory but this is the principal one.

Since a gas fills any space available to it, if a vessel, say, containing a given mass of gas is made larger, the molecules will have more space to move about in, encounters will be fewer, and pressure will fall. The converse is also true.

Three laws sum up the behaviour of all perfect gases, and a "perfect" gas may be defined as one which obeys these laws.

(*a*) **Boyle's Law** — this has already been referred to. It states that **when the temperature of a given mass of gas is kept constant, the volume varies inversely as the pressure.**

$$\text{i.e., } V = K \times \frac{1}{P}, \text{ where K is some constant}$$

$$\text{i.e., } PV = \text{constant},$$

$$\text{or } P_1V_1 = P_2V_2 = P_3V_3 \text{ and so on } \ldots\ldots \quad \text{VI}$$

WORKED EXAMPLE 5

Air is enclosed in a 24 cm long tube, closed at one end, the atmospheric pressure being 1006 mb. If the air is now compressed to 2½ cm in length, its temperature remaining constant, what is the pressure on the air?

In this case let the cross-sectional area of the tube be A.

$$P_1 = 1006 \text{ mb} \qquad P_2 = ?$$
$$V_1 = 24 \times A \qquad V_2 = 2{\cdot}5 \times A$$
$$P_1V_1 = P_2V_2$$
$$1006 \times 24 \times A = P_2 \times 2{\cdot}5 \times A$$
$$P_2 = 9658 \text{ mb } (9{\cdot}658 \text{ bar})$$

Note that A cancelled out.

(*b*) **Charles's Law** tells us what happens when the temperature varies, the pressure being kept constant. In other words, we have a given quantity of a gas, and without bringing any increased pressure to bear on it, we are going to heat it, or cool it — in short, a straightforward question of expansion and contraction like heating a block of iron or stone.

Different solids and liquids have different coefficients of expansion. **All gases have the same coefficient of expansion, and its value is 1/273.** This leads us to the following definition:—

Charles's Law:—Definition.

The coefficient of expansion of a gas under constant pressure is $^1/_{273}$, that is, if the temp. of a given quantity of a gas be raised 1° C, its volume would increase by $^1/_{273}$ of its volume at 0° C ..... **VII**

The same applies if a gas is cooled. Thus, if we started off with a given quantity of gas at 0° C, and cooled it, for each 1° C it would loose 1/273 of its original volume, (its volume at 0° C) so that after cooling it for 273 C°, i.e., down to a temperature of − 273° C, there would be nothing left of the original gas.

This temperature, − 273° C, is known as the **Absolute Zero**, and a scale of temperature which starts at the Absolute Zero is called the Absolute Scale, or Kelvin Scale, and temperatures on it are called Kelvin (K). Thus, 0° C corresponds to 273K, and 100° C to 373K, and so on.

In problems, it is convenient to express Charles's Law as follows:—

The volume of a gas is proportional to its absolute temperature, the pressure remaining constant.

$$\text{i.e., } \frac{V_1}{V_2} = \frac{T_1}{T_2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad \textbf{VIII}$$

WORKED EXAMPLE 6

A given mass of gas occupies 150 cm³ at 60° C. What will be its volume at 40° C?

$$\frac{V_1}{V_2} = \frac{T_1}{T_2}$$

Remember T_1 and T_2 must be in Kelvin.

$$\frac{150}{V_2} = \frac{333}{313}$$

$$\frac{150 \times 313}{333} = V_2$$

$$V_2 = 141 \text{ cm}^3$$

(*c*) **Law of Pressures.** The pressure of a gas whose volume is kept constant increases by 1/273 of its pressure at 0° C for each Celsius degree rise in temperature.

From this it follows that if the volume is kept constant, **the pressure varies directly as the absolute temperature.**

$$\text{i.e., } \frac{P_1}{P_2} = \frac{T_1}{T_2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad \textbf{IX}$$

WORKED EXAMPLE 7

A cylinder contains a fixed volume of oxygen at 40° C the gauge showing 2·2 bar. What would the gauge read if the temperature fell to 5° C?

$P_1 = 2{\cdot}2$ bar $\qquad P_2 = ?$

$T_1 = 40° \text{C} = 313 \text{ K} \qquad T_2 = 5° \text{C} = 278 \text{ K}$

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

$$\frac{2{\cdot}2}{313} = \frac{P_2}{278}$$

which gives $P_2 = 1{\cdot}954$ bar

Combination of Boyle's and Charles's Law — the **Equation of a Perfect Gas.**

If pressure, volume and temperature all vary, we may combine the two laws in the following convenient formula, viz:—

$\frac{PV}{T}$ is constant,

$$\text{or } \frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad \mathbf{X}$$

WORKED EXAMPLE 8

8 m^3 of air in a closed boiler exert a pressure of 60 kN/m^2 at 20° C. What pressure would it exert at 60° C?

$P_1 = 60\ kN/m^2$ $\quad$ $P_2 =$

$V_1 = 8\ m^3$ $\quad$ $V_2 = 8\ m^3$

$T_1 = 20°C = 293\ K$ $\quad$ $T_2 = 60°C = 333\ K$

Note that since the boiler is closed the volume of the air cannot change.

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

$$\frac{60 \times 8}{293} = \frac{P_2 \times 8}{333}$$

From which $P_2 = 68{\cdot}19\ kN/m^2$

Note that there are no set units for V_1 and V_2, the only requirement is that they are in the SAME units. The same is also true for P_1 and P_2. However T_1 and T_2 MUST be in Kelvin.

WORKED EXAMPLE 9

2000 cm^3 of air are at a temp. of 50° C, the pressure on it being 750 mm of mercury. It is cooled down to 10° C, the pressure now being 765 mm of mercury. What is its new volume?

Using the formula as at **X**, we have,

$$\frac{750 \times 2000}{323} = \frac{765 \times V_2}{283}$$

from which $V_2 = 1718\ cm^3$

EXERCISE B

1. A car tyre is at 2·1 bar pressure when the temperature is 6° C. What would be the pressure when the temperature rose to 22° C?
2. A gas is at a pressure of 760 mm of mercury and a temperature of 0° C. (Standard temperature and pressure — see below in the next section). Its pressure and temperature are changed to 830 mm of mercury and 45° C respectively. Calculate its new volume if its original volume was 59 litres.
3. 70 litres of air at a pressure of 1000 mbar and temperature of 10° C are allowed to expand to 103 litres with the new pressure then being 810 mbar. Calculate the new temperature.
4. A gas in a cylinder exerts a pressure of 240 kN/m^2 at 50° C. What pressure would it exert when compressed to one quarter of its former volume, the temperature meanwhile rising to 185° C?
5. The air in a diving bell occupies 26 m^3 at sea level, where the pressure is 1 atmosphere and temperature is 36° C. What volume would it occupy when sunk to a depth of 44 m in the sea, where the temperature was 28° C.
   (1 atmosphere = 10 m of sea water)

15.5 THE GAS EQUATION

Again, if $\frac{PV}{T}$ = constant, we may write

$$\mathbf{PV = rT}$$

in which r is called the **gas constant**. This is known as the gas equation. It is very nearly true for all gases.

The numerical value of the constant r in the gas equation depends on the particular gas being considered.

It also depends on the *mass* of that gas which is being considered in any given case. For instance, let us take two samples of the same gas under the same conditions of pressure and temperature, but let the second sample be *twice* the mass (and therefore twice the volume) of the first.

Since PV must equal rT in each case, it is clear that the second sample will give a value for r twice that of the first.

This may be overcome by *taking* 1 kg *of the gas*, in which case V is the volume occupied by 1 kg of the gas at Standard Temperature and Pressure (S.T.P.) i.e., at 273 K and a barometer height of 76 cm of mercury.

Used in accordance with this principle, the equation $PV = rT$ is known as the **Characteristic Gas Equation for 1 kg** of a given gas. In it, in SI units, if

P is the pressure in N/m^2.
V is the vol. in m^3 of 1 kg of the gas at S.T.P.
T is the Kelvin temperature.

then r can be shown (not proved here) to be in J/kg/K.

WORKED EXAMPLE 10

Find the gas constant for oxygen if oxygen has a density of 1·429 kg/m³ at S.T.P. (R.D. mercury = 13·6).

Since "S.T.P." signifies a height of the barometer of 76 cm, we must first convert this into SI units of pressure using the formula $P = h\rho g$

i.e. $P = 76 \times 10^{-2} \times 13{\cdot}6 \times 1000 \times 9{\cdot}81 \text{ N/m}^2$
$= 101\,396 \text{ N/m}^2$

Also for the oxygen 1·429 kg has a volume of 1 m³ so 1 kg has a volume of $\frac{1}{1{\cdot}429} \text{ m}^3 = 0{\cdot}6998 \text{ m}^3$

hence in the gas equation $PV = rT$ we have

$$101396 \times 0{\cdot}6998 = r \times 273$$
$$r = 260 \text{ J/kg/K}$$

The value of r thus found, though constant for a given gas, will still vary from one gas to another (because V will be different, gases being of different densities).

15.6 THE UNIVERSAL GAS CONSTANT

Note first a remarkable law of physics, **Avogadro's Law.**

This states that under the same conditions of pressure and temperature, **all perfect gases have the same number of molecules per unit volume.**

Thus, if a gas A has a density greater than that of B, it must be because a molecule of A has a mass greater than a molecule of B. In fact, the masses of any fixed volume (say 1 cm^3) of two gases A and B would be proportional to their molecular masses.

If we therefore take a volume of a gas such that its mass in grammes is numerically equal to its molecular mass, such a volume will be the same for all gases.

Such a quantity of gas is called a **gram-molecule**, or **mole**, (abbreviation mol) and its value is about 22·412 litre. This value is the same regardless of what system of units is employed.

Hence, in the gas equation ($PV = rT$), since we are now proposing to use a value for V which is the same for all gases, it is clear that we shall get a value for r which is the same for all gases. It is usually given the symbol R.

In the gas equation $PV = RT$, since 22·412 litre is equal to $22{\cdot}412 \times 10^{-3}$ m^3, we now have,

$$h\rho g V = RT$$

$$76 \times 10^{-2} \times 13{\cdot}6 \times 10^3 \times 9{\cdot}81 \times 22{\cdot}412 \times 10^{-3} = R \times 273$$

from which R is 8·32 J/mol/K

Finally, it can be shown that $R = mr$, where m is the molecular mass of the gas used for r, and the universal gas equation will sometimes be met with in the form.

$$\mathbf{PV = mrT}$$

In the examples quoted above, for oxygen, molecular mass 32, the reader may easily verify that

$$32 \times 0{\cdot}2605 = 8{\cdot}32 \text{ (very nearly)}$$

Further discussion on molecular masses is more appropriate to a text-book on chemistry.

WORKED EXAMPLE 11

A vessel of 1000 litre capacity contains air at a pressure of 1·0132 bar and temperature 0°C. Air is pumped in until the pressure is 8·40 bar and the temperature is 81°C. Find the mass of air pumped in. Take r for air $= 0{\cdot}287 \times 10^3$ J/kg/K.

Since 1 bar $= 10^5$ N/m^2, and 1000 litre $= 1$ m^3, then in the characteristic gas equation $PV = rT$, if m_1 is the mass of air in kg in the vessel initially, we have,

$$1{\cdot}0132 \times 10^5 \times 1 = m_1 \times 0{\cdot}287 \times 10^3 \times 273$$

(remember that the value of r given is "per kg", and we have m_1 kg).

$$\text{i.e., } m_1 = \frac{1{\cdot}0132 \times 10^2}{0{\cdot}287 \times 273} \text{ kg} = 1{\cdot}293 \text{ kg}$$

In the second case, if $m_2$kg is the mass of air now in the vessel, then

$$8{\cdot}40 \times 10^5 \times 1 = m_2 \times 0{\cdot}287 \times 10^3 \times 354$$

$$\text{i.e., } m_2 = \frac{8{\cdot}40 \times 10^2}{0{\cdot}287 \times 354} \text{ kg} = 8{\cdot}268 \text{ kg}$$

$$\text{Mass pumped in} = m_2 - m_1 = 6{\cdot}975 \text{ kg}$$

EXERCISE C

1. 2000 litre of air at 0°C are found to have a mass of 2·586 kg when the pressure is 1·013 bar. Find r for air in J/kg/K in the characteristic gas equation.

15.7 ISOTHERMAL AND ADIABATIC EXPANSION

In **isothermal** expansion or compression of a gas no change in temperature occurs. In short, when we have used the words "the temperature remaining constant", as for example in using Boyle's Law, we have been stipulating conditions of isothermal expansion or compression.

Heat must be supplied to a gas during isothermal expansion since unless this is done, the temperature of a gas will fall sharply if allowed to expand rapidly. This inflow of heat, for example, might come from the surrounding atmosphere. Owing to the fact that the gas must be contained in some vessel, such inflow of heat is never instantaneous. But when all has settled down, and cylinder (say) and contents are all at the same atmospheric temperature as before, the expansion has taken place isothermally.

Conversely, when a gas is compressed, its temperature rises. So for isothermal compression, heat must be removed from the gas. This would be done for example, in the diving bell, by the surrounding sea acting as a coolant.

In **adiabatic** expansion or compression, no heat is allowed to enter or leave the gas during the operation.

An approximation to these conditions may be obtained by the exact converse of the process just referred to, that is, by allowing the expansion or compression to take place sufficiently quickly for no heat to pass through the walls of the container. Under such conditions, the gas may be said to have "expanded adiabatically" — and of course its temperature would fall sharply. Examples of adiabatic expansion and compression occur in air masses and are of considerable importance in meteorology.

Neither isothermal nor adiabatic conditions can be achieved to perfection in Nature, but nevertheless, both concepts are important and calculations based on them provide close approximations to the truth under suitable conditions.

The formula $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$ applies for both isothermal and adiabatic changes. In the isothermal case, however, $T_1 = T_2$ and therefore the formula reduces to Boyle's Law, namely $P_1V_1 = P_2V_2$.

In the adiabatic case the pressures and volumes do not obey Boyle's law but are related according to the following law:

$$P_1V_1^{\gamma} = P_2V_2^{\gamma}$$

In this formula γ is a constant called the ratio of the principal specific heats. We shall not go into any further detail regarding this constant but simply note that it is a constant for a given gas and a typical value is 1·4.

WORKED EXAMPLE 12

A gas of volume 260 litres at s.t.p. is expanded adiabatically to 510 litres. Calculate its new pressure. Hence find its final temperature. Assume $\gamma = 1{\cdot}4$.

$$P_1 = 760 \text{ mm Hg} \qquad P_2 = ?$$
$$V_1 = 260 \text{ litres} \qquad V_2 = 510 \text{ litres}$$
$$T_1 = 0°\text{C} = 273 \text{ K} \qquad T_2 = ?$$
$$P_1V_1^{\gamma} = P_2V_2^{\gamma}$$
$$760 \times 260^{1{\cdot}4} = P_2 \times 510^{1{\cdot}4}$$
$$\frac{760 \times 260^{1{\cdot}4}}{510^{1{\cdot}4}} = P_2$$
$$P_2 = 295{\cdot}9 \text{ mm Hg}$$
$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$
$$\frac{760 \times 260}{273} = \frac{295{\cdot}9 \times 510}{T_2}$$
$$T_2 = \frac{295{\cdot}9 \times 510 \times 273}{760 \times 260}$$
$$T_2 = 208{\cdot}5 \text{ K } (-64{\cdot}5°\text{C})$$

WORKED EXAMPLE 13

The pressure of 1000 litres of a gas is changed from 140 kN/m^2 to 80 kN/m^2 by means of an adiabatic expansion. Calculate the gas's final volume given that $\gamma = 1{\cdot}3$.

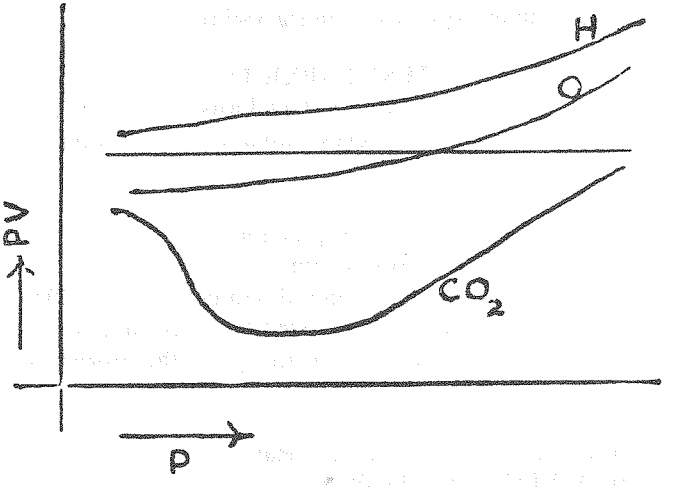
$$P_1 = 140 \text{ kN/m}^2 \qquad P_2 = 80 \text{ kN/m}^2$$
$$V_1 = 1000 \text{ litres} \qquad V_2 = ?$$
$$P_1V_1^{\gamma} = P_2V_2^{\gamma}$$
$$140 \times 1000^{1{\cdot}3} = 80 \times V^{1{\cdot}3}$$
$$\frac{140 \times 1000^{1{\cdot}3}}{80} = V_2^{1{\cdot}3}$$
$$V_2^{1{\cdot}3} = 13901$$
$$V_2 = 13901^{1/1{\cdot}3}$$
$$V_2 = 1538 \text{ litres}$$

EXERCISE D

1. 900 litres of gas at s.t.p. are expanded adiabatically to a volume of 1200 litres. Calculate its final pressure and temperature. Assume $\gamma = 1{\cdot}4$.
2. A gas at s.t.p. is isothermally compressed from 8 m^3 to 5 m^3. It is then adiabatically compressed to 3 m^3. Calculate the final pressure and temperature of the gas given that $\gamma = 1{\cdot}4$.

15.8 DEPARTURES FROM BOYLE'S LAW

If the product PV is plotted against values of P for a perfect gas, it will give a straight line parallel to the x (i.e., P) axis, since PV is constant.



Departure from Boyle's Law.

Most so-called permanent gases like oxygen (O), hydrogen (H), nitrogen (N) obey this law very closely at moderate pressures and at normal (and of course, constant during any given set of measurements) temperatures.

A notable exception, however, is carbon dioxide (CO_2) whose graph is illustrated. At low pressures, and particularly at a temperature approaching the temperature of condensation, the value of PV varies considerably from that for a perfect gas.

Joule and Thomson (later Lord Kelvin) carried out a long series of experiments on the behaviour of gases when the volume was allowed to increase sharply, the external temperature surrounding the gas remaining constant. Gas was forced slowly through a very small orifice, and it was found that on expanding on emergence from the orifice, **there was always a change of temperature.** (Theoretically, there would be no change in temperature for a "perfect" gas, but gases are not perfect).

For hydrogen, whose curve is above the *PV* line for a perfect gas, there was a rise in temperature. For all gases whose curves are below the *PV* line there was a **fall** in temperature, and the **cooling effect was greatest for carbon dioxide** which is the least perfect of the common gases.

This cooling effect was named the Joule-Thomson effect, and it is employed in many systems of refrigeration.

TEST PAPER 15
Expansivity, Gas Laws

1. *(a)* With the aid of sketches, explain how allowance is made for expansion in:
(i) Steam pipes.
(ii) Tankership cargo pipelines.
(iii) A ship's chronometer.

(b) An ideal gas at an initial temperature of 20° C and pressure 1 bar is compressed until its volume is halved. Find the final pressure of the gas if the process is:
(i) Isothermal.
(ii) Adiabatic.

Take the ratio of specific heats for the gas to be 1.4 (SCOTVEC March 1990).

2. *(a)* Define 'linear expansivity'.

(b) State the approximate value of 'linear expansivity' as compared to 'cubical expansivity' of a solid.

(c) It is required to thread a copper wire of 2·5 mm diameter at 15° C through a smaller round hole in a carbon block. If the process can just be carried out at − 100° C, calculate the diameter of the hole at 15° C.
Linear expansivity of copper = $17 \times 10^{-6} K^{-1}$
Linear expansivity of carbon = $5 \times 10^{-6} K^{-1}$

(SCOTVEC July 1990).

3. A parcel of air at a pressure of 1020 mbar rises and expands adiabatically to twice its original volume, when its pressure is found to be 380 mbar. Calculate the ratio of specific heat capacities of the air.
(SCOTVEC December 1990).

4. *(a)* Define the following terms:
(i) 'Cubical expansivity'.
(ii) 'Apparent expansitivity'.
(b) A steel ullage tape is used to measure the length of a copper wire and the length is found to be 10 metres when both tape and wire are at a temperature of 5° C. If the temperature of both tape and wire are raised to 25° C, calculate the reading which would show on the tape if it is again used to measure the length of the wire.
Linear expansivity of steel $= 12 \times 10^{-6}K^{-1}$
Linear expansivity of copper $= 17 \times 10^{-6}K^{-1}$
(SCOTVEC March 1991).

5. *(a)* State the meaning of the following terms:
(i) An adiabatic process.
(ii) An isothermal process.
(b) Two litres of gas enclosed by a piston is at an initial temperature of 20° C. The piston moves so that the pressure of the gas doubles whilst its temperature rises to 50° C. Calculate the final volume of gas in the piston.
(SCOTVEC July 1991).

6. A white metal collar is to be expanded so as to fit on to a ship's tail end shaft. If the shaft has a diameter of 1000 mm at a temperature of 15° C, calculate the inner diameter of the collar at 15° C if it just fits on to the shaft at a temperature of 215° C.
Linear expansivity of shaft $= 14 \times 10^{-6}K^{-1}$
Linear expansivity of collar $= 17 \times 10^{-6}K^{-1}$
(SCOTVEC December 1991).

7. *(a)* With the aid of a simple sketch, outline one method which is used to overcome the effects of expansion in the ship's structure or equipment.

(b) Two engine components, made of steel and copper, are each 10 cm in length and are fixed so that they are only free to expand towards one another, so closing the gap between them as the temperature rises. Calculate the clearance between them when the engine is at 15° C if there is to be a gap of 1 mm between them when the engine is at its running temperature of 150° C.

Linear expansivity of steel $= 11 \times 10^{-6}K^{-1}$

Linear expansivity of copper $= 17 \times 10^{-6}K^{-1}$

(SCOTVEC March 1992).

8. During the first stroke of a marine pump, an ideal gas is compressed isothermally from an initial pressure of 2 bar to a pressure of 8 bar. The gas is then expanded adiabatically so that its final pressure is 5 bar. If the initial volume of the gas was 300 cc, find its final volume. The ratio of specific heats for the gas is 1·32.
(SCOTVEC July 1992).

9. *(a)* A gas at an initial pressure 1 bar, initial temperature 47° C, initial volume 300 cc, is compressed in a cylinder by a piston to a volume of 100 cc, when its temperature is found to be 177° C. Calculate the final pressure of the gas.

(b) Assuming the process carried out in Q.9 *(a)* was adiabatic, calculate the *ratio of specific heats* for the gas.
(SCOTVEC December 1992).

10. *(a)* State the approximate mathematical relationship between *linear expansivity* and *cubical expansivity*.

(b) Two engine parts, one of brass and the other of steel, each 500 mm long, are fixed in an engine so that they are free to expand towards one another and are 4 mm apart at an ambient temperature of 30° C. If a minimum clearance of 3 mm must be maintained between them to prevent the engine seizing up, calculate the maximum allowable running temperature of the engine.

Linear expansivity of steel $= 15 \times 10^{-6}K^{-1}$

Linear expansivity of brass $= 17 \times 10^{-6}K^{-1}$

(SCOTVEC March 1993).

11. *(a)* (i) Define the term *isothermal process.*
(ii) Define the term *adiabatic process.*

(b) An ideal gas at an initial pressure of 0·75 bar is compressed until its volume is halved. Calculate the final pressure of the gas if this is done by EACH of the following:
(i) An isothermal process
(ii) an adiabatic process.

Take the ratio of specific heats for the gas to be 1·4.
(SCOTVEC July 1993).

12. *(a)* If the density of hydrogen at S.T.P. is 0·09 kg/m^3 calculate the value of the gas constant for hydrogen in J/kg/K.
(R.D. of mercury = 13·6).

(b) Taking 1 mol as 22·4 litre, calculate an approximate value for the universal gas constant R in J/mol/K. Hence deduce an approximate value for the molecular mass of hydrogen.

CHAPTER 16

HEAT

Change of state, latent heat, vaporisation

16.1 THE EFFECT OF HEAT IN CAUSING CHANGE OF STATE

Remember that while a substance is changing state, e.g., ice into water, although heat is continually being poured into the ice, **no rise in temperature takes place** until all the ice is melted. After that, of course, the temperature rises until the water boils, when it remains stationary again until all the water is converted into steam. What happens to the heat used in this way, in effecting the change of state? Heat used in effecting a change of state in a substance is called the **Latent Heat** (latent — hidden) of that substance. (See section 13.3 *(d)*).

We shall only consider water.

16.2 LATENT HEAT OF FUSION

The amount of heat or thermal energy required to convert 1 kg of ice at 0° C into water at 0° C is about 336×10^3 J.

This quantity is called the **Specific Latent Heat of Fusion of Ice** (fusion — melting).

Thus,

Specific latent of heat of fusion of ice $= 336 \times 10^3$ **J/kg** I

WORKED EXAMPLE 1

How much heat is required to bring 20 kg of ice at 0° C up to 30° C?

Heat required to melt it

$= 20 \times 336 \times 10^3$ J $= 6\ 720\ 000$ J

Heat required to raise 20 kg of water through 30° C

$= 20 \times 4200 \times 30$ J $= 2\ 520\ 000$ J

Total heat required $= 9\ 240\ 000$ J (9·24 MJ)

WORKED EXAMPLE 2

12 kg of ice at 0° C are dropped into 30 kg of water at 62° C. Find the resulting temperature, the containing vessel having a mass of 90 kg and a specific heat capacity of 700 J/kg °C.

The essential principle is,

Heat gained by ice = heat lost by warm water and vessel.

Now, it takes $12 \times 336 \times 10^3$ J to melt the ice, and then, if the final temp. is t° C, it will take $12 \times 4200 \times t$ J to raise it from 0° to t°.

So heat gained by ice $= (4\,032\,000 + 50\,400t)$ J ..... (1)

Heat lost by water and vessel

$= 30 \times 4200 \times (62 - t) + 90 \times 700 \times (62 - t)$ J ..... (2)

Equating (1) and (2) and solving,

$t = 32{\cdot}1°$ C = final temperature.

WORKED EXAMPLE 3

Find the heat required to raise 10 kg of dry ice at − 12° C to water at 15° C. (specific heat capacity of dry ice = 2100 J/kg° C.).

Heat required to raise ice from − 12° C to 0° C

$= 10 \times 2100 \times 12 = 252\,000$ J.

Heat required to convert ice into water at 0° C

$= 336\,000 \times 10 = 3360\,000$ J.

Heat required to raise water from 0° C to 15° C

$= 10 \times 4200 \times 15 = 630\,000$ J.

and adding these quantities, total heat required

= 4 242 000 J (4·242 MJ).

Further note

We have seen that 336×10^3 J of heat has to be used in turning 1 kg of ice into water. This latent heat forms a sort of "reserve of heat" — it is hidden, latent, but it is still there; a kind of "reservoir" of heat stored up in the water, which has all to be released again as water freezes into ice.

Without this "reservoir", water would instantly solidify as soon as the temp. reached 0° C, a phenomenon which would have many inconvenient consequences. As it is, the whole of the reservoir of latent heat has to be used up before the mass of water can freeze. Thus, freezing takes place gradually, commencing with the surface layers (important for the fish) and moreover, no matter how cold its

surroundings may be, the water under the ice will remain at 0° C until all water is frozen. Fortunately, this seldom occurs.

Thus, not only the fish, but all other forms of marine growth live in a medium which never falls below 0° C.

16.3 LATENT HEAT OF VAPORISATION

The amount of heat, or thermal energy, required to convert 1 kg of boiling water at 100° C into steam (vapour) at the same temperature is called the **Specific Latent Heat of Vaporisation of Water**, and is about 2250×10^3 J/kg.

Thus,

Specific latent heat of vaporisation of water
$= 2250 \times 10^3$ J/kg II

Note. For short, this is sometimes called the "latent heat of steam".

This term is just as good, because, of course, it is in the steam that the thermal energy is stored, and it must be given up again when steam is condensed back into water.

WORKED EXAMPLE 4

20 kg of steam at 100° C are passed into a mixture of 30 kg of ice and 100 kg of water at 0° C. Find the resulting temperature, the containing vessel having a mass of 48 kg and a specific heat capacity of 875 J/kg °C.

Heat given up by the steam
= heat gained by ice and water and vessel.

Heat given up in converting steam into water at 100° C
$= 20 \times 2250 \times 10^3$ J ..... (1)

If t° C is the final temp, heat given up in falling to this temp.
$= 20 \times 4200\ (100 - t)$ J ..... (2)

Heat used in melting ice
$= 30 \times 336 \times 10^3$ J ..... (3)

Heat used in raising 130 kg ice and water and vessel from 0° to t° C
$= (130 \times 4200 \times t) + (48 \times 875 \times t)$ ..... (4)

Hence, since (1) + (2) = (3) + (4), we have, solving,
$t = 64{\cdot}5$° C = final temperature.

WORKED EXAMPLE 5

Find the heat required to convert 100 kg of water at 90° C into steam at 100° C.

Heat required to raise water from 90° C to 100° C
$= 100 \times 4200 \times 10$
$= 4\ 200\ 000$ J

Heat required to convert water into steam
$= 100 \times 2\ 250\ 000$
$= 225\ 000\ 000$

and adding,

Total heat required
$= 229\ 200\ 000$ J (229·2 MJ).

EXERCISE A

Use the data given below and neglect all losses to the surroundings.

Specific heat capacity of copper = 400 J/kg °C
Specific heat capacity of water = 4200 J/kg °C
Specific Latent Heat of Fusion of Ice = 336 000 J/kg
Specific Latent Heat of Vaporisation of water = 2 250 000 J/kg
Specific Heat capacity of brass = 380 J/kg °C

1. A copper vessel of mass 12 kg contains 60 kg of water, both being at room temperature, 36° C. Into it is passed 4 kg of steam at 100° C. Find the resulting temperature of vessel and contents.
2. 6 kg of ice at 0° C is dropped into 11 kg of water in a vessel of mass 1·5 kg and specific heat capacity 800 J/kg° C, both at 72° C. Find the temperature of the vessel and contents at the instant all the ice is melted.
3. A drum of negligible mass contains 26 kg of water at 25° C. Steam is passed through it until the temperature of the water is 85° C. What mass of steam has been passed through?
4. 6 kg of steam at 100° C is passed through a brass vessel of mass 32 kg, containing 125 kg of water at 12° C. Find the resulting temperature of vessel and contents.
5. 4 kg of steam at 373 K are passed into 20 kg of water and 5 kg of ice, both at 273 K, in a vessel whose mass is 48 kg and specific heat capacity is 700 J/kg° C. Find the resulting temperature.

6. 150 g of a molten metal alloy which is just at its melting point, 1120° C, is allowed to solidify at the same temperature and gives out 17·2 kJ of heat. What is the specific latent heat of fusion of the alloy?

16.4 EVAPORATION AND CONDENSATION

The **Vaporisation**, or **Evaporation**, of water can (and *does*) take place at all temperatures. The reverse process, of converting water vapour back into a liquid is called **Condensation**.

In both processes, latent heat is involved, just as in vaporisation at the temperature of boiling, though the quantity of latent heat increases at lower temperatures. At 0° C, for instance, the latent heat required to convert 1 kg of water into water vapour at 0° C is about 2·5 MJ.

However the important thing to remember is that whenever evaporation takes place, heat is required, and this heat is taken from the surroundings.

A large bowl of water in a room, appreciably cools the room because as it evaporates (in fact, to enable it to evaporate) it absorbs heat from its surroundings.

A porous earthenware water jar acts on the same principle, only in this case it also absorbs heat from itself and so cools the water inside as evaporation takes place on the outside.

The wet bulb of the hydrometer is another instance, and one which draws attention to a further point, namely, that **the more rapidly evaporation is taking place, the more rapidly** will be heat drawn from the surroundings. Hence, on a dry day, the "depression of the wet bulb" is most pronounced.

16.5 TRANSFER OF HEAT FROM TROPICAL TO TEMPERATE REGIONS

(a) By Ocean Currents

We have already referred (section 14.2) to the **high specific heat capacity of water**. This is responsible for the equable climates enjoyed by most maritime countries. The water acts as a kind of storehouse of heat, releasing it slowly in the winter, and conversely,

being slow to heat up in the summer. Hence extremes are avoided — such extremes of temperature as are found for instance, in the interior of vast continents.

Similarly, the high specific heat of water enables vast quantities of heat to be transferred from the tropics into higher latitudes by means of **Ocean Currents**. The notable example is the *Gulf Stream*, which has its origin in the Gulf of Mexico and, after flowing across the North Atlantic finally expends itself on the North Western shores of Europe, many months after it left the Gulf, **still releasing vast quantities of heat** — enough to raise the average winter temperature of the British Isles, for instance, some 15° C above that of Labrador, in the same latitude.

(b) **By Moving Air Masses**

We have also spoken (section 16.3) of the **latent heat of vaporisation**, the quantity of heat which is drawn from the surroundings when water goes into vapour from. The water vapour thus forms a sort of "storehouse" for heat — and in a large air mass, on a vast scale. For example, about 2·5 MJ joule are absorbed for each kilogramme of water evaporated at 20° C, an **equal quantity being released** when condensation occurs.

Now, the most vigorous evaporation from the earth's surface takes place in the tropics, and much of the water vapour so formed ultimately finds its way to the middle and higher latitudes, where it provides the moisture for the clouds and rain of a cyclonic depression for example.

In the process of evaporation, however, heat is drawn from the tropics and on condensation, is released in higher latitudes, thus helping to reduce extremes of temperature in both regions.

The heat thus released in higher latitudes is believed to provide most of the energy needed for such meteorological phenomena as tropical revolving storms, cyclonic depressions, thunderstorms and even for the towering cumulo-nimbus cloud — all phenomena, in fact, which depend on **powerful ascending currents of air**. The latent heat released during condensation tends to make the air where the condensation is taking place **warmer than its surroundings**, so that it **rises**. The strong convectional currents thus caused give rise in turn to the phenomena named above.

16.6 USE OF WATER IN HEAT ENGINES

We have already spoken of water as "slow to heat up, slow to cool down". In other words, a great deal of heat has to be passed into water to raise its temperature — it is said to have a **High Thermal Capacity**.

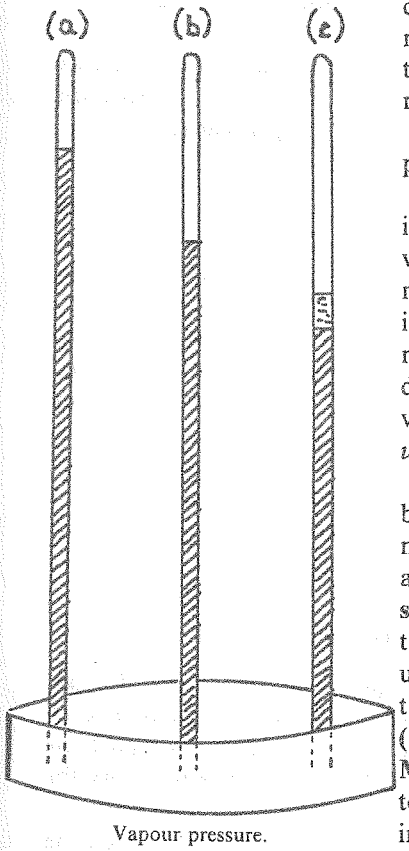
The thermal capacity of water adds very greatly to its value as a source of energy. Heat is energy. A mass of water (say several tons in a marine boiler) when raised to a high temperature, contains an enormous quantity of energy. And moreover, this energy is easily converted into mechanical energy in the steam engine.

Other obvious advantages of water as the working medium in a heat engine are its cheapness, the fact that it is available everywhere, is easy to handle, and finally, is inexpendable, for remember, having done its work, the steam turns back into water again.

16.7 VAPOURS AND VAPOUR PRESSURE

Although reference so far has been to water vapour, all liquids give off vapour. Some, such as ether, benzine, gasoline are very volatile and give off vapour freely, whilst from others, such as mercury, the vapour given off at normal temperatures is negligible.

From what has been said about the kinetic theory of gases, it will be appreciated that a vapour, which consists of molecules (of its liquid) in rapid motion, will exert a pressure just like a perfect gas, and this is termed the **vapour pressure**. This may be illustrated, by consideration of the diagram which represents three barometer tubes, the lower end being immersed in a mercury bath.



Vapour pressure.

(a) therefore shows the atmospheric pressure.

(b) if a few drops of water are introduced at the bottom of *(b)*, they will of course float to the top of the mercury and will then evaporate into the space. The level of the mercury is pushed down slightly, due to the vapour pressure. The vapour in this condition is said to be *unsaturated*.

In *(c)* more and more water has been introduced until finally, no more will evaporate. The space above the mercury is now said to be **saturated** (with water vapour) and the pressure exerted by the vapour under these circumstances is called the **Saturated Vapour Pressure. (S.V.P.)** or sometimes, the **Maximum Vapour Pressure,** at that temperature. It is usually expressed in mm of mercury.

Any *enclosed* space above a liquid is always saturated with the vapour of that liquid. Also.

different vapours exert different S.V.Ps

If *(c)* is now heated, more water will evaporate, and the mercury level will be pushed down still further.

The S.V.P. increases with temperature

The final state is reached when the boiling point of the water is reached (100° C) when the mercury level will be pushed right down to the level of the mercury in the trough.

The boiling point of a liquid is the temperature at which its S.V.P. is equal to the atmospheric pressure.

Note that this vapour, which is in contact with its liquid, is in the saturated condition.

The steam leaving a boiler is saturated. It may then be passed through tubes surrounded by hot gases, which raise its temperature, it becomes dry steam and is **unsaturated** or **superheated.**

16.8 EFFECT OF PRESSURE ON BOILING POINT

Consideration of the previous paragraph will show that if we increase the pressure on the surface of a liquid, a higher S.V.P. must be reached for the liquid to boil, i.e., the temperature of boiling is raised. In other words, the boiling point of a liquid becomes higher when the pressure on the liquid is increased, and conversely.

This fact is utilised in the distillation of crude oil, for example, where the various components are distilled off in turn at their respective boiling points. (See section 14.7).

16.9 DALTON'S LAW OF PARTIAL PRESSURES

This states that if two or more vapours, or a vapour and a gas, occupy a space, then providing there is no chemical action, each exerts the pressure which it would exert if it alone occupied the space, and the total pressure exerted by the mixture is simply the sum of the two individual pressures.

This is not difficult to understand if the reader has grasped the concept of a gas (section 15.4) as a loosely arranged collection of molecules, completely filling the space into which it is introduced and, by the energy of its molecules, exerting a pressure, p_1.

There is plenty of space between the molecules, so that when a second gas is introduced, its molecules can freely move about between the others, and completely fill the space and this gas will exert it own pressure p_2.

Hence the total pressure, $P = p_1 + p_2$ III

Note that perfect gases obey Boyle's and Charles's Laws and so do unsaturated vapours. Saturated vapours do not obey either law.

Heating a saturated vapour simply makes it unsaturated.

Cooling a saturated vapour simply causes some vapour to condense back into liquid.

If we increase the volume of the enclosing space above the liquid it becomes temporarily unsaturated. More molecules enter it, however, until it is again saturated with S.V.P. as before.

If we decrease the space, it momentarily contains more molecules than it can manage, some are condensed back into the liquid and once again, the S.V.P. is as before.

i.e., the S.V.P. is not dependent on the volume.

These matters are referred to again below.

16.10 CRITICAL TEMPERATURE

The critical temperature may be defined as the highest temperature at which a substance in the gaseous state can be liquefied by the application of pressure.

A substance which is in a gaseous state and at a temperature higher than the critical temperature cannot be liquefied merely by pressure. It must first have its temperature reduced, as is done for example, in the carriage of liquid methane gas in bulk.

The term **gas** is usually employed when the substance is above its critical temperature and **vapour** when it is below. Hence it may be said that vapours can be liquefied by pressure alone, gases can not.

The critical pressure is the pressure which will just cause liquefication at the critical temperature. Of course, at temperatures lower than the critical temperature, lower pressures will suffice.

Approximate values for three substances used in refrigeration are as follows—

	Crit. Temperature	Crit. press. in atmospheres
Carbon dioxide	31° C	73
Ammonia	130° C	115
Freon (type R.12)	112° C	40

Note Freon is a collective term for a group of organic liquids widely used as refrigerants.

These will be referred to again in the following paragraph.

16.11 REFRIGERATION

Refrigeration is a branch of applied physics which deals with the production of low temperatures.

Most systems of refrigeration rely on two principles, both of which have been referred to previously, but which are briefly reiterated here—

(a) When evaporation occurs freely from the surface of a liquid, that is, without the application of heat, the energy to perform the vaporisation must come from the liquid itself and consequently its temperature is reduced.

(b) When a gas or vapour expands freely, that is, its confining space is suddenly enlarged, its temperature is reduced.

Most refrigerating plants incorporate both these principles. Vapour is used as the working fluid, or refrigerant, and alternatively is converted into liquid form, evaporated and expanded again as a vapour, condensed again into liquid, and the whole cycle is repeated.

This cycle, compression and condensation-vaporisation-expansion (as a vapour) — compression, is basic to most refrigerating systems.

Refrigerants commonly employed are carbon dioxide, ammonia, freon, and sulphur dioxide, of which the first two are the ones most extensively used for industrial purposes.

Carbon dioxide can be liquefied by applying pressure, provided the temperature does not exceed 31° C, and at this temperature about 73 atmospheres is required. This working pressure, although

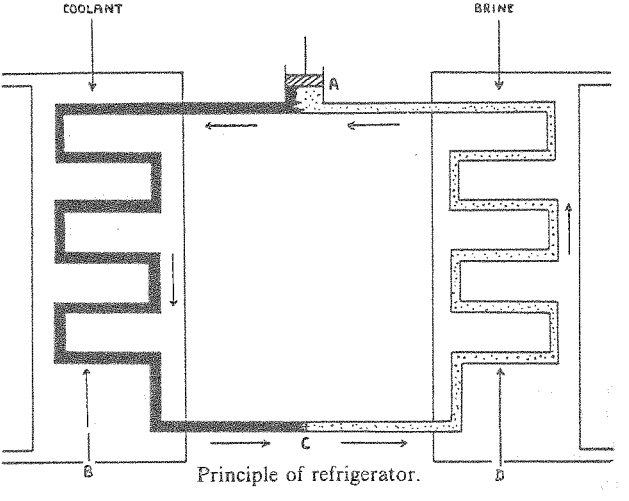
high, presents no difficulty; and also, for example, at a temperature of 13° C, 50 atmospheres pressure is sufficient. Also the actual volume of CO_2 vapour required is small and this enables the machine to be fairly compact.

The working pressures in an ammonia machine are lower (for example, at 13° C only some 7 atmospheres) but the volume of vapour is approximately twenty times as great as in a CO_2 machine. This makes it more suitable for shore installations.

Sulphur dioxide suffers from the disadvantage that its working pressures are low — below atmospheric, and so there is a possibility of air leaking *into* the machine and so affecting its working. It is only suitable where a low working pressure is a positive advantage.

Freon is the refrigerant most commonly used in a domestic refrigerator.

The refrigerating principle as used in industrial refrigeration is illustrated in the following sketch.



Principle of refrigerator.

It will be appreciated that compression of the refrigerant actually causes *warming*, so from the compressor at *A*, the working fluid passes through the coil *B*, in order to cool it slightly. *B* might be surrounded simply by a circulation of cool air, or in a bigger plant, by a continuous flow of cold water. At this point, as a result of the combined effect of compression and condensation, the refrigerant is a liquid.

It then passes through the finely adjusted valve *C*, which causes it to vaporise and also allows it to expand into coil *D*, and in doing so, its temperature falls.

Coil *D* is surrounded by a liquid, usually strong brine which has a freezing point of about − 10° C, so that, as the cycle continues, the brine may be cooled to a quite low temperature and still remain liquid. The brine is circulated through the chamber to be chilled.

Finally, the vapour from *D* is drawn back into the compressor at *A* and the whole cycle is repeated.

The compressor is usually an electrically driven pump, with a thermostat incorporated so as to shut off the electricity when a predetermined temperature is reached.

TEST PAPER 16
Change of State, Latent Heat, Vaporisation

Where appropriate use the data given at the beginning of EXERCISE A

1. 50 kg of ice at 0° C are dropped into 240 kg of water at 80° C. The mass of the container is 168 kg and its specific heat capacity is 400 J/kg° C. Find the final temperature.
2. 20 kg of steam at 100° C are passed into 206 kg of water at 12° C. Find the resulting temperature given that the container has a mass of 280 kg and a specific heat capacity of 420 J/kg° C.
3. 80 kg of water at 62° C are poured into a vessel of mass 231 kg which contains 30 kg of water and 4 kg of ice, both are at 0° C. Find the resulting temperature if the vessel's specific heat capacity is 400 J/kg° C.
4. *(a)* Define 'specific latent heat of vaporisation'.
   (b) Draw a block diagram of a basic refrigeration cycle, naming all the parts and explaining their action.
   (SCOTVEC December 1989).
5. What are *(a)* critical temperature: *(b)* Joule-Thomson effect? What is their importance in refrigeration?

CHAPTER 17

HEAT

Water vapour in the atmosphere

17.1 EVAPORATION

Evaporation is constantly taking place from the surface of water or ice over the earth's surface. It follows that all air contains water vapour.

The rate of evaporation naturally varies greatly under different conditions. Warm dry air in motion will cause rapid evaporation. If the air is cold, evaporation is much slower.

The converse is also true. If by means of a suitable instrument the rate of evaporation can be shown to be high, it follows that we are in a dry, or relatively dry, air-stream. If the rate of evaporation is low, the air stream must be moist.

To sum up:

Always assuming there is a water (or ice) surface available, rapid evaporation requires a warm dry air-stream.

Evaporation will be slow if the air-stream is cold or relatively moist.

17.2 WATER VAPOUR IN THE ATMOSPHERE

Atmospheric air always contains a certain amount of water vapour. Since warm air can contain more water vapour than cold air, in warm moist climates the actual amount of water vapour contained in, say, a cubic metre of air is relatively large (perhaps as much as 4 per cent of the volume). In cold climates, or in desert regions, the proportion of water vapour may be very small.

The state of the atmosphere at any moment in this respect, i.e., as to how much or how little water vapour it contains, is called the **humidity** of the atmosphere, and the amount of water vapour actually present in 1 cubic metre of air at any given moment is called the **absolute humidity**.

Definition:

The Absolute Humidity (of the atmosphere) is the amount of water actually present in 1 cubic metre of air. It is expressed in grammes per cubic metre I

It will be noted that this definition speaks of "water", whereas previously we have spoken of "water vapour". Thus, we are assuming that in some way the water vapour is condensed out of our cubic metre for the purpose of measurement. In a laboratory, this is not difficult, but for the practical man, the absolute humidity is not easily obtained. For many meteorological purposes is it found preferable to use the **relative humidity**. We shall refer to this again.

17.3 SATURATION OF AIR

Air which contains very little water vapour is generally spoken of as "dry" air, whilst air which contains a lot of water vapour is called "moist air". The quantity of water vapour present in a given sample of air depends, amongst other things, on the temperature of the air. The warmer the air is, the more water vapour it can hold.

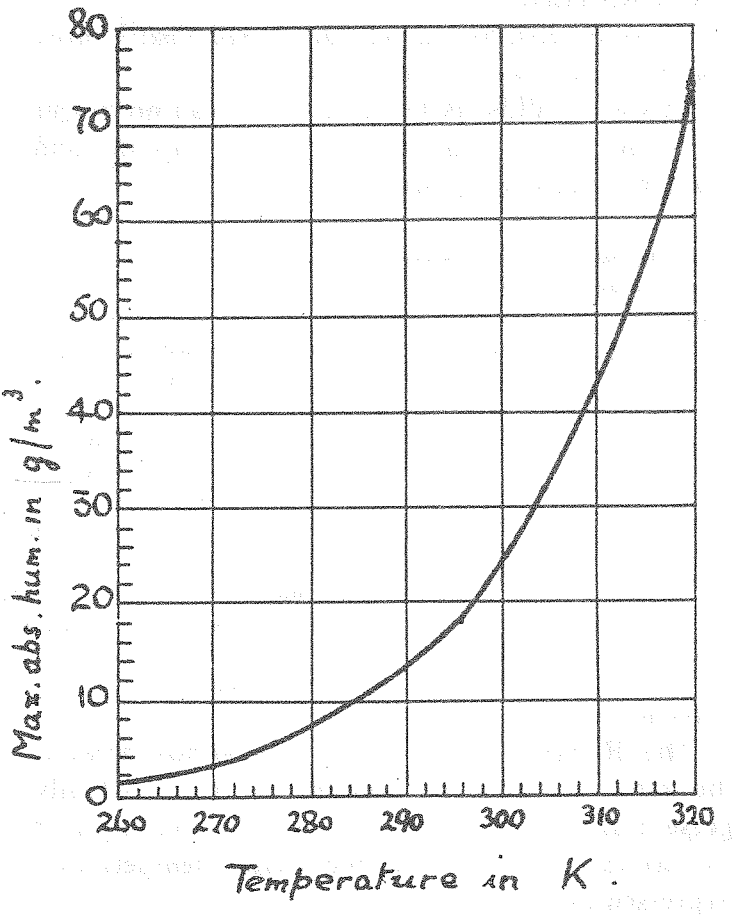
Thus, air at a given temperature, in contact with a water surface, will continue to absorb water in vapour form (a process called **evaporation**) **until it can contain no more**. It is then said to be **saturated** — it is holding all the water vapour it can hold at that temperature.

The only way we could persuade it to contain yet more water vapour would be by **raising its temperature**. It would then evaporate more water until again it was saturated at its new (higher) temperature.

Conversely, if we **lower its temperature**, it can no longer hold as much water vapour, and it must get rid of some of it. It does this by converting the surplus back into water, a process called **condensation**. The particles of water thus formed are of course **visible** (water vapour is not) and they may be deposited on the sides or surfaces of whatever is containing the air. In nature this would be **dew**, in a ship's hold **sweat**.

Or the minute particles of water may remain in suspension in the air, becoming visible as **mist** or **fog** or **cloud**.

The accompanying graph, which shows maximum absolute humidity at different temperatures, will help to fix ideas.



The curve shows the maximum value of the absolute humidity at any given temperature, i.e., the value of the absolute humidity when the air, at that temperature, is saturated.

For instance, at 280 K the maximum absolute humidity is only about 7·8 g/m³, whereas at 300 K it is about 24 g/m³.

To sum up:

Air at a given temperature can evaporate water, and contain it in vapour form, only up to a fixed maximum for that temperature. It is then said to be SATURATED.

If we RAISE its temperature it will absorb more water vapour to a new maximum.

If we LOWER its temperature, it can no longer hold the same amount of water vapour and condensation takes place II

17.4 RELATIVE HUMIDITY

Of course, in Nature all air is not saturated, yet it is the saturated state which is perhaps of most interest to us in meteorology. It is only when air becomes saturated that condensation can occur giving rise to rain or fog or dew.

A given sample of air contains only a certain proportion of the maximum amount of water vapour it might hold at that temperature, i.e., its **actual** absolute humidity is only some fraction of its **maximum** absolute humidity at that temperature.

Remember that absolute humidity is just the number of grammes of water present in 1 cubic metre.

So again, the amount of water **actually** present in a cubic metre is only a fraction of the amount of water that **would be there if it were saturated.**

This fraction is called the **Relative Humidity.**

Definition:

The Relative Humidity (of the atmosphere) is the ratio between the amount of water actually present in the air and the amount of water required to saturate the air at the same temperature, expressed as percentage III

i.e.,

$$\text{Relative Humidity R.H.} = \frac{\text{amount of water actually present}}{\text{amount of water required to saturate (at same temp.)}} \times 100\% \quad \text{IV}$$

WORKED EXAMPLE 1

The absolute humidity of a sample of air at 294 K was 13 g/m³. What was its relative humidity?

From the graph, saturation value of the absolute humidity at 294 K is about 17·8 g/m³.

$$\text{Hence, relative humidity} = \frac{13}{17{\cdot}8} \times \frac{100}{1} = 73\%$$

17.5 IMPORTANCE OF KNOWING THE RELATIVE HUMIDITY

One may see at once from the previous example what a useful index the relative humidity is to us. A statement such as "absolute humidity 13 g/m³" does not convey much information by itself.

"Relative humidity 73%", on the other hand, tells us at once that a given sample of air is 73% saturated — that is, fairly moist. It would not have to cool very much for it to be completely saturated. In fact, by referring again to the graph, with a vapour content of 13 g/m³ it would be saturated at about 289 K i.e., 16° C.

In short, this particular sample of air would only have to cool to 16° C for **condensation to occur**, causing **fog, mist, dew**, or in a ship's hold, **sweat**.

Of course, we can have the relative humidity 90%, 95%, under which conditions, a drop in temperature of a few degrees might cause precipitation — in a mist or fog the relative humidity is 100%, that is, the air is saturated.

Conversely, we can have the air only 50%, 25%, saturated or even less.

In every case, however, the relative humidity figures gives us at once a mental picture of the condition of a given air mass, and for this reason is particularly useful to the meteorologist, amateur or professional.

A further advantage is that the relative humidity is fairly easily found in practice, as explained below.

17.6 DEWPOINT

We have already seen that if we take a given sample of air and **cool** it, we shall ultimately reach the temperature at which it is **saturated**, and **condensation** will occur. This temperature is called the **dewpoint**.

Definition:

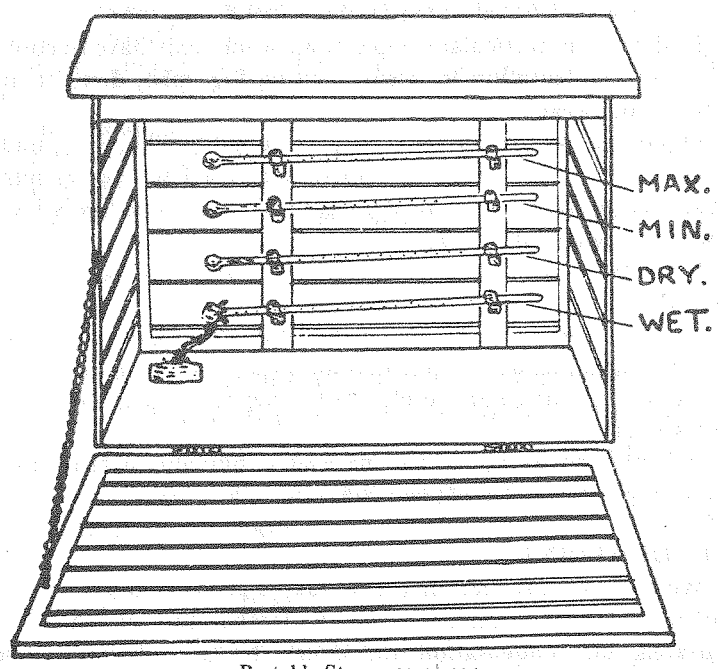
The Dewpoint for a given air mass is the temperature at which that air mass is saturated. Once the temperature falls below the Dewpoint, condensation will commence V

For instance, the dewpoint for the air in WORKED EXAMPLE 1, as explained, is 16° C.

Knowledge of the dewpoint is clearly of an importance equal to knowledge of the relative humidity and in fact, the same hygrometric tables that give us the relative humidity give the dewpoint also for the same air mass.

17.7 DETERMINATION OF RELATIVE HUMIDITY — THE HYGROMETER

This instrument is used to determine the hygrometric state of the atmosphere at any moment.



Portable Stevenson screen.

The type commonly employed is called **Mason's hygrometer.**

It consists of two identical thermometers side by side. One thermometer is used merely to give the temperature of the air, and is called the **dry bulb**. The other has its bulb covered with muslin which is kept moist by means of a wick which communicates with a vessel containing distilled water. This one is called the **wet bulb**.

The two thermometers, securely mounted as shown, are contained in a kind of box with louvred sides, called a **Stevenson Screen**, after its designer. This allows a free circulation of air round the bulbs, but protects them from direct sunlight and from the entry of rain.

The evaporation which takes place from the wet bulb causes a fall of temperature and thus the wet bulb always reads lower than the dry bulb. Moreover, this difference in temperature, called the **depression of the wet bulb**, will be proportional to the rate of evaporation, which in turn depends on the amount of moisture already present in the air.

Hence:—

In dry air, evaporation will be rapid, and the depression of the wet bulb will be correspondingly large. A normal depression might be 4 or 5 Celsius degrees, but in very dry air it may be considerably more than this.

In moist air, evaporation is slow, and the depression of the wet bulb may be as little as 1 Celsius degree. In saturated air the readings would be the same.

Thus, the relative humidity of the air, and its dewpoint, are closely related to the readings of the wet and dry bulbs.

Unfortunately however, it is not possible merely by noting the readings to say what the humidity actually is. For this, tables are necessary.

They are quite simple to use. Enter the tables with the temperature of the dry air, and the depression of the wet bulb. In one table, the relative humidity (per cent) is read off, and in the other, the dewpoint (°C) is obtained.

17.8 CARE IN USE

The instrument calls for little attention. The screen must be out of doors, away from heat from engine-room or other unwanted sources. The bowl of water must be kept replenished. In freezing

conditions, the wet bulb will work equally well as long as it is coated with ice. If this is not so, and the water in the bowl is frozen, wet the bulb and allow it to freeze before reading.

17.9 CONDENSATION IN SHIPS' HOLDS

Quite apart from the meteorological point of view, the foregoing ideas on humidity and dewpoint have important applications in connections with the ventilation of hold spaces.

One of the reasons for ventilation of holds is to prevent damage to the cargo by sweating, that is, by condensation occurring on beams and girders and then dripping on to the cargo.

Condensation will occur if warm moist air above the cargo is cooled below its dewpoint.

Thus different cargoes require different treatment. Some, of course, such as iron ore, timber, will not be damaged even if there is sweating.

Others, such as coal, steel rails, machinery, general cargo in cases, do not themselves give out any moisture and is only necessary to ensure a reasonable surface ventilation so that any gases may be carried away and the air in the hold space kept fresh.

The greatest care however is required with grain cargoes especially rice, for these give out a great deal of moisture in vapour form into the air space in the hold. If the ship now proceeds into higher latitudes and the outside temperature falls, the condensation in the hold will be heavy and moreover will continue vigorously since the cargo is continuing to exude water vapour. This sweating, if allowed to persist, will cause severe damage to the grain, which will start to sprout.

The remedy is obvious. Even at the risk of getting a bit of spray down the ventilators, adequate ventilation must be provided so that the moist air in the hold will be carried off and replaced by cooler, drier air from outside.

Lastly, there are cargoes such as bales (wool, cotton, jute), flour in bags, rolls of paper, which, although they may not exude much moisture themselves, would certainly be damaged by sweating.

The main rule for prevention of sweating is to ventilate whenever the ship is moving from a warmer to a cooler climate. However, if this rule is applied when the dewpoint of the outside air is higher than the cargo temperature, moisture will be deposited on

the cargo. Therefore it is most important to know the dewpoint both inside and outside the hold and air taken in for ventilation should always have a lower dewpoint than the temperature of the cargo.

Mechanical ventilation systems are now in common use and these maintain the correct air temperature and relative humidity in the hold throughout the passage.

17.10 FORMATION OF CLOUD

(a) **What cloud is**

All cloud is formed by air being cooled below the dewpoint, so that condensation begins, and the water vapour in the air becomes visible as minute particles of water.

For reasons which are not fully understood, the formation of these droplets requires the presence in the air of minute solid particles, such as very fine dust or ice crystals. Each of these minute particles forms the nucleus or centre around which the water droplet forms. At the very low temperatures experienced at great heights, these droplets may be of ice.

The term **hygroscopic particle** is sometimes used to describe such a droplet. (Greek, *hygros*, moisture).

(b) **Method of cooling**

Whenever air is forced to **rise**, it **cools**, and, if the ascent goes on long enough, the condensation level will be reached and cloud will form. In short, with few exceptions, **the cooling is caused whenever an air mass is forced to ascend.**

To sum up

Cloud consists of hygroscopic particles which form when air is cooled below the dewpoint. This cooling occurs whenever air rises, and in general, this is the cause of cloud.

17.11 CAUSE OF RAIN

If the ascent of an air mass continues vigorously, a stage is reached at which the cloud becomes so dense with hygroscopic particles that these begin to join together, forming larger drops. Finally the drops become too large to be suspended in the air, even by a vigorous ascending current, and fall to the earth as rain.

Thus, rain does require a fairly thick cloud layer. It is only in freak circumstances that one experiences rain drops falling out of

an apparently flimsy layer of cloud — generally such drops have originated much higher up.

It is generally accepted that even for slight rain to fall, we need a vertical cloud development of at least 1000 m, and for appreciable rain, something approaching 3000 m thickness is needed.

17.12 FORMATION OF FOG

Fog is cloud at the surface. Thus, whereas cloud is formed when air is cooled by ascent, **fog is formed when air is cooled at the surface.**

On high ground of course, there may be no real distinction between cloud and fog — the cooling is caused by ascent, and cloud forms on the mountain side.

At sea, however, or near the coast, fog is formed by air at the surface being cooled below the dewpoint, and a classification of fog into different types simply derives from the different ways in which surface cooling can occur.

(a) **Radiation fog.** This is caused by the cooling of the ground surface at night. The conditions necessary are very little air movement and a clear sky. The loss of heat by radiation is then rapid. The air in contact with this cold surface is cooled below the dewpoint and fog forms. This is the common type of fog seen on low lying ground in the early morning after a cold night.

Radiation fog is thus exclusively a land fog, but remember the fog formed in this way may drift a few miles off shore. It usually clears up quickly in summer, as the sun's effect is felt, but in winter, it may persist for some time.

(b) **Advection fog.** The term comes from two Latin words which broadly speaking mean "moving towards". This fog is formed therefore when a warm moist stream moves into a colder region or over a cold sea, and is the commonest cause of fog at sea. In the approaches to the Channel for instance, a S.W. wind may cause fog of this type. The Newfoundland Banks is another region, where the warm air from the Gulf Stream Drift is cooled by contact with the colder waters of the Labrador Current, and fog forms.

Fog of this type may persist as long as the air stream is kept up. By day, the effect of the sun's heat may be to lift it a few hundred feet above the surface, when it might be described more properly as turbulence cloud. But in general, advection fog may persist for days.

17.13 MIST AND HAZE

By international agreement, in meteorology, fog is deemed to exist when visibility is **less than 1000 metres** (say 1100 yards).

When visibility is more than 1000 metres but less than 2000 metres, the condition is known as (*a*) **mist** or (*b*) **haze**.

Beyond 2000 metres conditions are regarded as **clear**.

(*a*) **Mist** is of exactly the same nature as fog, i.e., it consists of hygroscopic particles, and is caused in the same way. The difference is only one of intensity — roughly speaking, fog is dense, mist is not so dense.

(*b*) **Haze** is rather different, being a state of poor visibility caused by **dust** or **smoke** in the atmosphere.

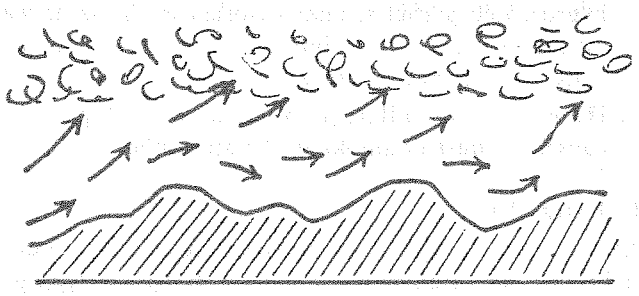
17.14 DRIZZLE

Drizzle is something halfway between fog and rain. A dense fog, as we know, can often be very damp. If the surface cooling is prolonged, the droplets may join and become distinguishable as such, with a tendency to fall to the ground at a noticeable speed. This is known as **drizzle**.

17.15 TYPES OF CLOUD AND RAIN

Four main types of cloud are recognised, according to the manner in which the cooling can occur. Rain in turn is classified under three broad headings.

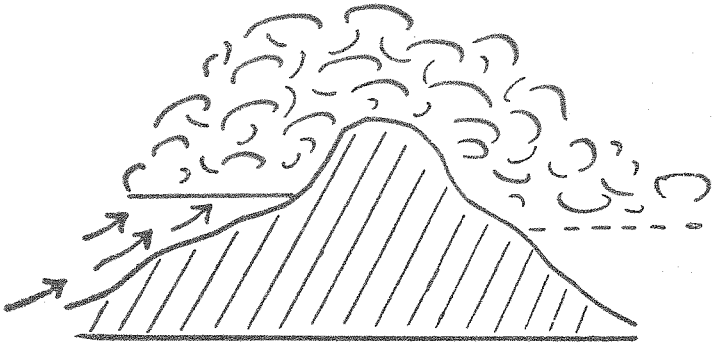
(*a*) **Turbulence cloud**



Turbulence cloud.

If the air stream is moving over rough ground or sea it is thrown into gusts and eddies, a state known as turbulence. In doing so, some of these eddies may reach a height of 500 m or 700 m and be cooled below the dewpoint, so that a layer of cloud is formed. This type is called turbulence cloud. It seldom gives rain.

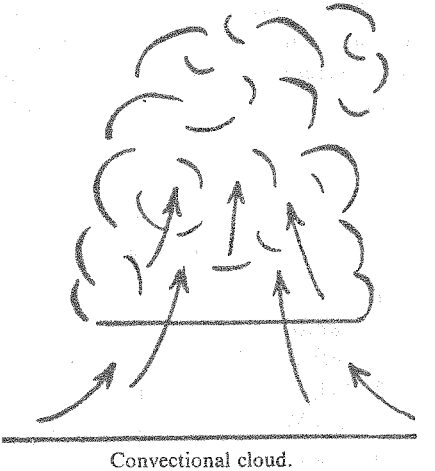
(*b*) **Orographic cloud**



Orographic cloud.

If air is forced by high ground to ascend cloud may form on the hills. This is known as orographic cloud and if the vertical development is sufficient it will give **orographic rain**. Otherwise it will give mist or drizzle.

A cloud cap of this type is frequently seen on the summit of the Rock of Gibraltar and also on Table Mountain, Cape Town.

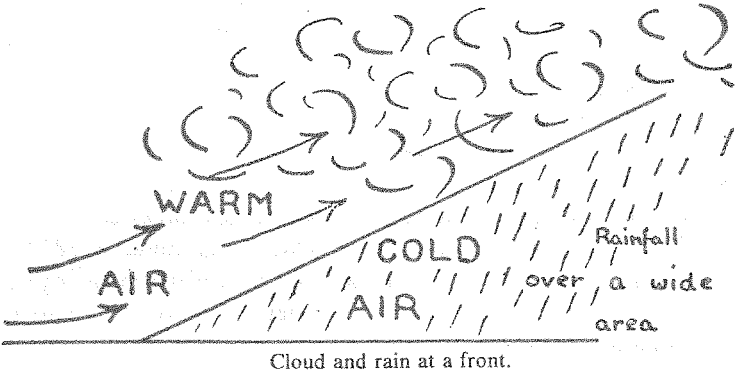


Convectional cloud.

(*c*) **Convection cloud**

If air is heated at the surface by contact with warm ground or warm sea, convectional currents will be set up. If the heating is pronounced the rising currents of air will become very vigorous, sometimes rising to a height of 10,000 m or more. This is the "wool-pack" type of cloud. It will give very heavy localised showers. The thunderstorms on a hot summer day are usually of this type. The towering clouds and heavy rain of the Doldrums are also caused in this way. Rain in these circumstances is known as **convectional rain**.

(*d*) **Coud and rain associated with the passage of a front**



Cloud and rain at a front.

In meteorology, a **front** is the line or zone along which two air masses of different characteristics meet.

Thus, a warm air stream meeting a cold (and therefore heavier) air mass will be forced to rise steadily over a region often extending for hundreds of miles. This is the type of weather associated with a "depression". It will give widespread cloud and rain of many hours duration.

TEST PAPER 17

Water vapour in the atmosphere

1. Define Relative Humidity. The absolute humidity of a sample of air at 29° C was 12 g/m^3. Using the curve in section 17·3, find the relative humidity.
2. Why is cloud often more widespread in the British Isles with a S.W'ly wind than with a S.E'ly wind?
3. State briefly how rain is formed. Name the main types of rainfall explaining how each is caused.
4. What is meant by saying that a given mass of air is "saturated"? What is the particular temperature of saturation called? Explain what happens when air is cooled below this temperature.
5. Define Relative Humidity and describe an instrument used for determining the relative humidity of the atmosphere.
6. Write short notes on the formation of;
   (a) fog, *(b)* mist, *(c)* haze, *(d)* drizzle.
7. Define Absolute Humidity.
   An enclosed volume of air has its temperature raised from 7° C to 15° C. What effect (if any) will this have on:
   (a) its absolute humidity
   (b) its relative humidity
   (c) its dew point?
   Give brief reasons in each case.
8. With the aid of the graph in section 17.3, find:
   (a) the absolute humidity of air whose dew point is 18° C.
   (b) the absolute humidity of air which is saturated at 38° C.
   Using these results, if the air in *(a)* is now warmed to 38° C, with no additional moisture entering, find its relative humidity at 38° C.

CHAPTER 18

MAGNETISM

General introduction. Terrestrial magnetism

18.1 NATURE OF MAGNETISM

From earliest times it was observed that certain ferrous ores possessed magnetic properties. That is, they possessed the property of attracting other small pieces of iron such as iron filings, and also, if freely suspended, a piece of this ore would always take up a particular direction relative to the earth's surface. For this reason the early navigators called it "lodestone", from the early word meaning "to lead", and the simplest forms of compass consisted of a piece of lodestone mounted on wood, floating in a bowl of water.

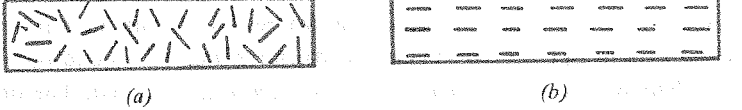
The precise nature of magnetism is not fully understood, even to this day, but the laws of magnetism have now been fully investigated. We shall, therefore, in these notes, state the facts as they are known, and avoid speculation as to "why" these things occur.

18.2 MAGNETIC SUBSTANCES

Magnetic properties (for all practical purposes) are confined to ferrous metals such as iron or steel. Such substances are therefore called **Ferromagnetic.**

18.3 MOLECULAR THEORY OF MAGNETISM

Some bars of iron are magnets and some are not. The molecular theory has provided a convenient explanation of what happens when a bar of iron is magnetised.



(a) *(b)*

Molecular theory of magnetism.

(a) Shows an unmagnetised bar. In it, the molecules, each one a tiny magnet with its own N. and S. poles, are lying at random. Hence the bar exhibits no resultant magnetic poles.

(b) Shows the same bar after being brought under a powerful magnetising force. The molecules have aligned themselves with the result that the bar is now a magnet with a resultant pole near each end.

Domain theory

More recently it is thought that the atoms within the molecules are themselves dipoles (i.e., having two magnetic poles) and form themselves into irregular closed circuits or groups. The name "magnetic domain" has been given to these groupings. Moreover, each domain will have a resultant magnetic axis. These could conveniently be represented by the above fig. *(a)*, with the resultant magnetic axes of the domains all lying in random directions. After magnetisation, the axes line themselves up as in fig. *(b)*.

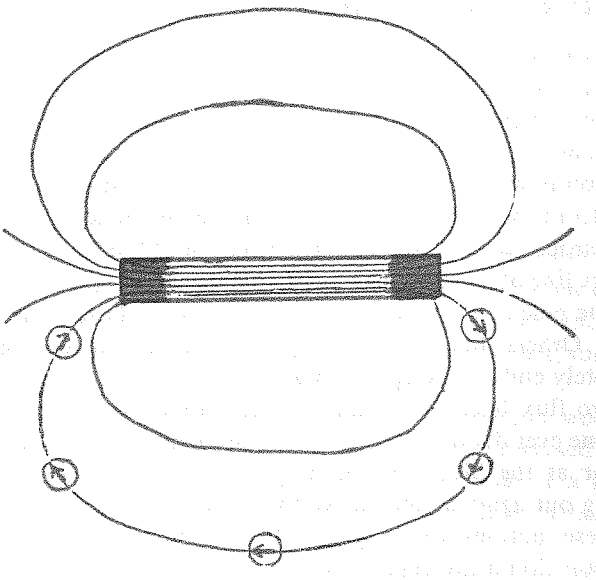
18.4 POLES OF A MAGNET

In a magnet, the magnetic force is concentrated at points near the ends, called the **Poles**, and the "quality of magnetism" concentrated there is called the **Pole Strength.** In any magnet, the poles are always equal in strength, though opposite in name.

For most practical purposes the poles are assumed to be at the ends of the magnet. A more exact result is obtained by assuming each pole to be $^1/_{12}$ of the length of the magnet in from each end. The distance between poles is called the **effective length.**

18.5 NAMING THE POLES

If a magnet is mounted (as in a compass) so that it is free to turn in a horizontal plane, it will come to rest with one end pointing approximately towards the North Magnetic Pole. This end is called the **North Seeking** or **Red** pole. It is sometimes marked. The other is called the **South Seeking** or **Blue** pole.



Field of a magnet — lines of force, for flux lines.

18.6 FIELD OF A MAGNET

Every magnet is surrounded by a region in which its influence is felt, called the **Field** of the magnet. Every field contains something which is called **magnetic flux.**

The diagram above shows the field surrounding a single magnet. Remember that the field completely surrounds the magnet (the sketch only shows it in the plane of the paper).

The field is shown by means of lines, called **Lines of Magnetic Force,** or **Flux Lines.** Every magnet is surrounded by these (imaginary) lines, and in fact, it is the flux lines that make the field.

The flux pattern may be traced by means of a small compass, moved into different positions as shown for example in the diagram above. Note particularly that the needle always lines itself up with a flux line. It does not necessarily "point towards the pole" — it lies along a flux line. This fact is of considerable importance in compass compensation on board ship.

18.7 DIRECTION OF THE FIELD

When we speak of the "direction of the magnetic field" at a point it is not enough to say that this is given by the direction of the flux lines at that point. A line runs in two directions.

It has been agreed therefore that the direction of field is the direction in which a north pole would be urged in that field, that is, away from the red, and towards a blue pole. Hence flux lines may be regarded as running "out at the red, in at the blue".

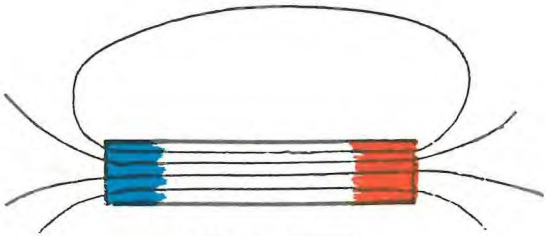
Any line of force leaving a red pole must end up on a blue, often the blue pole of the magnet itself, like some of those in the diagram above. Others, like those leaving the ends, go wandering off and ultimately end on the earth's blue.

Two flux lines never run together nor cross one another, nor does one ever divide. They are all separate and continuous, entering the bar at the blue end, running continuously through the bar, fanning out again as they leave the red end.

These notions on magnetic flux and the continuity of the magnetic circuit are very important.

18.8 BREAKING A MAGNET

If a magnet is cut into two parts, then two separate magnets are formed each with its own N. and S. poles.



Breaking a magnet.

This is quite apparent from our knowledge of lines of force, as shown in the foregoing diagram.

All that happens is that the lines of force, which run continuously through the bar, are severed but immediately form an independent "circuit", in a fresh magnet, as shown. This is true,

however many times the bar is cut. Neither pole can exist alone — the smallest magnet imaginable must still have two poles.

18.9 ATTRACTION AND REPULSION OF POLES

Magnetic poles brought into proximity attract or repel each other in accordance with the law.

Like Poles Repel Unlike Poles Attract

This affords us the standard method of testing whether a bar of iron is a magnet or not. If it is a magnet, with a pole in each end, then one end of the bar will attract one end of a known magnet, such as a small compass needle, and **repel the other end.**

At the other end of the bar the same phenomenon would be observed, though with the attraction and repulsion reversed.

If the iron bar is not a magnet, then either pole of a known magnet, brought close to any part of the bar, would be attracted to it.

In short, magnetic poles attract, and are attracted by, all iron.

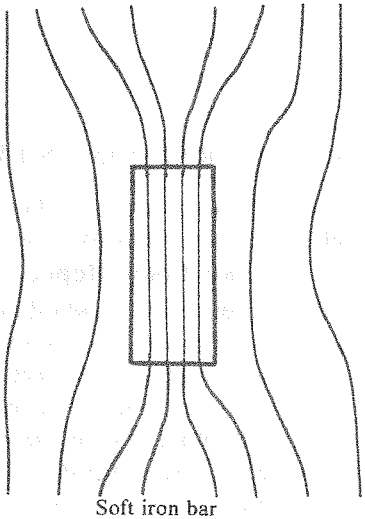
Magnetic poles are only attracted by other magnetic poles in accordance with the above law of attraction and repulsion.

It also follows from the above law that the **Earth's North Pole is Blue,** since all red poles of magnets are attracted towards the earth's north.

Conversely, the **Earth's South Pole is Red.**

18.10 MAGNETIC INDUCTION

The word induction is derived from a Latin word *duco*, (I lead), and induction means, literally, "leading into". The derivation will help us to remember the meaning of **magnetic induction**, namely, that certain ferromagnetic materials are so susceptible to a magnetising field that they draw the flux lines into themselves, resulting in a greatly increased concentration of lines within the material. This is illustrated for the case of a bar of "soft" iron.



Soft iron bar in the earth's field.

18.11 INTENSITY OF MAGNETISATION

Within the bar the intensity of magnetisation *(E)* is much greater than in the surrounding field in general. This is only another way of saying, what we have already said in other words, that the flux lines are densely packed within the bar and we shall see later that this packing of the flux lines affords us a means of measuring the intensity of magnetisation in the bar.

It should be noted that if a given magnetic substance, initially unmagnetised, is subjected to an increasing magnetising force the intensity of magnetisation E will increase until it reaches a maximum value. The substance is now "saturated" — its molecules are now fully aligned with the direction of the magnetising force and E cannot be increased further.

18.12 HARD AND SOFT IRON

Various qualities of steel are used in shipbuilding, and some of these are more susceptible to magnetic influences than others.

In magnetism, the terms "hard" and "soft" iron are used to express different magnetic properties.

Hard Iron is difficult to magnetise, but once magnetised, it retains its poles more or less permanently. Magnetism in hard iron is termed **Permanent Magnetism,** or quite often, just **Hard Iron Magnetism.**

In ships, most magnetism of this type is located in the shell and deck plating. The magnetic poles become permanently fixed by the amount of hammering the ship receives during construction. Their position thus bears some relation to the direction in which the ship is lying whilst building. A hard iron bar lying N. and S., if hammered, would acquire red polarity at its northern end (next to the earth's blue, that is) and of course blue at the other end. A ship is more complex.

Soft iron is easily magnetised. In fact, a bar of soft iron has only to be held in the earth's field for it to become a magnet. The poles are **not permanent,** however, but disappear the moment the bar is removed from the magnetising influence.

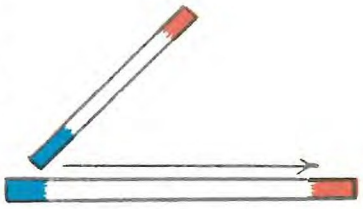
Because it is transient (non-permanent) and because it is induced instantly by a magnetising force, magnetism of this kind is called **Transient Induced Magnetism,** or quite often, just **Soft Iron Magnetism.**

In ships, magnetism of this type is located chiefly in beams and girders, such as frames, deck beams, masts, derricks and derrick posts, funnel, and so on. The important point to remember is that poles in these members are never constant for very long. They change whenever the position of the girder changes relative to the earth's field.

18.13 METHODS OF MAKING MAGNETS

(a) The Method of Single Touch

The bar to be magnetised is laid flat on the table and is stroked firmly many times from one end to the other. Do not rub back and forwards — stroke firmly one way only.



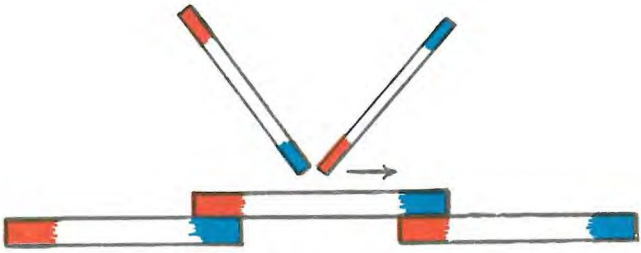
Method of single touch.

The poles induced in the bar will be as shown; for instance, if a

blue pole is used for the stroking, then a red pole will be caused in the end of the bar at which the stroking magnet leaves the bar.

This method will only produce a very weak magnet.

(b) **The method of Divided Touch**



Method of divided touch.

The method is as illustrated in the diagram above. The bar to be magnetised is mounted on two strong magnets arranged as shown, whilst two more magnets are used to do the stroking, this time simultaneously from the middle towards each end.

The poles induced will be as shown. Although better than *(a)* the magnets produced by this method are still fairly weak.

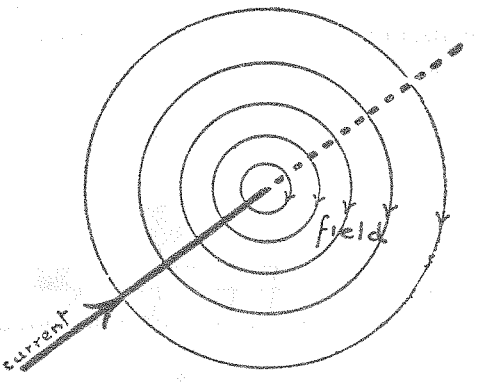
(c) **By Electrical Means**

This is the method generally used. It depends upon the fact that whenever a direct current of electricity flows in a wire, the wire is surrounded by a magnetic field.*

The direction of this field is clockwise when viewed end on, looking the way the current is flowing.

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* *A conductor carrying A.C. is surrounded by a magnetic field also, but as the direction of the field is changing, like the current, many times a second, there is no resultant field.*

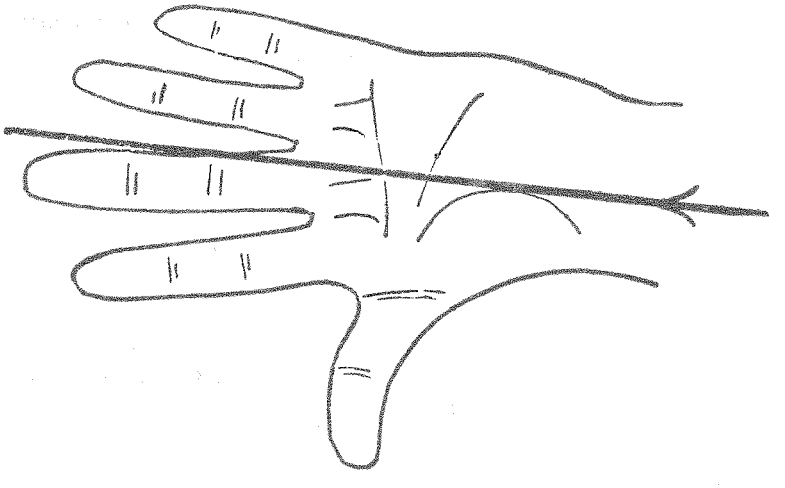


Magnetic field surrounding a wire (current flowing into the paper).

Or the direction of the field may be determined by the following rule, which some find easier to apply, viz:

Lay the wire on the palm of the left hand, with the fingers pointing the way the current is flowing.

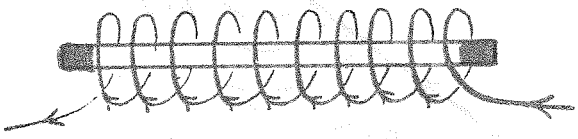
Then the thumb indicates the direction of the field, i.e., the direction in which a N. pole would be urged.



Direction of field round a wire (left hand rule).

If a wire like this is wound into a long coil, called a **Solenoid,** as shown in the diagram below, there will be a strong field inside the coil. A bar placed inside the solenoid would be magnetised as shown.

Very strong magnets can be produced by this method, using of course a hard iron bar for the purpose.



Bar in a solenoid.

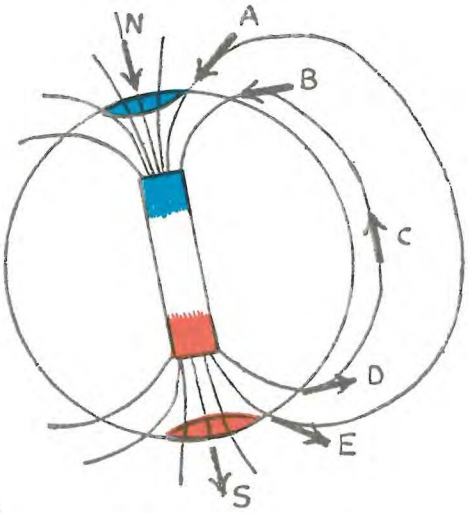
Note. in practice, a solenoid consists of hundreds of turns of insulated wire.

18.14 THE EARTH AS A MAGNET

When considering the nature of the Earth's magnetic field it is sometimes convenient to regard the Earth as if it had a large magnet inside it, as shown.

We said earlier (section 18.7) that the direction of the magnetic field is the direction in which a North pole (red) would be urged in that field. For a bar magnet in the Earth's field its North pole points towards what we call North. However such a North pole would of course be attracted towards the south pole of another magnet. Hence we consider the large magnet inside the Earth to have a South pole in the northern hemisphere as shown below.

A freely suspended magnet will line itself up with the Earth's lines of force wherever it happens to be. For instance, such a magnet would be vertical at *N*, red end pointing into the earth. At *A*, it would be inclined at the angle shown; similarly at *B*. At *C* it would be horizontal, whereas at *D* and *E* it would be inclined again, but this time with the red end upwards.



The Earth as a magnet.

Finally, at *S*, it would again be vertical, but with red end pointing upwards.

Thus, in a narrow zone running round the earth roughly midway between the magnetic poles the Earth's field is horizontal. Anywhere else on the Earth's surface the field is inclined at some angle to the horizontal.

These facts are considered in more detail below.

18.15 MAGNETIC POLES

The areas illustrated in the diagram above, where a freely suspended needle would be vertical are called the **Magnetic Poles.**

The **North Magnetic Pole** is situated in a region north of Hudson Bay, in the general vicinity of 70° N., 97° W.

The **South Magnetic Pole** is in the Antarctic, in the general vicinity of 73° S., 155° E.

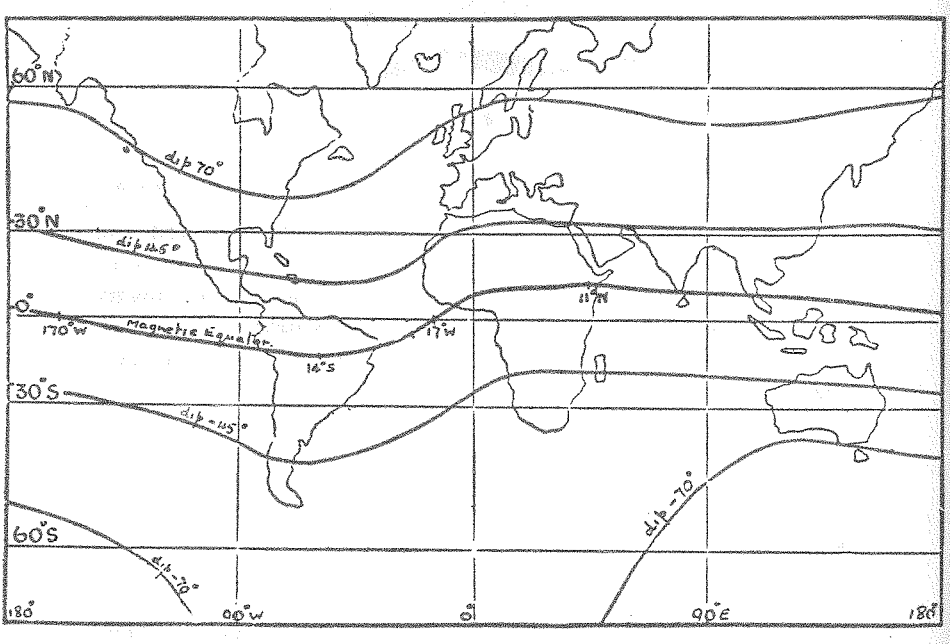
It should be noted that these are very far from being at opposite ends of a diameter. In fact, the earth's field itself is far from being

symmetrical — on no account should it be regarded as a nice tidy arrangement of great circles from north to south magnetic poles.

The two poles, as we can see, are not diametrically opposite to each other and all we can say about the lines of force is that they are smooth curves joining the north and south magnetic poles.

18.16 MAGNETIC EQUATOR

The narrow belt surrounding the earth roughly midway between the magnetic poles where the earth's field is horizontal is called the **Magnetic Equator.**



Magnetic Equator.

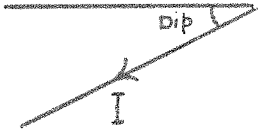
The approximate position of the magnetic equator is shown in the diagram above. From a maximum of about 14° S. latitude in Brazil, it crosses the Equator in about 17° W., reaches its maximum

of about 11° N. latitude near Cape Guardafui, and crosses the Equator again in about 170° W.

Alternatively, the magnetic equator may be defined as the line joining places where there is no dip, and for this reason it is sometimes known as the **Aclinic Line** (Greek *A — without, klino — bend).* The next section explains the significance of dip.

18.17 DIP

We have already seen that a freely suspended magnet will in general point at some angle down into the ground. The **angle of dip** is the angle a freely suspended magnet makes with the horizontal, or what is the same thing, the angle between the direction of the earth's field at any place and the horizontal.



Angle of dip.

This is illustrated in the diagram, where *I* (for field Intensity) is the Earth's **total field strength.**

In north magnetic latitudes, i.e., north of the magnetic equator, the dip is given a + sign, and in south magnetic latitudes a − sign, to remind us that although the angle of dip may be the same at two places, at one the north end is tilted down, at the other up.

The value of the dip may be found by reference to the diagram in section 18.16 where lines have been drawn through places of equal dip. Such lines are called **Isoclinic Lines** (Greek *Iso* — equal, *klino* — bend). The one showing dip = 0° is of course the **Aclinic Line**, or line of no dip, or magnetic equator, as previously mentioned.

18.18 COMPONENTS H AND Z

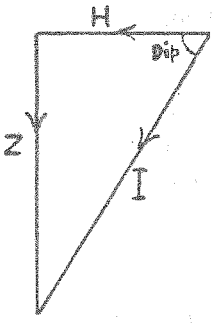
Since the compass needles are so mounted that they can only rotate in a horizontal plane, they can only be moved by a horizontal force, or at least, a force that has a horizontal component.

For this reason the Earth's total field intensity I is split up into two components, viz.:

H, the **Horizontal Component** of the Earth's field, and

Z, the **Vertical Component** of the Earth's field.

From the right-angled triangle in the diagram opposite the following formulae can easily be deduced, giving the relation between **Z**, **H**, **I** and **Dip**, viz.:



Components of the Earth's field.

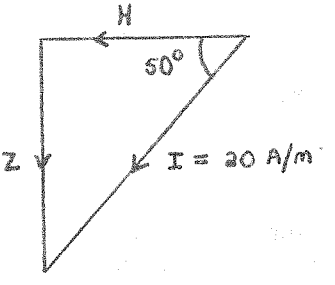
$$\left.\begin{aligned} \frac{Z}{H} &= \tan \text{dip} \\ I^2 &= Z^2 + H^2 \\ H &= I \cos \text{dip} \\ Z &= I \sin \text{dip} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{I}$$

These are easily written down if one remembers the figure from which they are derived.

Field intensity, I, and its components H and Z are measured in units called **amperes per metre**, but discussion of these is reserved till later.

WORKED EXAMPLE 1

Calculate the horizontal and vertical components of the Earth's magnetic field at a place where the angle of dip is 50° and the Earth's total field strength is 20 A/m.



$$H = I \cos \text{dip}$$
$$= 20 \times \cos 50°$$
$$= 12{\cdot}86 \text{ A/m}$$

$$Z = I \sin \text{dip}$$
$$Z = 20 \times \sin 50°$$
$$Z = 15{\cdot}32 \text{ A/m}$$

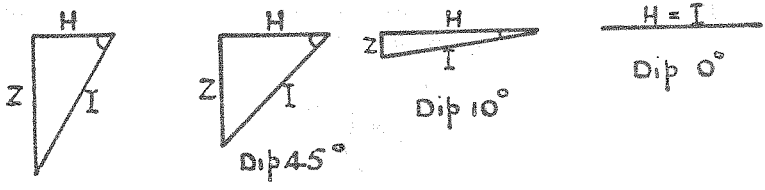
EXERCISE A

1. Given that the Earth's total magnetic field strength at a certain place is 25 A/m calculate its horizontal and vertical components. The angle of dip is 40°.
2. At a place where the horizontal and vertical components of the Earth's field are 7 A/m and 24 A/m respectively calculate the Earth's total field strength and the angle of dip.
3. Calculate the Earth's total magnetic field strength at a place where the angle of dip is 35° and the horizontal component of the field is 20 A/m.

18.19 IMPORTANCE OF H

H is the component that operates the compass, that is, gives the needles their **Directive Force**, their ability to point north. Its importance can therefore hardly be over emphasised.

At the magnetic equator, there is no dip, so H is equal to I, the earth's total force. Z is zero.



Change of H and Z with latitude.

As we move towards either pole, the dip increases, H diminishes and Z increases. With dip 45°, H and Z are equal. With dip 60°, H is only equal to half the earth's total force, until finally, at the poles, H is zero, and the whole of the earth's force is vertical.

It is easy to see from this that the compass is most efficient where H is greatest, i.e., at the magnetic equator, and is virtually useless as we get near the magnetic poles.

This is usually summed up in the words:

the **Directive Force (of the compass needle) varies directly as H.**

18.20 IMPORTANCE OF Z

From our point of view the vertical force Z has only a nuisance value. It plays no part in giving directive force to the compass. On the contrary, by inducing poles in vertical soft iron (V.S.I.) such as masts, derrick posts, frames and funnel, it causes deviation of the compass.

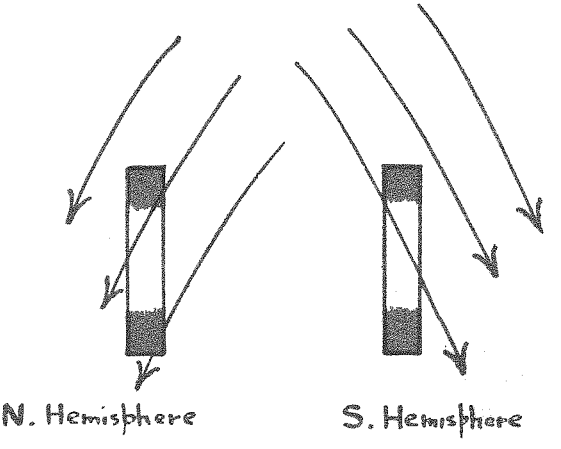
However, for this very reason it is obviously important to know the value of Z, so that we may know, not only the strength of the poles in V.S.I., but also which is the blue end and which the red.

The value of Z in turn depends on the value of the dip, i.e., **the greater the dip, the greater the force Z will be.**

Thus, at the magnetic equator dip is zero, Z is zero, and there are no poles in V.S.I.

As we move into north or south magnetic latitudes, dip increases, Z increases, and the strength of the poles in V.S.I. increases, until finally, at the magnetic poles, Z has its maximum value, being then equal to the earth's total field I.

Further remember that the polarity induced in V.S.I. will always be **red** next to the earth's **blue**, and *vice-versa.*



Polarity in vertical soft iron.

Thus in the northern hemisphere, a bar of V.S.I. will have a **red** pole at its lower end. In the southern hemisphere, a **blue** pole at its **lower** end.

18.21 MAGNETIC VARIATION

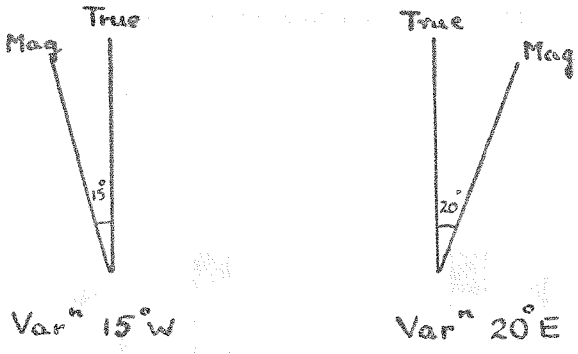
We have already seen that a compass needle does not point to true north, but lines itself up with the direction of the earth's field at that place. This direction is called the **Magnetic Meridian** at that place.

Definition. The magnetic meridian at any place is the direction assumed by a compass needle, free to move only in a horizontal plane, and under the influence of the earth's field alone.

Once the above the definition is learnt it is easy to define **Magnetic Variation**, as follows:

Definition. The magnetic variation at any place is the angle between the true and magnetic meridians at that place.

It is named westerly if magnetic north is to the westward of true north, and easterly if magnetic north is to eastward of true north.



E. and W. variation.

The value of the variation for any locality is found on the Admiralty Chart of that place. Also, a special chart can be obtained which shows **lines of equal variation,** or **isogonic lines** (Greek, *Iso* — equal, *gonia* — angle) or **isogonals.**

The line joining places where the variation is zero is called the **agonic line.**

18.22 SECULAR CHANGE

The value of the variation at any place is not constant. It is subject to a slight daily swing of a few minutes of arc to the westward, and back again, during the 24 hours. Also magnetic storms cause a slight swing. These are both negligible, as far as navigation is concerned.

The only change of importance to the navigator is the **secular** change, that is, a slow but fairly regular change which goes on persistently over a long period — centuries — and accumulates to a considerable amount.

The annual amount of the secular change is always quoted on the chart, in a statement such as **"variation 14° W. 1936, decreasing 11′ annually". This change must always be allowed for,** i.e., always work, whether on the chart or in calculations, with the variation for the current year.

18.23 DEVIATION

The deviation of the compass is caused by the iron in the ship. Without any such iron, the compass needle would lie in the magnetic meridian, and **deviation** may be very simply defined as the amount in degrees the compass needle is deflected to East or West of the magnetic meridian.

If the north end of the needle is deflected to the **eastward** of the magnetic meridian, the deviation is named **east**; if to the westward, it is named **west**.

TEST PAPER 18
Terrestrial Magnetism

1. *(a)* Explain what is meant by the following terms used in terrestrial magnetism:
   (i) The angle of variation.
   (ii) The angle of dip.
   (b) State the reason why the angle of variation is not constant at a given location on the Earth's surface.
   (c) Calculate the total strength of the Earth's magnetic field at a place where the horizontal field strength is 20 Am^{-1} and the angle of dip has been found to be 50°.
   (SCOTVEC December 1989).
2. Explain, with the aid of suitable sketches, what is meant by the 'domain theory of magnetism'.
   (SCOTVEC March 1990).
3. *(a)* Sketch the magnetic field pattern established by the dipole which is considered to exist within the Earth, showing clearly the orientation of the dipole and the direction of the field set up.
   (b) At a place on the Earth's surface where the total magnetic field strength has been found to be 20 Am^{-1} and the dip angle is 80°, calculate:
   (i) the vertical component of the Earth's magnetic field.
   (ii) the horizontal component of the Earth's magnetic field.
   (SCOTVEC March 1991).

4. *(a)* Define the term *the angle of dip.*
   (b) state the value of *the angle of dip* in the following locations:
   (i) on the magnetic equator.
   (ii) at the Earth's magnetic poles.
   (c) Define the term *the angle of variation.*
   (d) State the reason why the variation of the magnetic compass is not constant over a period of time at a particular location.
   (e) At a place on the Earth's surface where the angle of dip is $-40°$, the horizontal component of the Earth's magnetic field is found to be 25 Am^{-1}.
   (i) Calculate the total force of the Earth's magnetic field.
   (ii) Calculate the vertical component of the Earth's magnetic field.
   (iii) State whether the place on the Earth's surface is North or South of the Earth's magnetic equator.
   (SCOTVEC March 1992).
5. What is meant by the term "deviation" as applied to the magnetic compass.
6. What do you understand by the terms "soft iron" and "hard iron" with regard to their properties of acquiring and retaining magnetism?
7. Describe how you would test an iron bar to ascertain if it was a magnet or not.
8. Write short notes on, *(a)* direction of magnetic field, *(b)* agonic line, *(c)* isoclinic lines, *(d)* aclinic lines, *(e)* transient induced magnetism, *(f)* ferro-magnetic.

CHAPTER 19

MAGNETISM

Units, Magnetic properties of materials, Calculations

We have discussed the various magnetic elements in a general way. In this chapter we shall try to deal with them quantitatively. For this we shall need some definitions, not all of them easy!

Also it is relevant to point out that in this subject, magnetism, many slight variations in terminology are used which are often confusing to the student. It may help if we can point out, where appropriate, alternative expressions which may be met and which mean the same thing.

Finally, these paragraphs will be more easily understood by the reader who has some knowledge of electro-magnetism. This section should therefore be read, for preference, in conjunction with some study of electricity as given from Chapter 21 onwards.

19.1 FLUX

Let us first consider this mysterious "something" that is "contained in" every magnetic field, flux. Its existence may be demonstrated, as we have seen, by flux lines, giving a flux pattern.

Flux is a collective term — the sum total of all the magnetic influence within the field. The symbol is Φ (Greek letter phi) and it is measured in **webers (Wb).***

It is not easy to define a weber without reference to electro-magnetic induction (see section 24.1) and the common definition is as follows:

Definition:

1 weber is the flux which, linking a circuit of 1 turn produces in it an e.m.f. of 1 volt as the flux is uniformly reduced to zero in 1 second.

**In honour of the German physicist, Wilhelm Edward Weber (1804-1891).*

It has to be admitted that this is not too easy to comprehend at first. No harm will be done at this juncture if we use, as an alternative, the concept of flux lines, and think of **1 Wb as represented by 1 flux line.**

19.2 FLUX DENSITY

The sum total of all the flux in a field is not a very useful concept, and much more important is the flux density.

Definition:

Flux density is the quantity of flux per square metre cross section taken at right angles to the direction of the flux.

It is measured in **webers per square metre (Wb/m²)**. Clearly, this is the important thing, the flux density (symbol *B*).

So unit flux density has 1 Wb/m² (or, as a useful concept, 1 flux line per square metre). This unit has its own name, the **tesla** (*T*).

And to sum up, if a magnetic conductor is of cross section A (m²) and *B* and Φ have the meanings assigned to them above, then clearly,

$$\Phi \text{ (webers)} = \text{A(m}^2\text{)} \times \text{B (teslas)} \quad \ldots\ldots\ldots\ldots \text{ I}$$

19.3 POLE STRENGTH

The strength of a magnetic pole is measured in webers.

So a "unit pole" (which is a theoretical idea because in practice there is of course no such thing as an isolated magnetic pole) is the pole of strength 1 weber and we think of it as radiating a flux of 1 weber i.e. one flux line emanates from a unit pole.

A pole of strength *m* webers would radiate flux of *m* webers. This would of course be distributed in all directions around the pole.

19.4 INVERSE SQUARE LAW

The force between two poles of strengths m_1 and m_2, a distance *d* apart is given by

$$F \alpha \frac{m_1 \times m_2}{d^2}$$

$$F = \text{constant} \times \frac{m_1 \times m_2}{d^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{ II}$$

This force can be attractive or repulsive depending on whether the two poles are unlike (i.e. a north and a south) or like (2 norths or 2 souths). F will be in newtons.

The constant's value depends on the medium separating the poles. For free space the constant turns out to have a value of

$$\frac{1}{16\pi^2 \times 10^{-7}}$$

We can assume it is the same for air.

WORKED EXAMPLE 1

Two magnetic poles of pole strength 0·52 Wb and 0·28 Wb are 20 cm apart in air. Calculate the force between the poles.

$$F = \frac{0{\cdot}52 \times 0{\cdot}28}{16\pi^2 \times 10^{-7} \times 0{\cdot}20^2} \text{ N}$$

$$F = 2{\cdot}305 \times 10^5 \text{ N}$$

19.5 FIELD STRENGTH

This is a quantity which is variously referred to as intensity of field, field intensity, magnetising force, field strength, all meaning the same thing; it is given the symbol H. It is customary to speak of "a bar magnet lying in a field of intensity so-and-so". Sometimes even it is simply referred to as "the field" — meaning field strength. The reader should learn to recognise these different shades of meaning.

For the unit of field strength, or magnetising force, we must have recourse to the realms of **electromagnetism**.

It is well known that there is a strong field inside a long coil, or solenoid, when a current is passed through the coil. It can also be shown that the strength of this magnetic field is directly proportional to the current in amperes (I) and to the number of turns (N) in the solenoid.

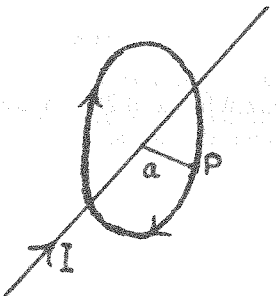
Restricting the coil for purposes of definition to 1 metre in length, and with the correct units for I and N, we may therefore write, for a solenoid L metres in length,

$$\text{Field strength} = I \times \frac{N}{L}, \text{ or in words,}$$

Field strength (H) = amperes × number of turns per metre N.B.

This is therefore the unit chosen for field strength, field intensity, magnetising force and so on, namely, the product amperes × number of turns per metre.

It is spoken in electromagnetism as "ampere turns per metre", and abbreviated to AT/m, an expression which is liable to be thought obscure if one did not know something of its origin.



In *pure magnetism* an alternative approach is preferred. It was explained in chapter 18 that a long straight wire carrying a current I (amps) is surrounded by a magnetic field.

It can be shown that if P is any point distant a (metres) from such a wire, then the magnetic field intensity H, at P, is directly proportional to the current and inversely proportional to the distance from the wire. In fact,

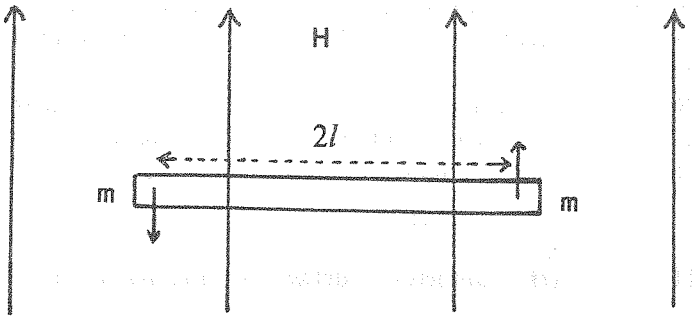
$$\text{Field strength } (H) = \frac{I}{2\pi a} \quad \text{N.B.}$$

Consideration of this formula will show that it is current (amperes) divided by distance (metres); that is, the unit of field strength in this form is "amperes per metre" (amp/m). This is the form we shall use in pure magnetism.

Of course, there is only one kind of field intensity — but two ways of expressing it!

19.6 MAGNETIC MOMENT

Consider a bar magnet of length $2l$ and pole strength m lying at right angles to a uniform magnetic field of strength H A/m.



A COUPLE will act on the magnet so as to try and align it with the field. This couple is known as the restoring couple and its magnitude will depend on H, m and l.

To determine the nature of this relationship first consider the following.

The moment of each pole about the central axis is given by

$$\text{pole strength} \times \text{distance} = m \times l$$

Therefore the total moment of both poles $= 2 \times m \times l$

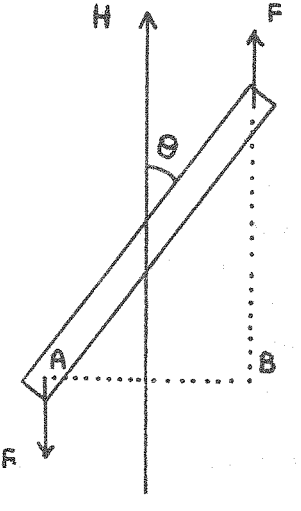
This total moment is called the MAGNETIC MOMENT, M.

i.e. $\mathbf{M = 2ml}$ ..... **III**

With m in webers and l in metres it follows the unit for magnetic moment is weber metre (Wb m).

Now the magnetic moment is NOT the restoring couple since it takes no account of the strength of the magnetic field.

To take it into account consider the bar magnet to have been deflected through an angle θ as shown below:



The force F acting at each end is given by $H \times m$.

These two forces, F, constitute a couple (section 5.1) whose moment is given by $F \times AB$.

Now $F = Hm$ and, using trigonometry on the above diagram, $AB = 2l \times \sin \theta$.

$$\text{Moment of couple} = F \times AB$$
$$= Hm \times 2l \sin\theta$$
$$= 2mlH \sin\theta$$

But $M = 2ml$

Moment of couple = MH sin θ IV

WORKED EXAMPLE 2

A bar magnet has a distance of 150 mm between poles and has a pole strength of 6×10^{-6} Wb. It is placed at 10° to a magnetic field of strength 20 A/m. Calculate the moment of the restoring couple.

$$M = 2ml$$
$$= 2 \times 6 \times 10^{-6} \times 0{\cdot}15 \text{ Wb } m$$
$$= 1{\cdot}8\ \mu\text{Wb } m$$
$$\text{Moment} = MH \sin\theta$$
$$= 1{\cdot}8 \times 10^{-6} \times 20 \times \sin 10°$$
$$= 6{\cdot}25 \times 10^{-6}\ Nm$$

19.7 INTENSITY OF MAGNETISATION

It has already been suggested that the "packing" of the flux lines provides us with a means of measuring the intensity of magnetisation (E) in a bar (section 18.11).

Definition:

The intensity of magnetisation is the pole strength or flux per unit area near the pole, taken at right angles to the direction of the flux lines V

An alternative definition, if the body be uniformly magnetised, is,

$$\text{Intensity of magnetisation (E)} = \frac{\text{magnetic moment}}{\text{volume of magnet}} \quad \text{..... Va}$$

or in other words,

E = magnetic moment per unit volume **N.B.**

V and Va are easily seen to be equivalent definitions.

Let the magnet be of length $2l$ and cross-section a and have pole strength m.

Then from the first definition

$$E = \frac{m}{a}$$

The volume is $2la$ and the magnetic moment $2ml$, so from the second definition,

$$E = \frac{2ml}{2la} = \frac{m}{a} \text{ as before.}$$

The unit of intensity of magnetisation is the weber per square metre (Wb/m²).

19.8 PERMEABILITY

This quantity describes the ease with which lines of magnetic force can pass through a material. It is the ratio of the magnetic flux density produced in the material to the strength of the magnetic field causing it. It is denoted by "μ".

$$\mu = \frac{\mathbf{B}}{\mathbf{H}} \text{ or } \mathbf{B} = \mu\mathbf{H} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{ VI}$$

Note that the second form will give us the flux density **B** (in Wb/m² remember) though H the magnetic intensity, is in amp/m. It is clear therefore that μ cannot be dimensionless — cannot be a pure number — and note:

The unit of permeability is the henry per metre (H/m) N.B.

The henry is the unit of inductance, and a fuller treatment of inductance will be found in Chapter 24. However, briefly, when the magnetic flux linking with a circuit (coil) changes, an e.m.f. is caused in the circuit. This phenomenon is called **inductance**, and:

Definition:

A circuit has an inductance of 1 henry when an e.m.f. of 1 volt is produced when the current changes at the rate of 1 ampere per second.

For the present however, let us note, for example that μ_0, the permeability of free space, is $4\pi \times 10^{-7}$ henrys per metre. Other substances have their own permeabilities.

RELATIVE PERMEABILITY

This is the absolute permeability of a substance (μ) relative to the permeability of free space (μ_0), i.e.,

$$\textbf{Relative permeability} = \frac{\mu}{\mu_0} \quad \ldots\ldots\ldots\ldots\ldots \text{ VII}$$

Relative permeability is dimensionless — a pure number. The relative permeability of soft iron naturally varies considerably, but a typical value would be 6000, i.e.,

Absolute permeability (μ) for soft iron $= 6000 \times 4\pi \times 10^{-7}$ H/m.

19.9 SUSCEPTIBILITY

By the susceptibility (k) of a substance we mean the ratio of the intensity of magnetisation to the magnetic field strength.

$$\textbf{Susceptibility (k)} = \frac{\textbf{Intensity of magnetisation (E)}}{\textbf{Magnetising force (H)}} \quad \textbf{VIII}$$

k is dimensionless — a pure number or ratio.

WORKED EXAMPLE 3

A bar of soft iron of length 150 cm and cross section area 2 cm^2 is in a magnetic field of total intensity $3{\cdot}5 \times 10^3$ amp/m in the direction of its length, when its pole strength is found to be 13·0 Wb. Find the magnetic moment of the bar, the intensity of magnetisation, and the susceptibility of the iron.

With all the dimensions in metres, we have,

M (mag. mom.) $=$ length $\times$ pole strength $= 1{\cdot}5 \times 13 = 19{\cdot}5$ Wb-m

E (intensity of magnetisation) $= \dfrac{\text{magnetic moment}}{\text{volume}} = \dfrac{19{\cdot}5}{1{\cdot}5 \times 2 \times 10^{-4}}$

$= 6{\cdot}5 \times 10^{-4}$ Wb/m^2

k (susceptibility) $= \dfrac{E}{H} = \dfrac{6{\cdot}5 \times 10^{-4}}{3{\cdot}5 \times 10^3} = 18{\cdot}6$

WORKED EXAMPLE 4

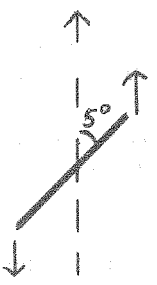
A specimen of iron of relative permeability 3000 is placed in a field of magnetising force $2{\cdot}0 \times 10^2$ amp/m.

Calculate its absolute permeability (μ) and the flux density in the iron.

$$\mu(\text{abs.}) = \mu(\text{rel.}) \times \mu_0 = 3000 \times 4\pi \times 10^{-7} \text{ H/m}$$
$$= 12\pi \times 10^{-4} \text{ H/m} = 37{\cdot}7 \times 10^{-4} \text{ H/m}.$$

For the flux density, we have $B = \mu H$, so

$$B = 12\pi \times 10^{-4} \times 2{\cdot}0 \times 10^2 = 24\pi \times 10^{-2} = 75{\cdot}4 \times 10^{-2} \text{ Wb/m}^2.$$



WORKED EXAMPLE 5

A bar magnet of length 36 cm has pole strength 500×10^{-8} Wb. Calculate the restoring couple acting on the magnet when deflected by an extraneous force 5° from the magnetic meridian, the horizontal component of the earth's field H being 18·0 amp/m.

Restoring couple $= MH \sin \theta = 500 \times 10^{-8} \times 0{\cdot}36 \times 18{\cdot}0 \times \sin 5°$ Nm, which reduces to $2{\cdot}76 \times 10^{-6}$ Nm.

Consider this question a little further.

In the deflected position, since the magnet is in equilibrium, the restoring couple must be exactly equal to the deflecting couple provided by the extraneous force. If the latter is due, for example, to permanent magnetic poles in the ship structure, the deflecting couple will remain constant, provided of course, that the ship is on the same heading, on each occasion.

Hence, if θ_1 be the deflection when the HF is H_1*
and θ_2 be the deflection when the HF is H_2

we have

$$MH_1 \sin \theta_1 = MH_2 \sin \theta_2 \text{ and cancelling } M,$$

$$\frac{\sin \theta_2}{\sin \theta_1} = \frac{H_1}{H_2}$$

i.e., the sine of the deflection varies inversely as HF.

For small angles such as the deviation of a ship's compass, $\sin \theta$ is very equal to θ, and so we may say, (for small angles)

the deflection varies inversely as HF . . . . . N.B.

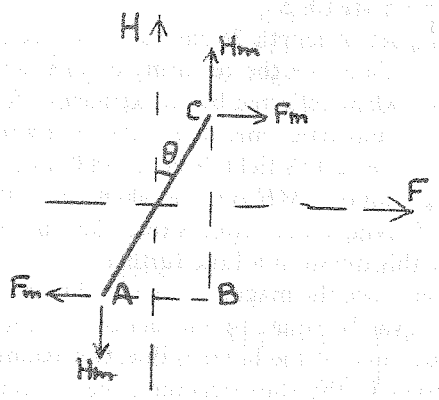
So for example, if the bar magnet were removed to a place where $HF = 12{\cdot}0$ amp/m, other things remaining equal find the new deflection.

$$\frac{\theta_2}{\theta_1} = \frac{H_1}{H_2}, \text{ i.e., } \theta_2 = \frac{5 \times 18{\cdot}0}{12{\cdot}0} = 7{\cdot}5° \text{ deflection at second place.}$$

WORKED EXAMPLE 6

A compass needle is of pole strength m Wb and length $2l$ and is at a place where $HF = 14{\cdot}6$ amp/m. It is subjected to a deflecting field acting at right angles to the magnetic meridian, of intensity 6·0 amp/m at the compass position. Calculate the deflection of the compass needle when in equilibrium, assuming the North end to be attracted by the extraneous force.

*Read HF as "the horizontal component of the earth's magnetic field".



The sketch illustrates the conditions. Let the intensity of the deflecting field be F; and of course, H is the earth's hor. field intensity.

The forces acting on the poles of the needle are as shown and the needle is in equilibrium under two couples.

Restoring couple $= Hm \times AB$

$= Hm \times 21 \sin\theta$ ..... (1)

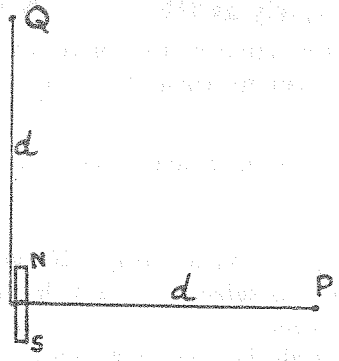
Deflecting couple $Fm \times BC$

$= Fm \times 21 \cos\theta$ ..... (2)

(1) = (2) i.e., $H \sin\theta = F \cos\theta$

and $\tan\theta = \dfrac{F}{H} = \dfrac{6{\cdot}0}{14{\cdot}6}$ and $\theta = 22°\ 20' =$ deflection

19.10 INTENSITY OF FIELD DUE TO A SHORT BAR MAGNET



Consider the magnet illustrated, of moment M. By a "short" magnet is meant one whose length is small in relation to the distance d.

P is a point in the field on a line at right angles to the axis of the magnet. This is called the "broadside-on" position.

Similarly, Q is said to be in the "end-on" position.

The proof of the following equations may be found in more advanced works. We shall have to state them without proof.

BROADSIDE-ON position

It can be shown that the field intensity F at P is given by

$$\mathrm{F\ (amp/m)} = \frac{\mathrm{M}}{\mathrm{K}\mu_0 \mathrm{d}^3}$$

where K turns out to have a value of 4π.

END-ON position

Similarly, it can be shown that

$$\mathrm{F\ (amp/m)} = \frac{\mathrm{2M}}{\mathrm{K}\mu_0 \mathrm{d}^3}$$

It will be observed that the field in the end-on position is exactly twice the field in the broadside-on position. It follows from what has been said in the previous section that for *small angles*, the deflection caused on a small compass needle at Q will be twice the deflection at P.

Also, consideration of the above equations, in which M, K and μ_0 are constants, will show that the field intensity is inversely proportional to d^3, i.e., for small angles,

the deflection is inversely proportional to d^3 **N.B.**

WORKED EXAMPLE 7

Find the field strength at P due to the short magnet shown, of length 10 cm and pole strength 30μWb.



The above shows a broadside-on position.

$$\mathrm{F} = \frac{M}{K\mu_0 d^3} \quad \text{where } M = 2ml$$

$$\mathrm{F} = \frac{30\times 10^{-6}\times 0{\cdot}1}{4\pi\times 4\pi\times 10^{-3}\times 2^3} \quad A/m$$

$$= 0{\cdot}0237\ A/m$$

WORKED EXAMPLE 8

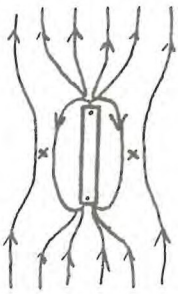
A magnet when end-on to a compass needle at a distance of 1·0 m deflects it 8°. What deflection will the same magnet produce when broadside-on at a distance of 1·2 m?

Using the formula,

$$\frac{\theta_2}{\theta_1} = \frac{d_1^3}{d_2^3} \text{ and so } \theta_2 = \frac{8 \times 1{\cdot}0^3}{1{\cdot}2^3}$$

whence new deflection = 4·6°, still in the end on position. So broadside-on, deflection will be 2·3°.

WORKED EXAMPLE 9



A short bar of magnetic moment 4500×10^{-8} Wb-m lies in the magnetic meridian with North end pointing North. By sketching the flux lines indicate the position of the neutral point(s) and calculate its position if HF = 15·0 amp/m at that place. (Assume $\pi^2 = 10$).

Neutral points will lie on the axis of symmetry at right angles to the axis of the magnet. At the neutral point, there is no resultant horizontal field, i.e., the field due to the magnet is equal and opposite to HF.

For the field due to the magnet, we have, for the broadside-on position.

$$F = \frac{M}{K\mu_0 d^3} \text{ i.e., } d^3 = \frac{M}{K\mu_0 F}$$

$$d^3 = \frac{4500 \times 10^{-8}}{4\pi \times 4\pi \times 10^{-7} \times 15} = \frac{30}{16\pi^2} = \frac{3}{16}$$

from which $d = 0{\cdot}57$ m = distance of neutral points from the magnet, along the axis of symmetry at right angles to the axis of the magnet.

WORKED EXAMPLE 10

A solenoid is made up in four layers, each having 12 turns per cm, and a current of 10A is passing. Calculate (*a*) the field intensity inside the coil (*b*) the flux density (*c*) the flux density when an iron core of relative permeability 4000 is placed inside the coil.

(*a*) Field intensity (H) = amps × no. of turns/m

$$= 10 \times 12 \times 100 \times 4 = 48 \times 10^3 \text{ AT/m}$$

(*b*) Flux density, $B = \mu_0 H$ $= 4\pi \times 10^{-7} \times 48 \times 10^3 = 192\pi \times 10^{-4}$ Wb/m^2

(*c*) With the core, $\mu = 4000 \times \mu_0$, so in this case,

flux density $= 4000 \times 192\pi \times 10^{-4}$

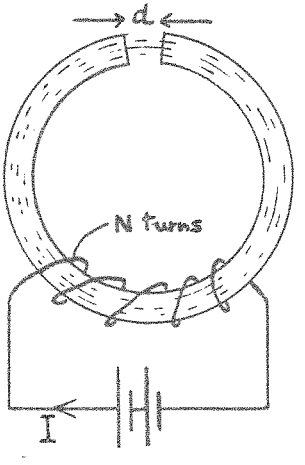
$= 768\pi \times 10^{-1}$ Wb/m^2

19.11 THE MAGNETIC CIRCUIT

The reader will be aware that every electrical circuit has three essential features. These are (1) a continuous path in which something called the current is flowing (2) a force which is causing the current to flow, namely the electro-motive force, or e.m.f. and (3) a resistance offered by the path of the flow of the current. The relation between these is summed up in Ohms Law, namely

$$\text{Current} = \frac{\text{e.m.f.}}{\text{resistance}}$$

Let us now consider the electro-magnet illustrated, consisting of a soft iron core of length L, cross section a, and permeability μ, with a small air gap of length d and permeability μ_0, surrounded by a coil of N turns in which a current I amperes is passing.



We have seen already that a magnetic field exists inside such a coil, and it can be shown that the intensity of such a field is proportional to the current I and to the number of turns N, i.e., the intensity or magnetising force in the core is *proportional* to IN ($= IN$ if the correct units are employed). This magnetising force is analogous to the e.m.f. and is called the **Magneto-Motive Force (MMF)** (Symbol F).

Since MMF is amperes × turns, the unit of MMF is the "ampere turn" (At).

Also this magnetising force will produce a flux in the soft iron, the total flux being Φ (in webers, as usual).

The essential properties of an electric current are (*a*) that it is the same all round the circuit and (*b*) that is caused by the e.m.f.

The magnetic flux is the same all round the circuit and is caused by the MMF. Thus the flux is analogous to the current.

What about the resistance? Well, we have said that the permeability is an indication of a material's willingness to accept magnetisation. So it seems reasonable to suggest that a material's "unwillingness' to accept magnetisation is inversely proportional to its permeability, i.e., is proportional to $1/\mu$.

Also it is shown in section 22.5 that the electrical resistance of a conductor is directly proportional to its cross section area.

Using these ideas, and assuming use of correct units to dispense with the proportionality, we may say that the unwillingness to accept magnetisation of the iron core is $L/\mu a$ and of the air is $d/\mu_0 a$.

This unwillingness is analogous to resistance in the electrical circuit, and in the magnetic circuit is called the **reluctance** (S).

Moreover, when reluctances are in series they can be added algebraically just like electrical resistances, so the total reluctance is

$$\frac{L}{\mu a} + \frac{d}{\mu_0 a}$$

The unit of reluctance is A/Wb.

Hence, we can now sum up by stating

$$\textbf{Magnetic Flux} = \frac{\textbf{Magneto-Motive Force}}{\textbf{Reluctance}}$$

$$\textbf{in the same way as current} = \frac{\textbf{e.m.f.}}{\textbf{resistance}}$$

For the magnetic circuit we have

$$\Phi = \frac{\text{mmf}}{\text{S}}$$

$$\Phi = \frac{\text{IN}}{\left(\frac{\text{L}}{\mu\text{a}} + \frac{\text{d}}{\mu_0\text{a}}\right)}$$

It must be remembered that the magnetic equation refers to statical phenomena, whilst the electrical equation refers to dynamical phenomena. Nothing is moving round in the flux.

In fact, the magnetic equation is only an analogy, but nevertheless, a very useful one at times.

TEST PAPER 19
Magnetic Properties of materials

Take $\mu_0 = 4\pi \times 10^{-7}$.

1. A ship's permanent magnetism causes 6° deflection of the compass needle at a place where HF is 12·0 amp/m. What will be the deflection at a place where HF = 16·0 amp/m?
2. Explain the various terms in the formula $F = m_1 m_2 / K\mu_0 d^2$ governing the force between magnetic poles in air.

   In air, two unlike magnetic poles of pole strength $320\pi \times 10^{-8}$ Wb and $400\pi \times 10^{-8}$ Wb respectively attract each other with a force of $2{\cdot}0 \times 10^{-4}$ newton. Calculate the distance apart.
3. If the Earth's field at a certain place has a horizontal flux density of $1{\cdot}4 \times 10^{-5}$ Wb/m^2, what is the value of HF at that place?
4. The flux density in a piece of iron is 0·80 Wb/m^2 when placed in a magnetising field of intensity $2{\cdot}4 \times 10^2$ amp/m. Calculate is absolute permeability and relative permeability, stating the units (if any) in which each is expressed.
5. A permanent magnet placed due West magnetic of a compass needle at a place where HF = 18·6 amp/m creates a uniform magnetic field of intensity 4·2 amp/m at the compass position, such as to attract the N. end of the compass needle. Calculate the deflection of the needle when at rest.
6. A small compass needle lies due east magnetic of a bar magnet at a place where HF = 24·5 amp/m and the deflection is observed to be 12·0°. Calculate the field intensity at the compass position due to the bar magnet.
7. When broadside-on to a compass needle a bar magnet at a distance of 1·6 m caused a deflection of 7°. At what distance end-on would the magnet have to be placed to cause the same deflection?
8. A short bar magnet of moment 820×10^{-8} Wb-m lies in the magnetic meridian with its N. end pointing South. By sketching the flux lines deduce the position of the neutral point(s) and calculate its position if HF = 22·0 amp/m at that place.

9. A magnet of moment M causes a deflection of 4° when placed 2·0 m from a compass needle. At what distance should a magnet of moment $2M$ be placed in the same manner of alignment to cause the same deflection?
10. A solenoid 80 cm long has 6 layers of wiring, each of 20 coils per cm, and is carrying a current of 8 A. What is (*a*) the magnetising force inside the solenoid (*b*) the flux density with an air core (*c*) the flux density with an iron core of relative permeability 3600?
11. The core of a solenoid bent into the form of a circle forms a closed magnetic circuit of length 60 cm and cross section 15 cm^2. It is wound with a total of 4000 turns of wire in which a 6 A current is passing. Calculate the MMF, the magnetising force, the flux density with an air core, the total flux and the reluctance.
12. A solenoid consists of 5000 turns of wire wound round an iron core 50 cm long, of relative permeability 3000. If the flux density in the bar is found to be 6π Wb/m^2, calculate the magnetising force, and the current in the coil.
13. A compass needle of length 16 cm and pole strength $8\pi \times 10^{-6}$ Wb is deflected 6° by an extraneous force at a place where the Earth's horizontal force (HF) = 26·0 amp/m. Calculate the restoring couple on the needle.
14. Two magnetic poles of strength $320\pi \times 10^{-8}$ Wb and $400\pi \times 10^{-8}$ Wb respectively are placed 40 cm apart in air. Calculate the force between them.
15. Find the field intensity due to a short magnet at a point end-on to it and distant 1·5 m from it, length of magnet being 12 cm and pole strength $1600\pi \times 10^{-8}$ Wb.
16. A short bar magnet of moment 3200×10^{-8} Wb-m lies in the magnetic meridian with North end pointing North. Sketch the flux pattern and deduce the position of the neutral point(s). Calculate the exact position if HF = 30·0 amp/m.

CHAPTER 20

MAGNETISM

Ship magnetism. Compass deviation

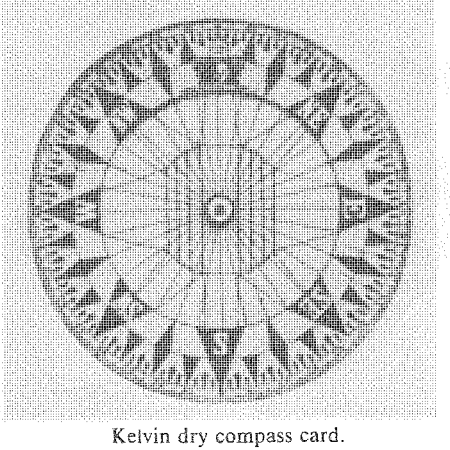
20.1 THE MAGNETIC COMPASS

In its simplest form, the magnetic compass consists of a magnetised piece of steel, generally referred to as the "compass needle", mounted on a smooth pivot so that it is free to rotate in a horizontal plane. It depends for its operation on the magnetic force of the Earth. When allowed to settle, such a needle will line itself up with the direction of the Earth's field at that place (in other words, with the magnetic meridian) and by means of a graduated rim, magnetic North may be determined.

The magnetic compass, which, notwithstanding the introduction of the gyro compass, is still widely used at sea, has been developed into an accurate scientific instrument.

20.2 THE STANDARD COMPASS (DRY CARD TYPE)

(*a*) The compass card and magnet assembly is illustrated in the diagram below.



Kelvin dry compass card.

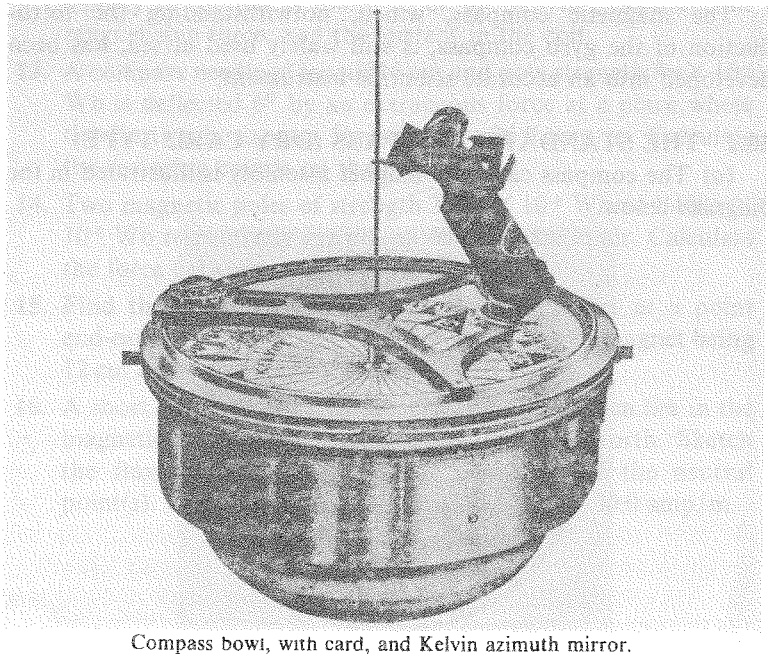
The card is of rice paper, stiffened by a very light aluminium ring, from which the magnets are suspended by silken cords. This construction keeps the card as light as possible consistent with strength. At the centre of the card is a metal socket containing the cap, a very hard semi-precious stone, usually agate. This rests on the pivot when the card is in position in the compass bowl.

There are eight magnets, arranged as shown. These are kept short and compact so that all will lie in a uniform magnetic field and so will function as one. The centre of gravity of the whole assembly is below the point of support and thus it preserves a horizontal position.

The card must be accurately divided into points and quarter-points as well as degrees. It must be adequately illuminated when in use.

(*b*) The bowl is of brass slung in gimbals and its centre of gravity is low so that it remains upright.

The gimbals are suspended on strings so that the bowl is protected from vibration.



Compass bowl, with card, and Kelvin azimuth mirror.

Centrally placed inside the bowl is the pivot, a vertical shaft with a very hard, polished tip of iridium. The jewel cap fits on to this pivot and so supports the card on a bearing which is almost frictionless and which also will give years of service without undue wear.

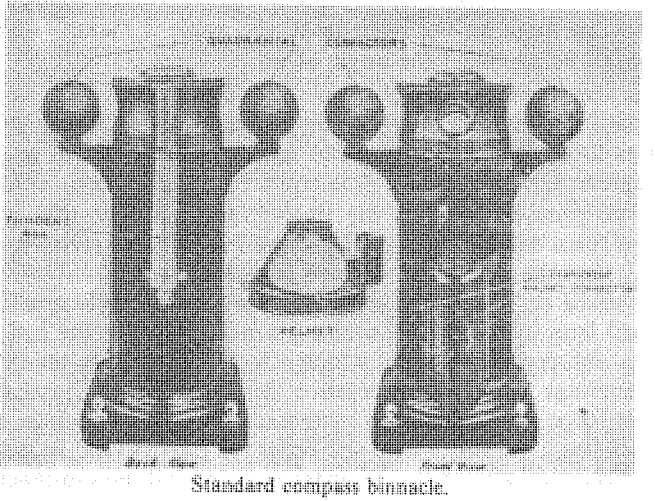
An important feature of the bowl is the lubber line, which of course indicates the ship's head.

(*c*) Finally, the bowl is mounted in the binnacle, illustrated in the diagram below.

The soft iron spheres, or quadrantal correctors are used for compensating against the effect of horizontal soft iron in the ship (e.g., the deck beams) and the Flinder's bar for compensating against vertical soft iron (e.g., the funnel). Further consideration of these however is beyond our scope.

The permanent magnets for compensating the compass against the ship's permanent poles are housed inside the binnacle, as indicated, where also there is a small bucket containing the heeling error correctors. The small doors enclosing these correcting magnets, shown open in the diagram below are of course kept locked except when the actual work of compass correcting is in progress.

Finally, it is only necessary to state the obvious — that the binnacle must be of robust construction and strongly secured to the deck.



Standard compass binnacle.

20.3 THE LIQUID COMPASS

This type is generally popular as a steering compass. The bowl is filled with a mixture of alcohol and water (pure water would freeze too easily) and is fitted with a flexible diaphragm in its base to allow for expansion and contraction of the liquid.

The card is much heavier than the dry card being of metal construction throughout, but it incorporates a float centrally placed, so that pressure on the pivot is still very slight.

There are usually just two fairly substantial magnets, enclosed in brass to protect them from corrosion.

The effect of the liquid is to damp out excessive swinging of the card, and for this reason the liquid compass is sometimes called "dead beat". In general, its main advantage is that it is steadier than a dry card, and so makes a good steering compass.

20.4 DIRECTIVE FORCE

We have already referred to the directive force on the compass needle in section 18.19 where we described it in general terms as "the force that gives the compass needles their ability to point North".

On the Earth's surface in a place free from any disturbing influences, the directive force is H, **the horizontal component of the Earth's total magnetic field**. It will be recalled that the Earth's total field is only horizontal at the magnetic equator. Everywhere else it acts at some angle to the horizontal, the angle it makes (with the horizontal) being called the **angle of dip**.

This total force is split up into horizontal and vertical components, H and Z, and the greater the value of H, the greater directive force the compass needles will possess. This will occur at the magnetic equator, where H is equal to the Earth's total force.

As we proceed towards the Poles, H diminishes, that is, the directive force diminishes, until near the Poles the directive force is so small that the magnetic compass is useless.

On board ship, the directive force is not in general equal to H. The hard iron in the ship contains permanent poles, the soft iron contains induced poles, all of which affect the compass needle. On some headings these will strengthen the Earth's field, so that the directive force of the needle is increased. As the ship swings on to other headings, the ship's field will resist the Earth's field so that the actual directive force at the compass position will be diminished.

Thus, in an uncompensated compass the directive force of the needle would vary considerably as the ship's head swung through a complete arc of 360° — that is, the compass would work very efficiently on some headings, not nearly so efficiently on others. This is one of the main reasons for compensating the compass, namely to **equalize the directive force on all headings.**

Unfortunately, when this has been done, it is found invariably to be the case that the mean value of the directive force at the compass position is somewhat less than *H*.

20.5 COEFFICIENT λ

The ratio between the mean directive force at the compass position and the directive force on shore is a very important quantity in ship magnetism. It is called **coefficient lambda (λ).**
Definition:

$$\text{Coefficient } \lambda = \frac{\text{Mean directive force at the compass position}}{\text{Directive force ashore}}$$

A knowledge of λ is very important in choosing a position for a compass on board ship. Obviously, we want it to be as high as possible.

A good value for λ would be 0·9 which is only another way of saying that the mean directive force at the compass position is 0·9 of *H*. As we have said λ is always less than 1, that is, the mean directive force at the compass position is always less than *H*.

20.6 THE PERMANENT MAGNETISM OF A SHIP

The permanent magnetic poles in a steel ship are located in the hull, and they are caused by the continuous vibration and hammering the hull and superstructure receive whilst under construction.

It was explained in section 18.12 in connection with the poles induced into hard iron by hammering, that the polarity of such a bar is determined by its position relative to the Earth's field whilst the hammering is going on:

red will be induced in the N'ly portions, adjacent to the Earth's blue.
blue will be induced in the S'ly portions, adjacent to the Earth's red.

A ship is a complex structure however. With modern methods of construction and particularly the increasing use of large prefabricated units which will possess poles of their own, she cannot be said to have been "built" in any one particular direction. The completed ship will have permanent, or nearly permanent, poles, but there is no simple connection between the position of these poles and the building direction.

These permanent poles are really the combined effect of a lot of individual poles, and the ship's total permanent magnetic force is simply the resultant of a number of individual magnetic forces. These permanent poles may be said to be "built" into the ship, and no great harm is done if, when meeting the term "built", it is interpreted as meaning "magnetically built", that is,

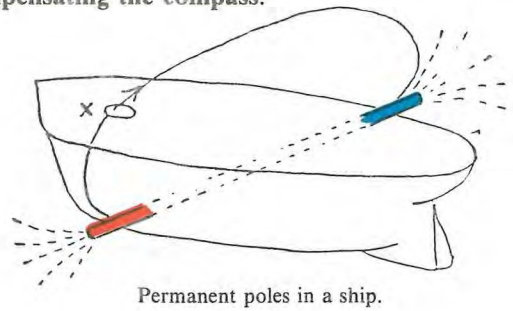
(*a*) possessing poles consistent with those we should find in a piece of iron hammered whilst lying in the direction stated, and

(*b*) "built" in the sense that these poles are more or less permanent in the ship and change very little during her lifetime.

In these Notes, however, we shall try to avoid using the term "built" at all. It will be enough to state for example that a ship has a "blue pole forward, a red pole aft", without considering how they got there.

20.7 FORCES P, Q AND R

When a ship goes into service she is amongst other things, a huge magnet, and the art of the compass adjuster lies in analysing, in splitting up, this total magnetic force on the compass needle into various parts which may then be conveniently dealt with. This is called **compensating the compass.**



Permanent poles in a ship.

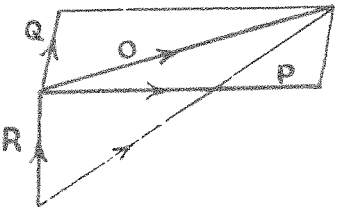
We have from time to time mentioned the induced poles in soft iron. These all affect the compass, of course, and have to be compensated, but we are not at the moment considering soft iron at all. We are only considering the permanent poles.

Consider a ship having a red resultant pole low on the port bow, a blue resultant pole high on the starboard quarter.

For many practical purposes, she can be regarded as having a huge magnet inside her, with poles as shown. The total magnetic force of the ship runs obliquely through the hull. Similarly, the direction of the ship's field runs obliquely through *X*, the compass position.

Now the first thing to remember is that the compass needle can only move in a horizontal plane, and therefore only a horizontal force can move it. We accordingly split up the ship's total force into **three components** (component forces, that is) as shown in the diagram.

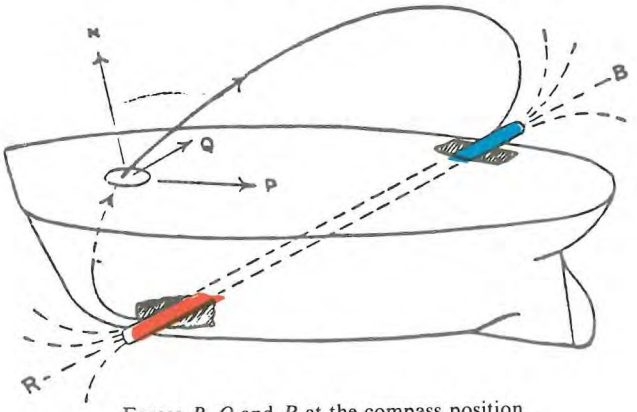
First we split the total force into a **vertical component**, *R*, and a horizontal component running obliquely across the deck. The symbol *O* has been given by some writers to this oblique horizontal component.



Resolution of total force into three components.

This force *O* is again split up into two component forces at right angles, namely, the **fore and aft component**, *P*, and the **thwartship component**, *Q*.

The effect of these components at the compass position is shown in the diagram below.



Forces *P*, *Q* and *R* at the compass position.

Since force *R* acts vertically, it can cause no deviation as long as the ship is upright. We shall therefore not consider *R* any further here, but confine our attentions to forces *P* and *Q*. These will cause deviation of the compass needle.

Of course, sometimes the blue pole will be forward, sometimes aft; sometimes to port, sometimes to starboard. To distinguish between the different cases, signs are given to these components as follows (remember that we are only using the word "component" as short for "component of a force", *P*, *Q*, *R* **are forces**).

Well then,

+ *P* means an attraction of the **N. end** of the compass needle towards the **Bow**.

− *P* means an attraction of the **N. end** of the compass needle towards the **Stern**.

+ *Q* means an attraction of the **N. end** of the compass needle to **Starboard**.

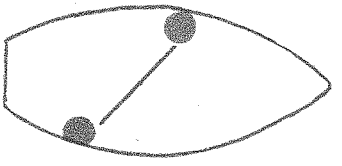
− *Q* means an attraction of the **N. end** of the compass needle to **Port**.

Or in other words,

+ *P* means the ship has a blue pole Forward.
− *P* means the ship has a blue pole Aft.
+ *Q* means the ship has a blue pole to Starboard.
− *Q* means the ship has a blue pole to Port.

Some practice must now be obtained in deciding what forces a ship will possess for a given position of the permanent poles.

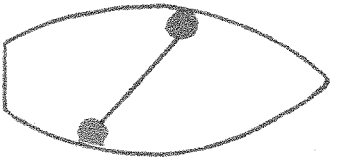
Example 1. Ship with red pole on starboard quarter, blue pole on port bow.



Let us fix our attention on where the ship's Blue pole is (because that is the one that attracts the N. end of the needle).

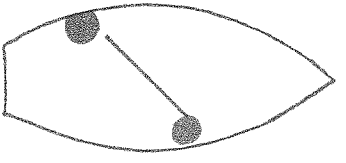
Well then, the blue pole is forward and to port. Thus, ship has $+P, -Q$.

Example 2. Red pole port bow, blue pole starboard quarter.



Blue pole is aft, and to starboard. Thus, ship has $-P, +Q$.

Example 3. Red pole port quarter, blue pole starboard bow.



Blue is forward and to starboard. Ship has $+P, +Q$.

We shall now consider the effects of forces P and Q in causing deviations.

20.8 ADVANTAGE OF USING COMPONENTS

It should be kept in mind that the main purpose in splitting the ship's total force into component forces is that each component may then be dealt with separately. We have already seen that R acts *vertically* through the compass and therefore cannot cause deviation as long as the ship is upright, that is, as long as R is vertical. We can thus proceed just as if R did not exist.

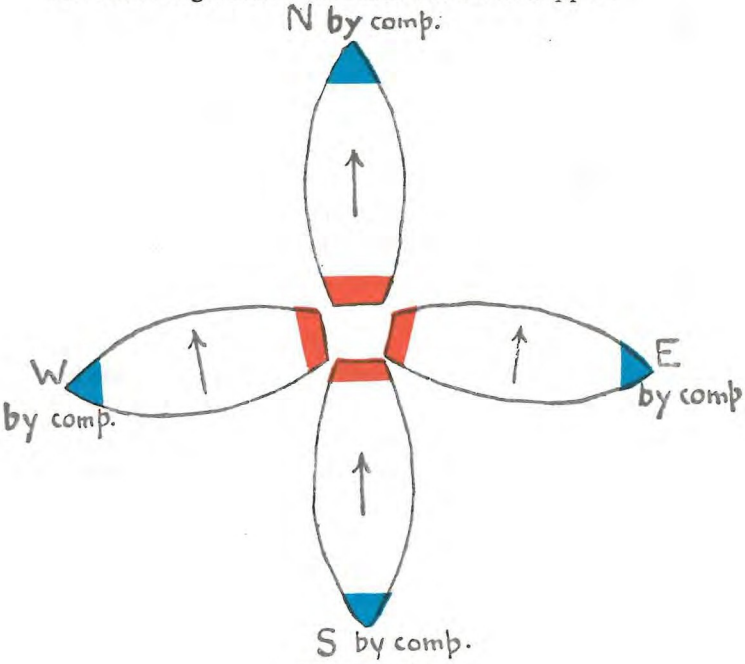
Similarly, we can consider P, or Q, just as if the other did not exist. The deviations caused by either may be deduced quite separately, and the means taken to compensate each are also quite separate.

20.9 FORCE + P

Consider a ship having a blue pole on the starboard bow ("forward and to starboard"), i.e., a $+P$ and a $+Q$.

We will take the $+P$ first and consider its effect on the compass needles. The effect will be exactly as if there were blue and red poles **in the fore and aft line the blue pole being forward.**

The following sketch illustrates what will happen.



Effect of $+P$ force.

HEAD N. BY COMPASS

The ship's poles are in line with the compass needles so there will be no deviation.

The ship's blue pole however, is acting in conjunction with the Earth's blue, so there is an **increase in directive force.**

Now, as the ship's head swings towards N.E. and beyond, the $+P$ (just a convenient way of saying "blue pole forward") will attract the N. end of the needles, causing E'ly deviation. This will be a maximum when the deflecting force is at right angles to the needles, i.e., on E. by compass. Hence:

HEAD E. BY COMPASS

The deflecting force is at right angles to the needles and is causing **maximum E'ly deviation.**

As the ship continues to swing towards S.E. the deviation is still E'ly, but reduced in amount, until finally, on South, deviation is again zero.

HEAD S. BY COMPASS

The deflecting force is again in line with the needles, so there will be no deviation.

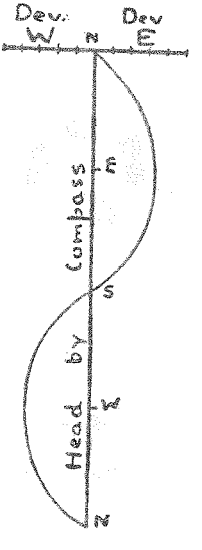
The ship's red pole however is interposed between the N. end of the compass needle and the Earth's blue, causing a **loss of directive force.**

As the ship's head swings into the S.W. quadrant, the deviation begins to increase again, but this time, W'ly in name.

HEAD W. BY COMPASS

It is easy to see the deflecting force is again at right angles to the needles, causing **maximum W'ly deviation.**

Finally, the deviation decreases again, though still remaining W'ly, until on north by compass, deviation is again zero.



Curve of deviations due to a blue pole forward (+ *P* force).

These deviations may conveniently be illustrated by a curve, as shown.

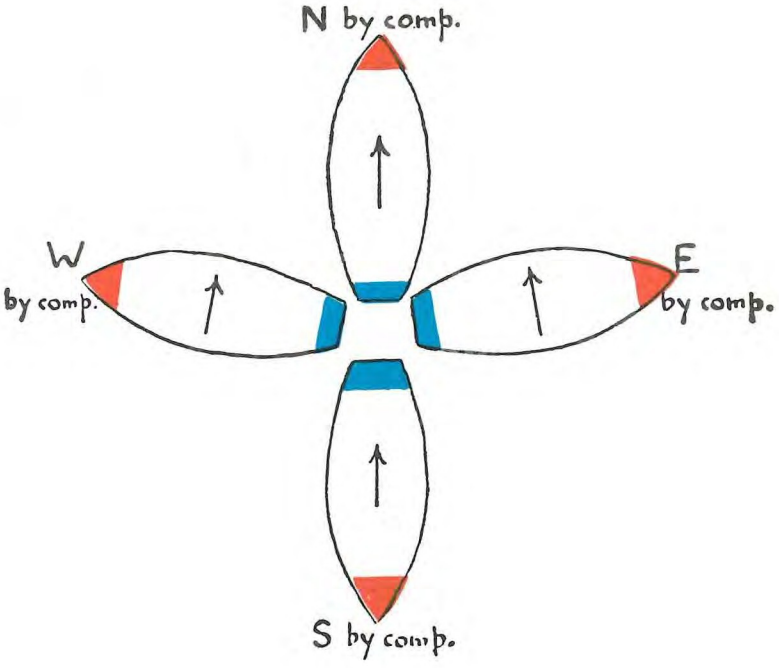
The centre line (straight) gives the direction of the ship's head by compass, and the curve shows the deviation due to a + *P* force, that is:

Zero on N., increasing to a maximum E'ly deviation on E.

Zero again on S., increasing to maximum W'ly deviation W. on and back, to zero again on 360°.

FORCE − *P*

The student who has followed the above carefully should have no difficulty in deducing a similar way what will happen in the case of a ship with a − *P* **(that is, a blue pole aft).**



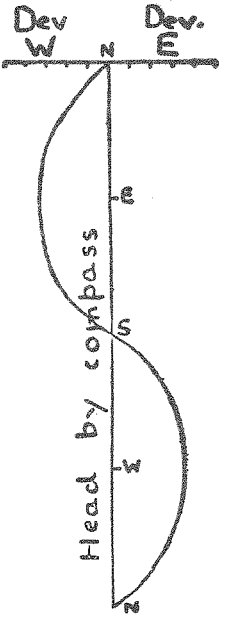
Effect of − *P* force.

The diagram above shows what happens during a complete swing, that is:

N. (comp.), no deviation, decrease in directive force.
E. (comp.), Maximum W'ly deviation.
S. (comp.), no deviation, increase in directive force.
W. (comp.), Maximum E'ly deviation.

The diagram below gives the curve of deviations that would result.

These simple curves are very important. Note that they are not drawn to any particular scale. Nor are they actual curves of the deviation that would be found on any ship's compass, because, of course, the compass is compensated against these effects and in any case, the resultant deviation on the compass is the combined effect of several different forces, some of which tend to cancel out.



Curve of deviations due to $-P$ force.

These are purely symbolic curves, showing the effect of a $+$ or $-P$ and as such they should be understood.

FORCE + *Q*

Similarly the effect of a + *Q* is easily seen.

Referring to the diagram below.

HEAD N. BY COMPASS

The thwartship deflecting force (blue to starboard) is at right angles to the needle causing **maximum E'ly deviation.**

HEAD N.E. BY COMPASS

Deviation is reduced in amount but is still E'ly.

HEAD E. BY COMPASS

No deviation, but **loss of directive force**, due to the ship's red being interposed between the N. end of the compass needles and the Earth's blue.

HEAD SE. BY COMPASS

Deviation increasing again but this time, W'ly.

HEAD S. BY COMPASS

The deflection force is again at right angles, causing maximum W'ly deviation.

HEAD S.W. BY COMPASS

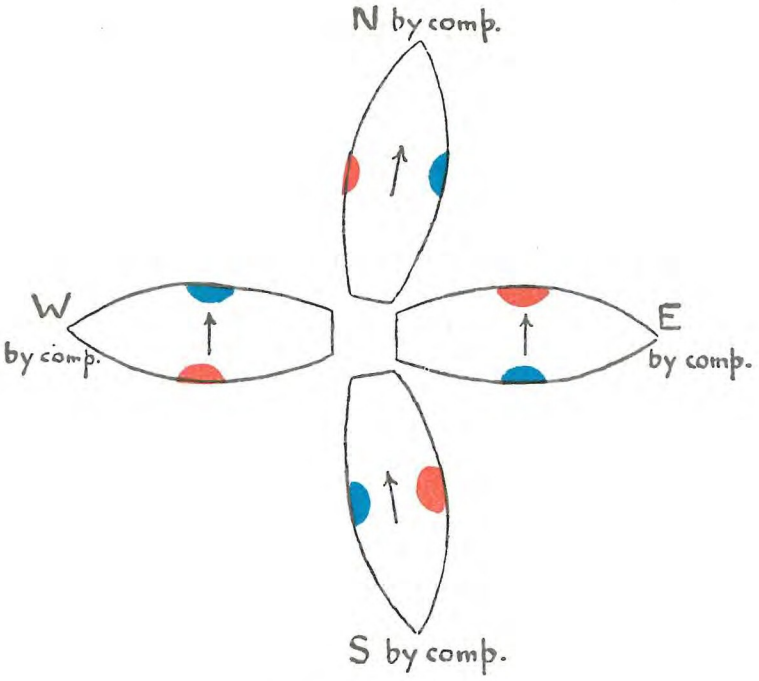
Deviation decreasing but is still W'ly.

HEAD W. BY COMPASS

No deviation, but increase in directive force.

HEAD N.W. BY COMPASS

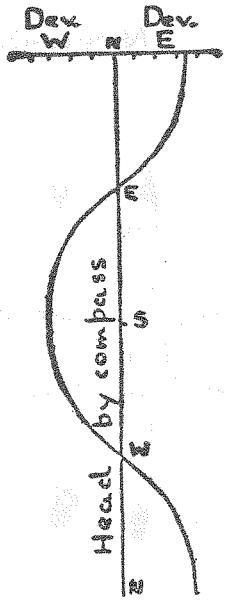
Deviation increasing, E'ly in name.



Effect of + Q force.

The curve of deviations is as shown, that is: 000°, maximum E'ly deviation decreasing to zero on 090°.

Maximum W'ly deviation on 180°, zero again on 270°, increasing again to maximum E'ly deviation on 360°.

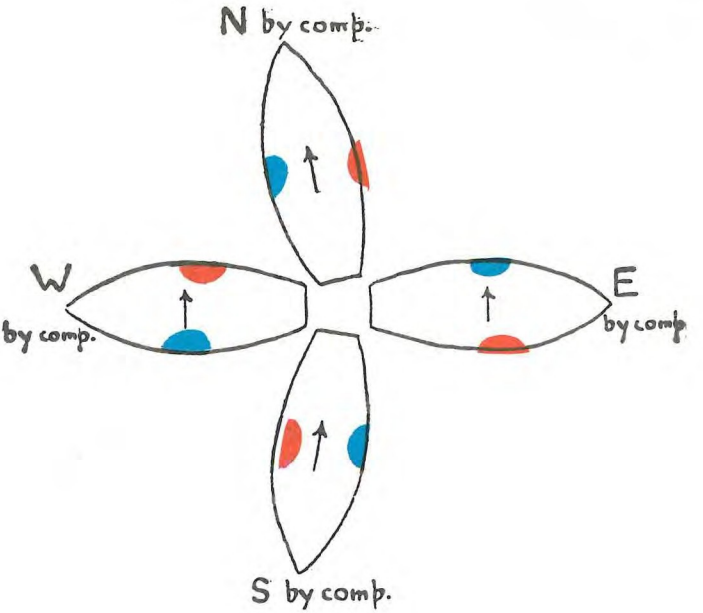


Curve of deviations due to $+Q$ force.

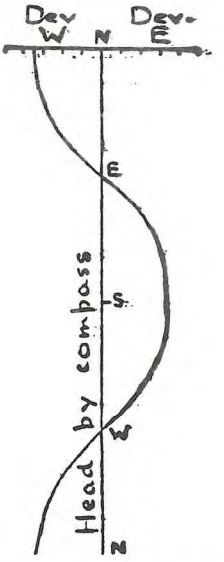
FORCE $-Q$

Similar arguments may be used in the case of a ship with a $-Q$ force.

The following two diagrams show what will happen, and in the light of what has been said previously, call for little explanation.



Effect of $-Q$ force.



Curve of deviations due to $-Q$ force.

20.10 COMBINED EFFECT OF P AND Q

Most ships will have both a P and a Q force acting on the compass needles, and the resultant deviation will be a combination of the deviations caused by these forces separately together with any deviation caused by other factors not considered here.

Enough has been said, however, to make it clear that as the direction of the ship's head changes, the magnetic poles in the hull change their position relative to the compass needles.

Thus remember that the deviation is peculiar to the particular heading. Each time we alter course, the deviation changes.

20.11 CHANGE OF DEVIATION WITH LATITUDE

We are speaking here of "magnetic latitude", i.e., our position on the Earth's surface relative to the magnetic equator.

Now it should be realized that **forces P and Q are fixed forces.** They are due to permanent magnetism in the ship, that is, to permanent poles of constant strength, always exerting the same deflecting force on the needles even though the ship may change her latitude considerably.

The Earth's horizontal component, H, on the other hand, is not fixed. It is maximum at the magnetic equator, zero at the magnetic poles.

Thus, for a given fixed deflecting force:

the deviation caused will be at least when the directive force is greatest, i.e., when H is greatest.

the deviation caused will be greatest when H is least.

In short, The Deviation due to forces P and Q varies inversely as H.

As a formula, this statement becomes,

$$\frac{\text{Dev}_1}{\text{Dev}_2} = \frac{H_2}{H_1}, \text{ or as it is sometimes preferred in words .. I}$$

$$\frac{\text{New Dev.}}{\text{Old Dev.}} = \frac{\text{Old } H}{\text{New } H} \text{ ..... I}$$

The unit of magnetic field strength is of course the amp/m, which has already been defined.

A few simple calculations will help to fix ideas.

WORKED EXAMPLE 1.

The deviation due to permanent magnetism is 15° W. at a place where $H = 18$ amp/m. What will it be on the same heading at a place where $H = 24$ amp/m.

$$\frac{\text{New dev.}}{15} = \frac{18}{24}$$

$$\text{New dev.} = \frac{18 \times 15}{24} = 11{\cdot}3°\ \text{W.}$$

(That is, H being greater at the new place, the deviation is less, as we would expect).

WORKED EXAMPLE 2

A ship has a $+ P$ which causes 14° E. deviation when heading E. at a place where $H = 16$. What will the deviation from the same cause be when heading W. at a place where H = 12? (Values of H in amp/m).

$$\frac{\text{New dev.}}{14} = \frac{16}{12}$$

$$\text{New dev.} = \frac{16 \times 14}{12} = 18{\cdot}7°\ \text{W.}$$

The deviation will be W'ly because the ship is now on the opposite heading.

It is increased in value. This is because H has diminished, that is, the directive force on the compass is less. The deflecting force on the other hand remains constant and so, with a weakened directive force it will cause greater deviation.

EXERCISE A (H is amp/m in each case)

1. A ship has a $- Q$ force which causes deviation of 8° E. at a place where $H = 14$. What deviation will it cause on the same heading at a place where $H = 21$?
2. If the deviation from hard iron on East is 10° W., when $H = 16$, find the deviation on the same heading when *(a)* $H = 20$, *(b)* $H = 8$.

3. A ship has 12° E. deviation from hard iron at a place where $H = 15$. Later on the same heading the deviation is observed to be 10° E. Find H at the second place.
4. A ship has a $-P$ which causes 6° W. deviation on East when $H = 8$. What deviation would it cause on West, where H = 12?

TEST PAPER 20
Magnetism

1. State the signs of the P and Q forces in the following cases.
   (a) Red pole port bow, blue pole starboard quarter.
   (b) Red pole starboard quarter, blue pole port bow.
   (c) Blue pole port quarter, red pole starboard bow.
   (d) Blue pole starboard bow, red pole port quarter.
2. A ship has a deviation of 6° E. due to permanent magnetism at A where HF = 20 amp/m. What would the deviation be on the same heading at B, where HF = 16 amp/m?
3. An electric lead carrying D.C. runs athwartships under the compass, current flowing port to starboard. What deviation will be caused on N., E., S. and W.?
4. The deviation due to permanent magnetism is 15° W at a place where HF = 22·6 amp/m. What will it be on the same heading at a place where HF = 18·0 amp/m?
5. If the deviation from hard iron on East is 10° W. when HF = 20·4 amp/m, find the deviation on the same heading when (a) HF = 23·2 amp/m (b) HF = 16·8 amp/m.
6. A ship has 12° E, deviation from hard iron at a place where HF = 24·6 amp/m. Later on the same heading the deviation is found to be 10° E. Find HF at the second place.
7. A ship has a $-P$ force which causes 6° deviation on East when HF = 19·2 amp/m. What deviation would it cause on West at a place where HF = 28·6 amp/m?
8. A ship has a $-Q$ force which causes a deviation of 8° when heading North at a place where HF = 27·5 amp/m. What amount and name of deviation will it cause when heading South where HF = 22·2 amp/m?

CHAPTER 21

ELECTRICITY

General Introduction — Current and Static Electricity

21.1 NATURE OF ELECTRICITY

There is one thing common to all electrical appliances. Whether we are thinking of a heater, lamp, motor, dynamo or what you will, there is one phenomenon common to all of them. That one thing is the **"electric current"**.

What is an electric current? There is something intangible about it. It cannot be seen — it can only be recognised by its results. What is it?

The smallest particle of matter is the **atom**. It was discovered by Sir J. J. Thomson, however, that every atom itself consists of **Protons**, which are relatively heavy, and **Electrons**, which are light. The electrons are spinning round the protons rather like planets in their orbits round the sun. This is because the electrons are considered to have a negative electric charge, the protons a positive electric charge and a force of attraction exists between the two types of charge. An atom is nearly all empty space, and there is plenty of room between the atoms for swarms of electrons to be flying about. Their motion is quite haphazard and on the whole they do not progress far in any given direction. Before they can do so we must apply a particular kind of force called an electro-motive-force (e.m.f.) to the unattached electrons.

To sum up: A piece of copper wire can be regarded as a vast number of heavy copper atoms. In the inter-atomic spaces are swarms of free electrons. If a cell is connected to the ends of the wire, the free electrons will be forced to drift steadily along the wire.

This stream of electrons is called an electric current I

21.2 POTENTIAL

A cell thus acts rather like a water pump forcing water along a pipe. Note that the cell does not make the electricity any more than the pump makes the water.

The cell provides the force (the e.m.f.) to drive the electrons along the conductor. This force that is required before any electric current can flow is variously called **Potential**, or **Potential Difference** (p.d.) or e.m.f. These terms are used in slightly different ways, as we shall see later. But in every case they mean that there is a force (an "electrical pressure") present which will cause the electron stream to flow.

N.B. For an electric current, we must have an e.m.f. II

We must also have a continuous path for the electron stream to flow in III

21.3 CONDUCTORS AND INSULATORS

It goes without saying that the atom is a very complicated structure.

(*a*) Some atoms are very compact. Their electrons are well behaved and move in well defined orbits, keeping close to the protons at the centre.

(*b*) Other substances have rather unstable atoms, with large numbers of unattached electrons wandering aimlessly about in the inter-atomic spaces, like the ones just referred to in the copper wire.

Substances such as (*a*) with no free electrons are called **Insulators** or non-conductors. If there are no free electrons, our e.m.f. cannot cause an electron stream to flow. Such things as ebonite, glass, mica, wax, rubber are insulators.

Substances such as (*b*) are called **Conductors**. They have plenty of free electrons, and thus a current is easily set up when an e.m.f. is applied. Metals are good conductors.

21.4 THE ELECTRIC CELL

The simplest source of an e.m.f. is the electric cell. A number of cells grouped together is called a **Battery**.

There are various types of cells, but the principle is similar in each case, viz., two poles (rods) of different materials are immersed in a chemical solution called the **Electrolyte**. Then owing to the chemical reactions which take place, one pole becomes positive

(electrically), the other negative. When connected externally by a conductor, the e.m.f. of the cell will cause a current to flow in the conductor.

The pole of the cell by which the current leaves is called the positive pole or terminal IV

The negative pole or terminal is that by which the current re-enters the cell V

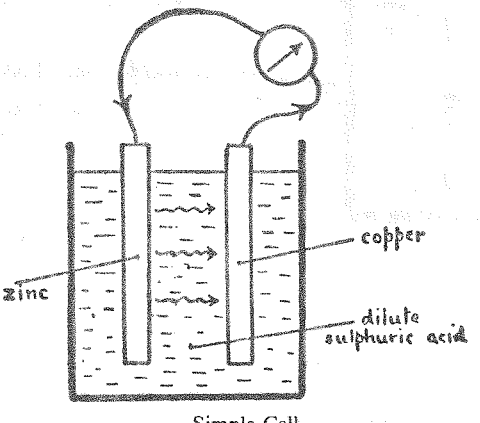
When considering the flow of current through an electrolyte, the pole by which current enters the electrolyte is called the **Anode** and that by which it leaves the electrolyte is called the **Cathode**.

Important. *It should be noted that the movement of the electrons is actually in the opposite direction to the above. This arises from the fact that many rules were established before the true nature of electricity was understood.*

By convention, current flow is from + ve to − ve. The electron flow is in the opposite direction. This latter point is important in some branches of electronics.

21.5 SIMPLE VOLTIC CELL

So called after its inventor, Volta. This consists of a zinc rod and a copper rod immersed in dilute sulphuric acid. The copper is the positive, the zinc the negative. Carbon might be (and often is) used instead of copper.



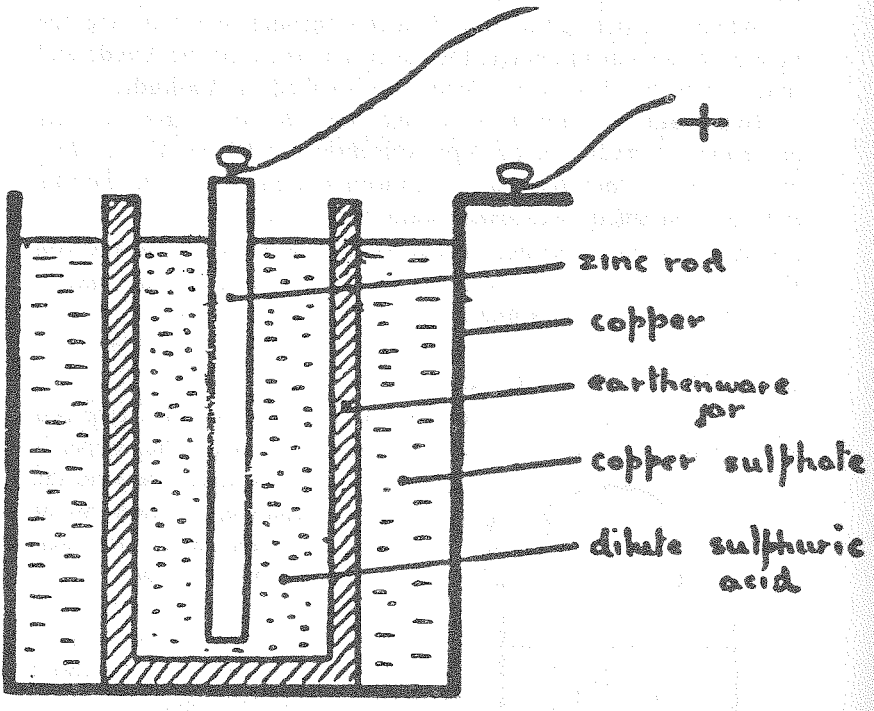
Simple Cell.

This type of cell is not very efficient. Initially it gives an e.m.f. of about 1·0-1·5 volts (we shall define the volt later).

But the e.m.f. quickly falls off, due to **Polarisation**. This is a result of the chemical reactions in the electrolyte, which result in particles of hydrogen being deposited on the copper. This deposit, in effect, "chokes" the cell, which is then said to be polarised.

21.6 DANIELL CELL

The general arrangement is as shown. The porous earthenware jar allows the two liquids to meet, but not to mix freely. The outside case itself is of copper and forms the positive.



Daniell Cell.

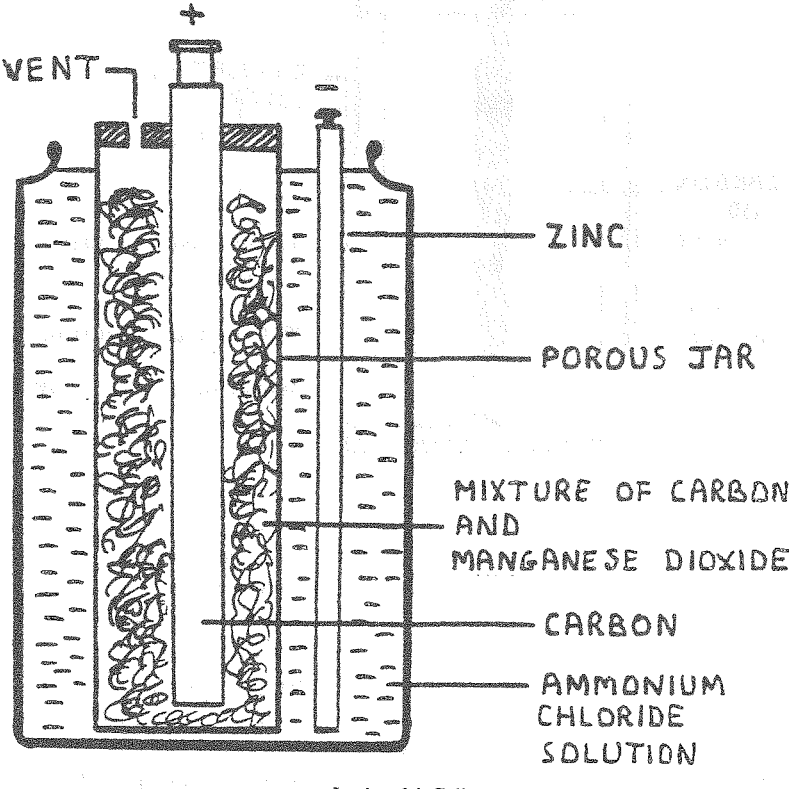
When hydrogen particles are liberated at the zinc and move towards the copper, they are absorbed by the copper sulphate solution. Thus the cell does not become polarised.

The e.m.f. of this cell is very constant, being about 1·1 volts.

However this type of cell is of limited practical use since it has to be assembled anew each time it is to be used. This is because if it is allowed to stand when not in use the copper sulphate solution will diffuse through the porous jar and deposit copper on the zinc rod.

21.7 LECLANCHÉ CELL

This is used occasionally, e.g. for household bell systems, where short intermittent periods of working are required.



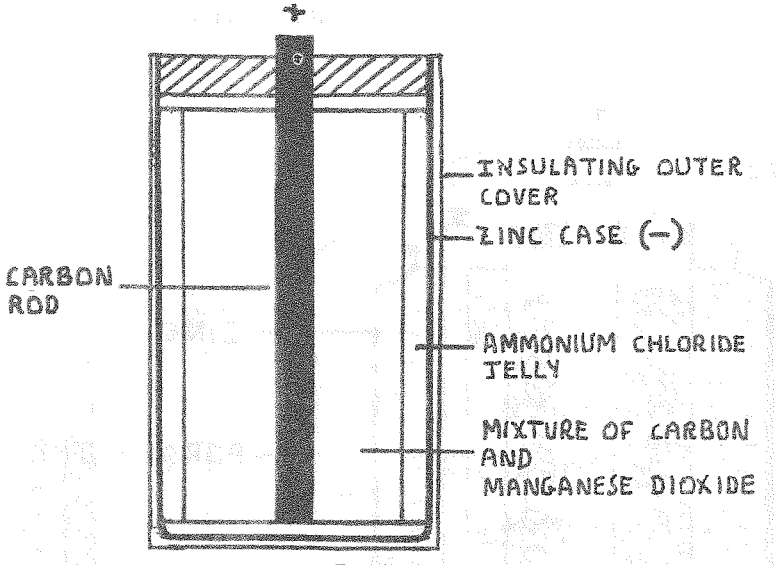
Leclanché Cell.

The arrangement is as shown. On making the circuit, hydrogen particles rapidly form on the carbon rod which becomes polarised. On standing idle, however, the manganese dioxide removes the hydrogen and the cell recovers its original voltage. This is about 1·5 volts.

Hence this type of cell is suitable for electric bells, telephones, etc., where continuous working is not required.

21.8 DRY CELL

This is just an adaptation of the Leclanché Cell, in a convenient form for carrying about in an electric torch.



Dry Cell.

It is not, strictly speaking, "dry". If it were it would not work.

The case itself is of zinc. The jelly is prevented from drying out by sealing the top of the cell with pitch or marine glue.

A 9 V battery, such as is found in many transistor radio receivers, is just a number of such dry cells joined together.

All the above cells are examples of **Primary Cells**. The chemical reactions taking place in such a cell result in the original chemicals being transformed into others. When this has occurred the cell is **Discharged** and an e.m.f. is no longer produced. It is not possible to reverse this process i.e. the cell cannot be **Recharged**.

21.9 SECONDARY CELL

It is possible to recharge a secondary cell. These are more commonly known as **Accumulators**, because they do not provide an e.m.f. themselves until they have been charged. They merely store "electricity", as it is sometimes expressed (though not quite correctly. What they really store is chemical energy). It is not

proposed here to go into the chemical explanation of this. But when a current is passed through an accumulator for a time, it is said to become **Charged**, i.e., it acquires chemical energy which can be released as electrical energy. If it is now connected through an external circuit current will flow in the circuit until such time as the accumulator is **Discharged**. It is then possible to recharge the cell again by passing a current through it.

One such accumulator gives an e.m.f. of about 2 volts. A large number joined together will give quite a large voltage — for example, enough to work the ship's W/T in an emergency.

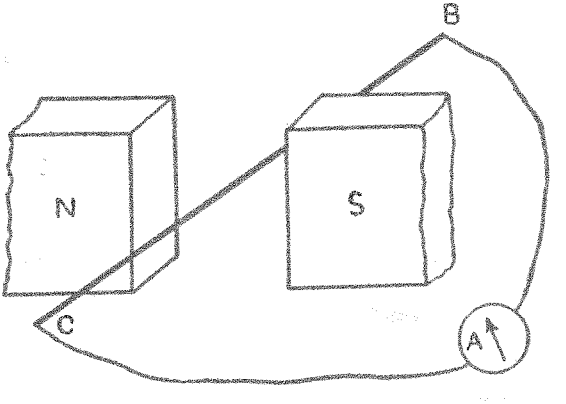
Examples of secondary cells are the lead-acid cell and the Nife cell.

21.10 DYNAMO, OR GENERATOR, AS A SOURCE OF CURRENT

Cells and accumulators produce their e.m.f.'s as a result of chemical reactions. The generator utilises an electro-magnetic principle as follows.

CURRENT IN A MAGNETIC FIELD

One of the most important discoveries made in the development of electricity was that whenever a conductor cuts across magnetic lines of force, an e.m.f. is produced in the conductor.



E.M.F. produced by cutting across magnetic field.

For example, in the big horseshoe magnet shown, the space between the N. and S. poles is full of magnetic lines of force. If the copper wire *BC* is now moved through the space, thus cutting the magnetic lines of force then a current will flow in *BC*, and could be made to show on a very sensitive ammeter at *A*.

Admittedly, such a current would be very minute. But if instead of one wire we had many hundreds (this would then be called an **Armature**) and it was made to rotate very rapidly cutting the magnetic lines all the time then a very large current would result.

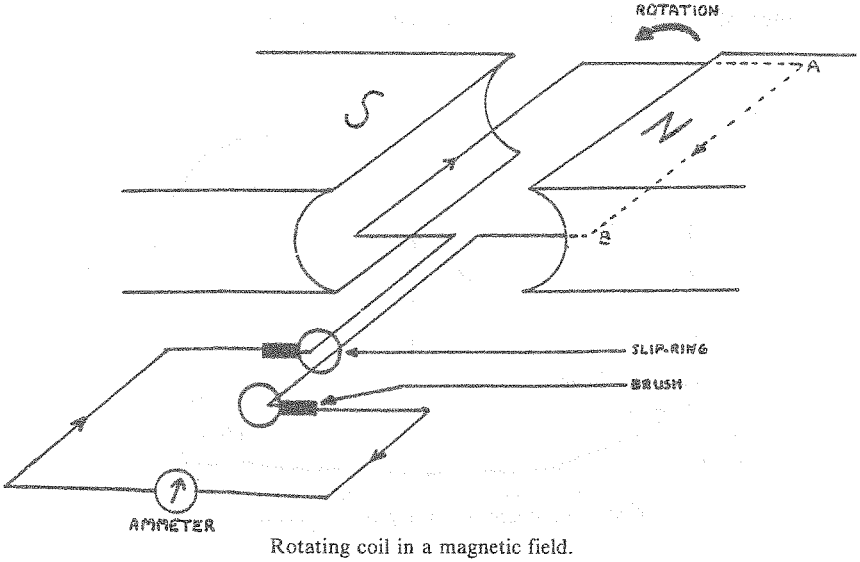
This is the principle of the generator which is explained in more detail below.

THE ROTATING COIL—PRINCIPLE OF THE GENERATOR

Consider the arrangement shown in the diagram below where the straight conductor of the previous paragraph has been bent into the form of a coil so that it can be rotated rapidly. The two ends of the coil as they rotate are in continuous contact with two **Slip-Rings**.

The coil rotates in the magnetic field between N. and S. which are the poles of what is called the **Field Magnet**.

During one half of a revolution the currents in the two portions of the coil would be as shown.



Rotating coil in a magnetic field.

During the second half revolution the currents are still in the same directions, as shown by the arrows, but the two sides of the coil have changed places.

Thus, in the arm *AB*, the current for one half revolution flows in the direction *A* to *B*; for the other half revolution, from *B* to *A*.

From the slip-rings the current is carried into the external circuit through two **Brushes**, as they are called. They are not really brushes but simply pieces of metal which bear on the slip-rings.

This little machine is a simple form of **Alternating-Current Generator**.

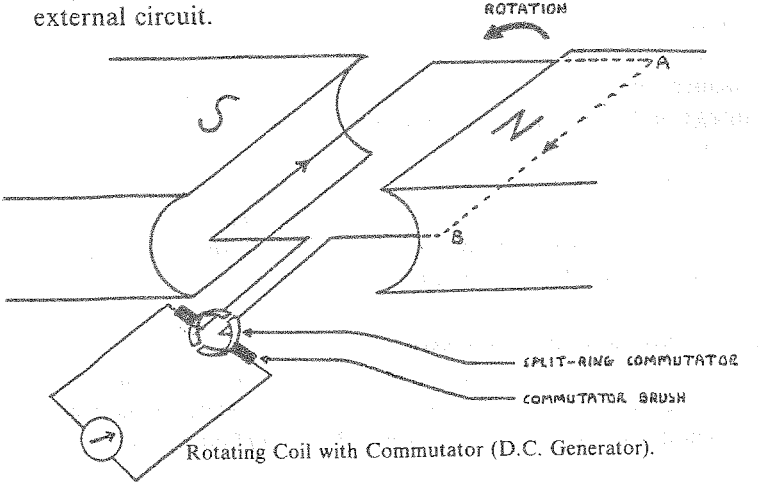
The current in the circuit is called alternating current (A.C.) because it flows first one way, then the other. In the ordinary A.C. domestic supply, the current may reverse its direction as much as 50 times in a second (50 cycles as it is termed).

Alternating current is suitable for heating, lighting and driving many kinds of machinery. It is not suitable however, for charging accumulators. For this purpose, **Direct Current** (D.C.) has to be used. In D.C. the current flows the same way all the time (see following paragraph).

THE COMMUTATOR

By a very simple device called a **Commutator**, the two halves of the current can be made to flow in the same direction. The slip rings of the previous diagram are replaced by **Split-ring** or **Commutator**.

The sketch illustrates the principle employed. The two halves of the split-ring are attached to the ends of the coil and rotate with it but the **Commutator Brushes** are fixed and connected to the external circuit.



Rotating Coil with Commutator (D.C. Generator).

A little consideration will show that half of the coil which is ascending is always connected to the brush *A*, and the descending half is always connected to the brush *B*. Hence the current will always flow the same way in the external circuit as shown.

The pole by which current leaves the generator is regarded as **Positive**, the other pole is **Negative**.

Current like this, always flowing the same way, is called **Direct Current** (D.C.).

D.C. can also be used for most purposes, and further, it is essential to use D.C. for charging accumulators.

21.11 USE OF A.C. AND D.C.

The main advantage of A.C. is in the transmission of electrical power through cables over considerable distances. For economy of working, long distance transmission should be at the highest practicable voltage (maximum is about 132,000 volts), because it can be shown (not here) that the higher the voltage, the less the loss of power in pushing the current through the cables ("line losses").

On the other hand most domestic supplies are around 200-250 volts.

Also, the economical generation voltage of electricity from a dynamo does not usually exceed 5000 volts.

Thus we have the process of producing it (say) at 5000 volts, stepping it up to 100,000 volts or more for transmission, then reducing it to 240 volts for consumption, and this is happening moreover, in many hundreds of localities, all the time.

This could not be accomplished with D.C. but by means of a **transformer** (see section 24.9) it is quite a simple matter to change the voltage of A.C. Hence A.C. is used almost everywhere on shore and on board ship.

A.C. is also used a great deal in W/T transmission.

D.C. on the other hand, is required to charge accumulators. However, D.C. is easily obtained from A.C. by means of a **rectifier**. One cannot convert D.C. into A.C. so readily.

21.12 DANGEROUS VOLTAGES

It is not easy to say just what voltage is safe and what dangerous. Of course, the 2 volts from an accumulator would not hurt anybody in any circumstances. The 100,000 volts of an

overhead transmission cable (**High Tension Cables**, as they are called) would prove fatal.

But what about the normal voltages of domestic or ship consumption say 100-250 volts?

Well, the first thing to realise is that for a current to flow there must be a continuous path.

Secondly, that the human body is a conductor of electricity — not a very good conductor perhaps, but still, if we place our body so as to close the gap in an electric circuit then current will flow through the body.

Thirdly, that water is also a conductor. Thus, if our skin is wet, the body is improved as a conductor and will pass more electricity.

All voltages are more dangerous with a wet skin ..... **VI**

The main points, therefore, are:

(*a*) Treat all voltages (except the very smallest) with respect. Try to ensure that the current is switched off first.

(*b*) Merely to touch two live wires with the fingers of one hand would hardly be fatal (we are speaking of medium voltages) just unpleasant perhaps, because the shock would be confined to one hand.

(*c*) The same two wires, however, if touched with two hands might easily prove fatal, because there is then a path for the current up one arm through the chest and down the other arm.

(*d*) Or again, standing in bare feet on the deck or ground would provide a path for the current right through the body. From time to time we read of fatal shocks to people in the bathroom, where a hand or foot is in contact with a metal water pipe to earth, whilst the other hand accidentally touches a live circuit.

Precautions—Switch off.
Dry hands.
Rubber soles are best, but even leather helps, if dry.
Never work in bare feet.
Always make sure that no part of the body is connected to earth.
Stand on a wooden box. Avoid touching anything with the free hand.

21.13 DEFINITION OF TERMS

(*a*) **Coulomb.** The unit of quantity of electricity is the **coulomb** (*C*).



Think for a moment of water flowing along a pipe. We are quite accustomed to speaking of a quantity of water in gallons, so that, for example, we might say there was 1 gallon of water between *A* and *B*.

Now a "current of electricity" is composed of electrons flowing along a conductor. (It is a "pipe" in a way, because remember there is plenty of space between the atoms for the electrons to stream along). It would be a difficult matter to count the electrons at any instant on the section *AB*; but it has been agreed that

$6{\cdot}24 \times 10^{18}$ electrons = 1 coulomb (very nearly).

$$\textbf{1 coulomb} = \mathbf{6{\cdot}24 \times 10^{18}\ electrons} \quad \ldots\ldots\ldots \quad \textbf{VII}$$

This is a *quantity of electricity*, and we might speak of a current of electricity as so many coulombs per second, and in fact we do, in some cases.

(*b*) **Ampere.** In general, however, it is easier to use another unit, the **Unit of Current**, called the **ampere** (*A*).

$$\textbf{1 amp} = \textbf{1 coulomb per second} \quad \ldots\ldots\ldots \quad \textbf{VIII}$$

To repeat: a current of 1 amp means that 1 coulomb per second is passing through. 10 amps would mean 10 coulombs per second, and so on.

10 amps is quite a big current — even 1 amp is a fairly strong current — a 100 watt bulb in a domestic circuit takes about ½ amp.

It also follows from this fundamental definition of current that the current must be the same all through the circuit (assuming this to be a single conductor). That is, notwithstanding resistances, lamps or whatever there may be in the circuit, if 1 amp is the current at any point, then 1 amp is the current *throughout the circuit.*

Further, if coulombs measure a quantity of electricity, and amps the quantity of passing per second, that is, the current (*I*), then clearly

Total quantity passed = amps × time in seconds for which it has been flowing.

$$\text{i.e., } \mathbf{Q = It} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad \textbf{IX}$$

(*c*) **Volt.** We have already referred to the volt as the unit of p.d. or e.m.f. — in fact, as an electrical force — the force that causes the electrons to start flowing.

In mechanics, we speak of a pressure as "so-many newtons per square metre".

In electricity, we have to be rather more careful, because the conductor offers a resistance to the flow of electrons. The unit of resistance is called the **ohm** (Ω) and we now arrive at the following definitions, which link up all three units.*

Definition:

A conductor has a resistance of 1 ohm if a p.d. of 1 volt causes a current of 1 amp to flow in it X

This is the international definition of the ohm. It should be studied carefully.

The symbol for e.m.f. is E and for p.d. is V

The symbol for current I

The symbol for resistance, R

and the above results can be summarized conveniently in a rule known as **OHM's Law.**†

21.14 OHM'S LAW

This states that in a single circuit, the current is directly proportional to the p.d., and inversely proportional to the resistance, i.e.,

$$I = \frac{V}{R}, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad \text{XI}$$

where I is in amps, V in volts and R in ohms.

Re-arranging equation XI we have

$$V = IR \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad \text{XIa}$$

This formula applies to each and every resistor in an electrical circuit.

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* *The fundamental definitions are given later.*

† *When Ohm, after many years of experiment, first put forward his theory in 1827, it was received with open incredulity and derision. For his "incompetence" he was removed from his teaching post, only to be reinstated some six years later.*

Note: An appliance deliberately inserted into a circuit in order to provide resistance, is called a *resistor*. Otherwise, a circuit is spoken of in a general sense as having *resistance*.

21.15 WORK, ENERGY AND POWER IN A CIRCUIT

We are already familiar in mechanics with the concept of work such as for example, when a quantity of material is moved against (or by) a given force. An example is that of a brick being moved to the top of a wall; work is done.

There is a close analogy in electricity, and by definition:

If a p.d. of 1 volt causes 1 coulomb of electricity to be moved then we say 1 joule of work has been done.

Clearly then, if the p.d. is V volts and Q coulombs are moved, the work done is QV joules. But $Q = It$ from IX, and so

$$\text{Work done in t seconds} = VIt \quad \ldots\ldots\ldots\ldots \quad \text{XII}$$

By definition, power is the work done in 1 second. The unit of power is the watt, as always, and so, dividing by t, we have

Work done in 1 second $= V \times I$, that is,

$$\text{watts} = \text{volts} \times \text{amps} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad \text{XIII}$$

This is a particularly important formula. It gives us the work done by a circuit, or alternatively, the energy taken out of a circuit by some appliance such as a heater, lamp or motor.

Again, since from XII, Work done $= VIt$, and by Ohm's Law $I = V/R$ and also $V = IR$, substituting, we have,

$$\text{Work done} = \frac{V^2t}{R} \text{ joules or } I^2Rt \text{ joules} \ldots\ldots \quad \text{XIV}$$

alternative forms which are sometimes useful. The latter shows that the work done is proportional to the *square* of the current.

In the type of circuit we are considering, the work done is transformed into thermal energy in the conductor. The whole conducting path of the current becomes heated as a result. When the portion of the conducting path is designed for the purpose, it can become very hot (a heater) or even white-hot (an electric lamp).

Reference has already been made to the advantage of high tension A.C. for transmission of electric energy over long distances

and we can now see this more clearly. Since watts = volts × amps, in order to transfer a *given quantity* of energy through a conductor, that is in order to have the same product on the L.H.S. of the above statement, we can either use high voltage and low amps, or low voltage and high amps.

Since line losses from heating are proportional to the square of the current, there is an obvious need to keep the current as low as possible — and hence it follows, the voltage as high as possible.

WORKED EXAMPLE 1

An electric kettle has a resistance of 40 ohm and it operates at 240 V. The kettle has a mass of 1 kg and a specific heat capacity of 420 J/kg°C. Calculate how long it would take to boil 2 kg of water in the kettle if the water's initial temperature is 20°C and we ignore all heat losses to the surroundings. Assume the specific heat capacity of water is 4200 J/kg°C.

Heat provided by electric supply
= heat gained by water and kettle

$$\text{Heat provided by electric supply} = \frac{V^2t}{R}$$

$$= \frac{240^2t}{40} \text{ J}$$

$$= 1440t \text{ J}$$

$$\text{Heat gained by water} = mcT$$

$$= 2 \times 4200 \times 80 \text{ J}$$

$$= 672000 \text{ J}$$

$$\text{Heat gained by kettle} = mcT$$

$$= 1 \times 420 \times 80 \text{ J}$$

$$= 33600 \text{ J}$$

$$\text{Therefore } 1440t = 672000 + 33600$$

$$1440t = 705600$$

$$t = 490\text{s}$$

21.16 THE SIMPLE CIRCUIT

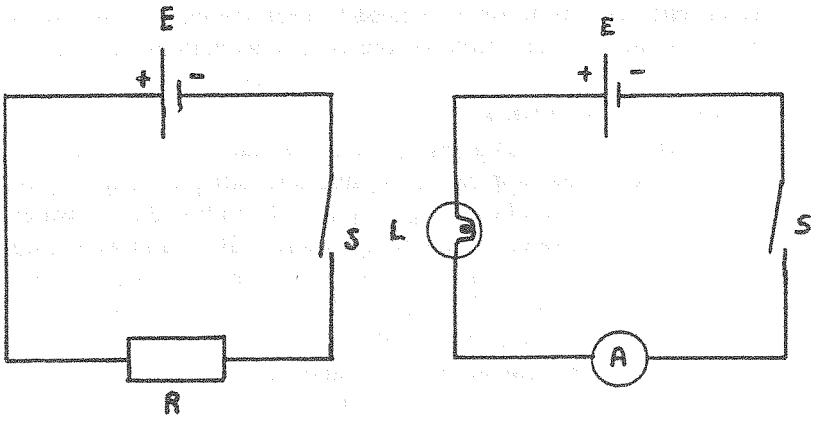
A "simple" circuit is one which does not contain such things as capacitors, induction coils, transistors, etc.

All simple circuits obey Ohm's law.

The correct symbols are as shown in the following diagram;

these should always be adhered to. Note particularly that the positive pole of the cell is always shown by the longer stroke.

Essentially, a simple circuit consists of a cell E, a resistor R, a switch S, and of course a continuous conducting path for the current to flow in.



Simple circuits.

It might also contain a lamp L, which of course is just a resistor, and an ammeter A, to register the amount of current (amps) passing.

21.17 ELECTRICAL APPLIANCES — VOLTAGE

Some knowledge of the working voltages, current and power consumption of various appliances is essential.

Firstly, remember that very many appliances have their working conditions stamped on them. A cabin lamp might carry the words "40 W 100 V". If not actually attached to the machine, then the working voltages, amps, watts, etc., will certainly be found in the operating manual of the particular machine.

The student should make a point of trying to find out some of the working conditions of the electrical gear on board his own ship.

Secondly, the significance of the rated voltage should be understood. This is the voltage for which the machine was constructed and in which it will work most efficiently. Domestic voltages range from 200 V, to 250 V.

On board ship with D.C., 100 V to 150 V is the usual for cabin lighting and heating, and 500 V for power.

The advantage of the lower voltage lies in there being less danger from shock. On the other hand more current would be needed to give the same energy, since watts = volts × amps.

E.g., 900 W = 150 V × 6 A, or alternatively 900 W = 100 V × 9 A, that is, 6 A at the higher voltage and 9 A at the lower voltage to give the same power.

In ships using A.C., maximum permitted voltages are 150 V to 250 V for lighting and heating respectively, and up to about 450 V for power.

Any appliance should be used with its correct voltage.

In general, if used with a *lower* voltage no harm will be done, but it will not develop full power. A lamp will not burn so brightly, an iron, kettle or fire not get so hot, a winch will have less power. However, there are exceptions, e.g. fluorescent tubular lighting will go out if the voltage falls below the rated value. Broadly speaking, however, no harm is done.

If used with too *high* a voltage, however, the lamp, heater or winch will take too much current, and get too hot and may burn out.

21.18 WATTAGE

Assuming that everything is being used with correct voltage the important piece of information we require is the "wattage".

Everyone is familiar with a 40 W lamp, a 100 W lamp, and so on.

A kettle or electric iron might be 750 W. A one-bar electric fire 1000 W.

A radio receiver might be rated as low as from 5–10 W. The average radio receiver uses very little current. A transmitter, however, uses more, e.g., a 1½ kW transmitter.

Motors may require anything from quite small to very large power.

Quite a small motor such as in a vacuum cleaner, or electric fan might take as little as 100 W to 500 W, hardly any more than a lamp.

An electric winch or windlass might require from 25 kW to 35 kW or even more.

To sum up a few comparative figures.

Lamp (say)	100 W
Small motor	100–150 W
Iron, kettle	750 W
One-bar fire	1000 W
Large motor (winch)	20, 25, 35 kW.

Try to add to a list of this type for the appliances on board ship.

21.19 AMPERAGE

The amps (current) flowing can be found at once from the rated watts and volts, since watts = volts × amps.

Hence, for a 100 W, 200 V lamp current is given by $\frac{100}{200} = 0{\cdot}5$ A

For a 20 kW 500 V motor, current is $\frac{20{,}000}{500} = 40$ A and so on.

21.20 GENERATORS

Where does all this power come from? Obviously, in a modern ship there is a lot of it, and ships' generators are constructed to give a considerable output of electrical energy. A quite normal output from one generator would be 60 kW. A more powerful one might generate 300 kW and a ship might have several generators.

21.21 STATIC ELECTRICITY

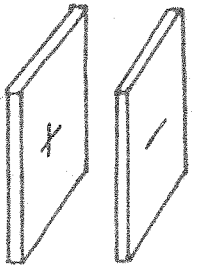
In the earlier part of this chapter we have been considering a type of electricity which manifested itself by a "flow" along a conductor — what we have called a current of electricity — and the means by which it could be produced.

It is now necessary to recognise a different type of electricity known as **static electricity**. We say a different type only in as much as the electricity is moving in the one case already discussed, whilst in this case, we are about to consider a stationary form, or static form of electricity.

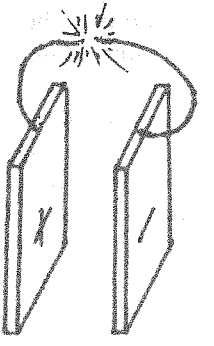
All materials normally contain equal numbers of + *ve* and − *ve*'ly charged particles. These are evenly distributed throughout the material and the material as a whole possesses no charge.

Electrostatic charging occurs whenever this equality of distribution is disturbed and charges of one sign are separated from those

of the opposite sign. This may happen for example, when there is a drift of electrons through an electrolyte, caused by chemical action in the plates.



(*a*) Two plates in an electrolyte.



(*b*) Release of energy.

Consider a metal plate which by some means or another has been robbed of its free electrons (for example, by chemical reaction). Such a plate is left predominantly positive since the atoms contained in it are out of balance due to the loss of the electrons, and we can say the plate has a positive charge on it.

If a second metal plate has the free electrons taken from the first plate inserted into it then the atoms contained in this plate are again out of balance, but now we say the surplus of electrons has created a negative charge.

Now let the two plates be brought adjacent to one another. In this state we can say there is static electricity between the two. The energy is not moving — it is potential energy, and is waiting to be released. Once it is released by connecting a wire between the two, the free electrons in the negative plate are attracted towards the positive plate and a steady flow will continue for as long as the supply of electrons is available. Current flow has taken place.

The current will often be able to flow through the external conductor as in a simple circuit connected to the terminals of a cell or charged accumulator. But where the voltage between the separate charges is substantial, it may as the conducting path is about to be joined, be sufficient to break down the resistance of the atmosphere in the vicinity and discharge across the gap in the form of a spark.

21.22 ELECTRICITY BY FRICTION

The method by which we produced a positive charge on one plate and a negative charge on the other was by chemical reaction. This of course is the principle underlying a simple cell. There is however another and perhaps simpler fashion of bringing about this displacement. It is a method which may be tried by the reader with very little inconvenience — **friction.**

If we rub an ebonite rod with fur, then electrons in the fur are transferred on to the ebonite rod. The ebonite has then assumed a negative charge since it has a surplus of electrons.

Also, if we rub a glass rod with silk we find that some of the free electrons are transferred from the glass on to the silk. The rod in this case has assumed a positive charge since it is deficient in electrons.

The ebonite and the glass are now at different potentials and if brought together will spark across until a balanced state is reached. This will occur very quickly and the spark can be seen and heard.

This occurs not only in these two materials but is merely an example of how static electricity can be brought about. In modern man-made fibres such as terylene, static electricity is sometimes produced simply by the action of the wearer rubbing himself, say, on a metal framed chair. A slight shock may be experienced on rising and then touching the metal handle of a door or chair back.

21.23 STATIC ELECTRICITY HAZARDS

Apart from the aspect of feeling a shock created by static electricity, there is a more serious side to this. This of course is the danger of fire, which may be created by the action of discharging static in the vicinity of inflammable gas. This could occur particularly where oil or petrol is concerned, and in oil tankers, forms a particular hazard. The precautions taken when loading and unloading oil are extensive and a summary is given below.

It is absolutely essential that these precautions should be strictly adhered to in the interest of safety and the prevention of fire. But first let us consider how petroleum may become charged.

21.24 ELECTROSTATIC CHARGING OF PETROLEUM

The prime cause of sparking is the fact that static charges are built up wherever there is friction between two materials, whether

the materials are oil and water, oil and metal, or oil and the impurities contained in oil.

All commercial petroleum products contain at least some trace of impurities and it is these that tend to split up, under friction, into positively and negatively charged molecular components and so give rise to the electrostatic charging of petroleum. This can easily occur in the normal process of cargo handling, as follows:

(*a*) **Water.** Charging will occur whenever there is relative motion of water and petroleum in turbulent flow in pipelines, and to a greater degree, when water droplets settle through a depth of petroleum in a tank. Such charging due to settlement can continue long after pumping has ceased.

(*b*) **Splashing and spraying.** When petroleum is splashed off other material and so creates a spray or mist, these droplets may carry a charge. When such spray settles on an unbonded conductor a concentrated charge can accumulate and result in a spark discharge to a bonded structure. This type of charge can occur even in "black" oils.

When liquid particles are sprayed at high velocity through a nozzle the particles may be charged as in splashing. Thus, spraying involves the same static producing mechanism as splashing.

(*c*) **Steam.** A jet of steam is a potential generator of statically charged mist. Where steaming out is found necessary, as when tank cleaning for special grades, the steam should be introduced at very low velocity to lessen the charging effect.

Broadly speaking, all petroleum distillates should be regarded as prone to electrostatic generation and should be handled accordingly.

21.25 STATIC CHARGE CONDITIONS

The existence of a static charge may be suspected in any of the situations that follow:

1. On the vessel herself.

   When she arrives alongside after a voyage. She may possess a different potential from that of the breastworks or jetty.
2. On the flanges or other metal parts of pipelines when clean petroleum is being pumped through and hoses are not properly bonded.

3. Within the body or on the surface of any clean petroleum.
   (*a*) during or for some time after loading if a static producing mechanism is in operation during loading (such as in (*a*) above).
   (*b*) whenever any liquid is loaded "overall" (i.e., through the tank lids), thus giving rise to splashing.
4. Within an empty but steam filled tank, a suspended metal object is introduced.

21.26 PRECAUTIONS

(*a*) On every jetty used for discharging or loading petroleum **there should be an earth lead**. This should be connected to the vessel upon berthing so that the ship is thoroughly bonded to the jetty. The jetty connection should be made last, so that if any arcing takes place it will do so in relative safety.

(*b*) **Cargo hoses must be bonded** so that there is adequate earthing. As an additional precaution, a lead similar to that in (*a*) should be used connecting the ship's manifold to the jetty.

(*c*) **Low initial loading or discharging rates** must be observed. Water in suspension in petroleum is a potent static producing agent and it is most likely to be present at the commencement of cargo operations.

(*d*) During loading, **no earthed conducting probes** capable of conducting electrical stress, and no unearthed conductor capable of gathering charge from the liquid must be allowed inside the tank. An example of the first is a metal weight on the end of a metal tape — of the second, a sampling can floating on the surface of the incoming oil.

(*e*) **When loading overall**, if petroleum has been charged in the pipeline, the charge will be delivered to the bulk of the liquid and create a hazard in the gaseous atmosphere.

21.27 CONDUCTOR INTRODUCED INTO STEAM FILLED TANKS

The hazard of ignition when tanks have been discharged but are not gas free arises when a conductor is suspended in a tank and steam is also present. Earthing, though desirable, does not remove this hazard.

No tank washing machine, whether earthed or unearthed,

should be permitted in a tank if a steam cloud is present. Full ventilation must be carried out before washing commences and continued during the washing operation.

21.28 DISCHARGING CARGO

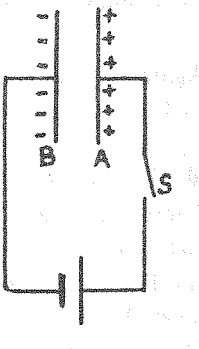
The same precautions should be observed during discharge, with the effect upon the shore tanks in mind. Pumping speed should be kept low at first until the shore lines are full of the product and the cargo has actually entered the shore tanks.

21.29 CAPACITANCE AND CAPACITORS

We have spoken earlier about the manner in which electricity may become stored — may accumulate — in unwanted places. This storage of static electricity occurred by some means or other, generally speaking beyond our control. Nor were we always able to control its subsequent action which might, in certain circumstances, constitute a hazard, for example, if a discharge took place in proximity to petroleum.

However, there is often a need to store electricity, even for a few microseconds, in some part of a circuit, and moreover, to be able to control the storage and the release of such electrical energy as we wish.

A **capacitor** (formerly known as a condenser) is a device for storing electricity. In its simplest form, it consists of two parallel plates as illustrated, completely insulated from their surroundings. If the circuit is made by closing the switch *S*, current will flow for a very short time, plate *A* acquiring a positive charge, *B* a negative charge. As things are, this is now static, and if the plates are disconnected from the battery, the positive and negative charges will remain stored in the capacitor until released. This might be done, for example, by touching them with a wire.



Charge on a capacitor.

In the position illustrated above, however, two things are clear:

(*a*) **there is a difference of potential between the plates.**

(*b*) **there is a charge on each plate. (Equal in amount, but opposite in sign).**

The **capacitance** (of a capacitor) is a term that is almost self-explanatory. It means the ability of a capacitor to store a charge. By their capacitances we can differentiate between one capacitor and another.

Hence we need a scientific definition of capacitance (C) and this is afforded us by means of (*a*) and (*b*) above. If Q is the quantity of electricity on either plate, and V the p.d. between the plates, then the capacitance C is given by

$$\mathbf{C\ (farads) = \frac{Q\ (coulombs)}{V\ (volts)}} \qquad \text{XV}$$

As we have already implied in this formula, the unit of capacitance is the **farad** (*F*).

Definition:

A conductor has a capacitance of 1 farad if a charge of 1 coulomb changes its potential by 1 volt.

This seems quite straightforward. The farad is actually a very large unit, and it is more customary to speak of the capacitance of capacitors (e.g., in radio circuits) in "millionths of a farad", i.e., in microfarads (μF).

Rearranging equation XV we have

$$\mathbf{Q = CV}$$

Again, as with Ohm's law, $V = IR$, for resistors, $Q = CV$ can be applied to each and every capacitor in a circuit.

21.30 FACTORS AFFECTING CAPACITANCE

There are three main features which affect the capacitance of a parallel plate capacitor.

(1) **The distance apart of the plates.** The capacitance is increased by bringing the plates closer together, and conversely.

(2) **The area of plates.** Increasing the size of the plates increases the capacitance.

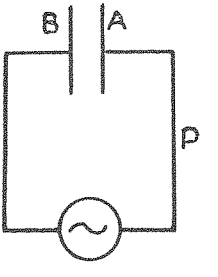
(3) **The dielectric** — that is the medium occupying the space between the plates. Most commonly the dielectric is air. It must in all circumstances be an insulating substance.

Since in practical use it is essential for the operator to be able to adjust a capacitor, it is clear that these provide the means of doing so, especially (1) and (2) where by simply rotating a milled knob it is possible to alter the area of plates opposed to one another, or (less frequently) their distance apart.

21.31 CAPACITOR IN AN A.C. CIRCUIT

We saw above that when a capacitor was connected to a battery, that is, a D.C. circuit, there was a sudden rush of current until the capacitor was fully charged. A stationary situation then existed.

But consider what happens if the current is "alternating", that is, reversing its direction many times a second. There will be a rush of current into *A*, back into *B*, and so on, in accordance with the frequency of the generated A.C.



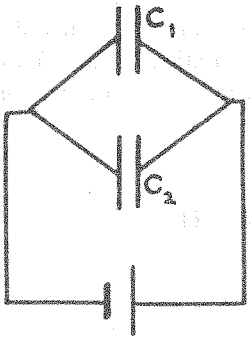
Capacitor in an A.C. circuit.

The effect for all practical purposes, at some intermediate point *P*, is that **current is flowing in the circuit**. This is one of the most important — perhaps unique — features of the capacitor. Notwithstanding what we said earlier (in regard to D.C. circuits) about "the need for a continuous path", it is clear from the sketch that there is not a continuous path — in fact, there is a big gap in it — yet current is flowing in the circuit (though not, be it noted, across the gap).

Moreover, by using a variable capacitor, as suggested in the previous paragraph, the current flowing can be varied at will. We shall return to these ideas later.

21.32 GROUPING OF CAPACITORS

It is frequently necessary to have two or more capacitors connected together, and the resulting capacitances of such arrangements must be known.



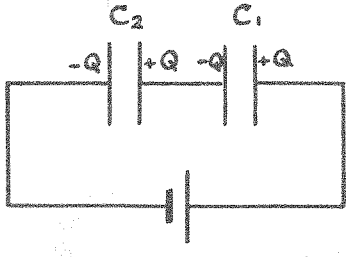
Capacitors in battery, or parallel.

In parallel (or in "battery")

Two or more capacitors joined as shown are said to be in parallel. The effect is simply that of enlarging the plates of the capacitor, and hence the capacitance of the combination is simply the sum of the individual capacitances, i.e.

$$C = C_1 + C_2 + \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad \textbf{XVI}$$

Note C_1 and C_2 will have the same p.d. across them.



Capacitors in cascade, or series.

In series (or in "cascade")

Joined as illustrated, two or more capacitors are said to be in series. It is not quite so easy to see what the combined effect will be, but consider the following.

From the two capacitors C_1 and C_2 we have (from XV)

$$V_1 = \frac{Q}{C_1} \text{ and } V_2 = \frac{Q}{C_2}$$

Note that the charge on each plate will be equal in magnitude, opposite in sign, as shown, in accordance with the laws of induction.

Also, V, the volts drop in the whole circuit will be $V_1 + V_2$. Hence

$$V = \frac{Q}{C_1} + \frac{Q}{C_2}$$

$$\frac{V}{Q} = \frac{1}{C_1} + \frac{1}{C_2} \left(\text{and} \frac{V}{Q} = \frac{1}{C}\right) \text{ and so}$$

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots\ldots\ldots\ldots\ldots\ldots \textbf{XVII}$$

These two formulae are quite easy in application and enable us to calculate the resulting capacitance of any combination of capacitors.

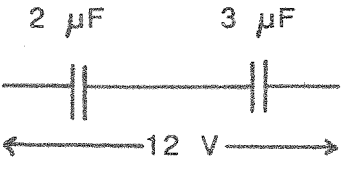
WORKED EXAMPLE 2

For the arrangement shown opposite calculate

(*a*) the total capacitance of the circuit.

(*b*) the total charge.

(*c*) the p.d. across each capacitor.



(*a*) For capacitors in series $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$

Working in μF $\frac{1}{C} = \frac{1}{2} + \frac{1}{3}$

From which $C = 1{\cdot}2\ \mu F$

(*b*) For the circuit as a whole $Q = CV$

$= 1{\cdot}2 \times 10^{-6} \times 12\ C$

$= 14{\cdot}4\ \mu C$

(*c*) Since the capacitors are in series they will EACH have a charge of 14·4 μC.

For 2 μF

$Q = CV$

$\frac{Q}{C} = V$

$V = \frac{14{\cdot}4 \times 10^{-6}}{2 \times 10^{-6}}\ V$

$V = 7{\cdot}2\ V$

For 3 μF

$Q = CV$

$\frac{Q}{C} = V$

$V = \frac{14{\cdot}4 \times 10^{-6}}{3 \times 10^{-6}}\ V$

$V = 4{\cdot}8\ V$

Note that these two p.d.s add up to 12 V.

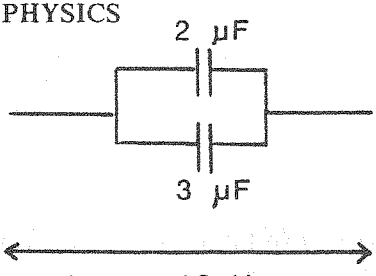
WORKED EXAMPLE 3

For the arrangement shown opposite calculate

(*a*) the total capacitance of the circuit.

(*b*) the total charge.

(*c*) the charge on each capacitor.



(*a*) For capacitors in parallel $C = C_1 + C_2$

Working in μF $C = 2 + 3\ \mu F$

$C = 5\ \mu F$

(*b*) For the circuit as a whole $Q = CV$

$= 5 \times 10^{-6} \times 12\ C$

$= 60\ \mu C$

(*c*) Since the capacitors are in parallel they will have the same p.d. (12 V) across them.

For 2 μF	For 3 μF
$Q = CV$	$Q = CV$
$Q = 2 \times 10^{-6} \times 12\ C$	$Q = 3 \times 10^{-6} \times 12\ C$
$Q = 24\ \mu C$	$Q = 36\ \mu C$

Note that these two charges add up to the charge in the circuit as a whole.

WORKED EXAMPLE 4

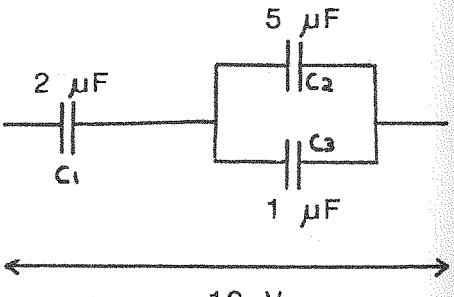
For the arrangement shown opposite calculate

(*a*) the total capacitance of the circuit.

(*b*) the total charge.

(*c*) the p.d. across C_1.

(*d*) the charge on each of C_2 and C_3.



(*a*) For the parallel arrangement $C = 5 + 1\ \mu F$

$C = 6\ \mu F$

This 6 μF is in series with the 2 μF.

Thus the total capacitance is given by $\frac{1}{C} = \frac{1}{2} + \frac{1}{6}$

from which $C = 1{\cdot}5\ \mu F$

(*b*) For the circuit as a whole $Q = CV$
$= 1{\cdot}5 \times 10^{-6} \times 12\,C$
$= 18\ \mu C$

(*c*) For C_1 $Q = CV$
$\frac{Q}{C} = V$

Now, since C is in series with the parallel arrangement, it and th parallel arrangement will be charged to 18 μC.

$$V = \frac{18 \times 10^{-6}}{2 \times 10^{-6}}\ V$$
$$V = 9\ V$$

(*d*) Since there are 9 V across C_1 there must be $12 - 9 = 3$ V across the parallel arrangement. Thus each of C_2 and C_3 have a p.d. of 3 V across them.

For C_2

$Q_2 = CV$
$= 5 \times 10^{-6} \times 3\ C$
$= 15\ \mu C$

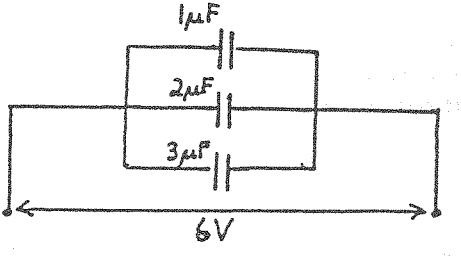
For C

$Q_3 = CV$
$= 1 \times 10^{-6} \times 3\ C$
$= 3\ \mu C$

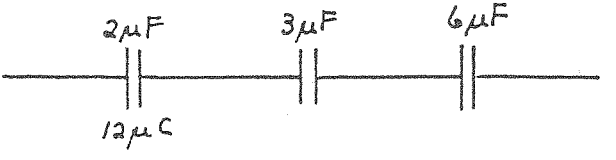
Note $Q_2 + Q_3 = 18\ \mu C$ as we would expect.

EXERCISE A

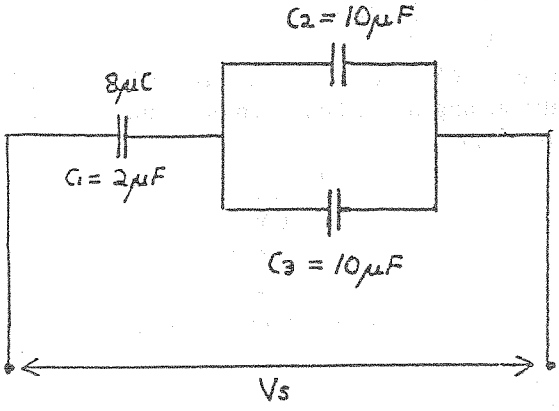
1. For the circuit shown below calculate the total capacitance and the charge on each capacitor.



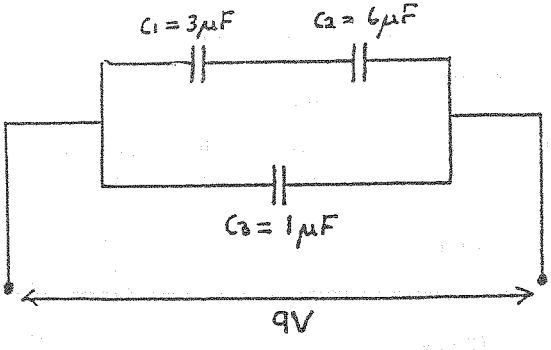
2. For the arrangement shown below calculate the total capacitance and the p.d. across each capacitor.



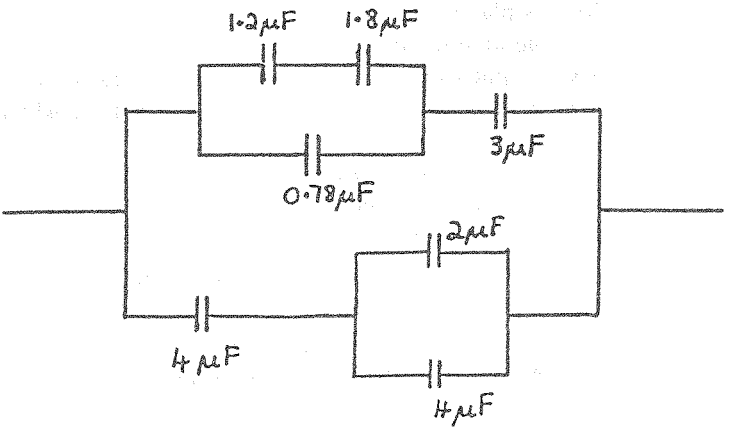
3. For the network below calculate
   (*a*) the p.d. across C_1.
   (*b*) the total charge stored on C_2 and C_3.
   (*c*) the charge stored on each of C_2 and C_3.
   (*d*) the equivalent capacitance of C_2 and C_3.
   (*e*) the p.d. across C_2 and C_3.
   (*f*) the supply p.d. Vs.
   (*g*) the total capacitance.



4. For the arrangement below calculate
   (*a*) the total capacitance.
   (*b*) the total charge stored.
   (*c*) the charge stored on each of C_1 and C_2.
   (*d*) the p.d. across C_1.



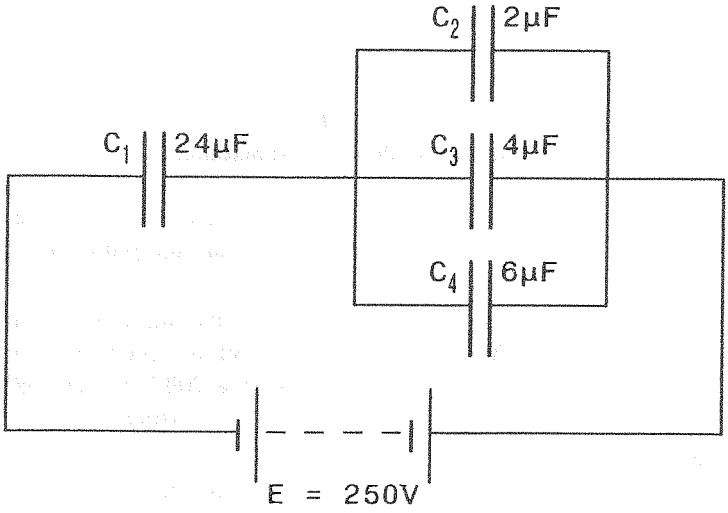
5. Find the total capacitance of the arrangement shown below



TEST PAPER 21
Ohm's Law, Power, Capacitance

1. The p.d. between the ends of a conductor of resistance 50 ohms is 250 volts. What current is it taking? What quantity of electricity would pass in 1 minute?
2. State the formula connecting watts, volts and amps. A lamp is rated at 100 W for a 240 V circuit. What is the resistance of the lamp? It is now connected up in a 200 V circuit. What current is it passing and what power is it using?
3. A lamp is rated at 60 W for a 240 V circuit. If it is now connected up in a 216 V circuit, what current is it passing and what power is it taking?
4. Find the capacitance of a capacitor if a current of 5 mA flowing for 600 microseconds causes a p.d. of 600 mV between the plates.
5. Three 2·5 μF, 3·0 μF, 4·0 μF capacitors are connected (*a*) in parallel (battery) (*b*) in series (cascade). Find the resulting combined capacitance in each case.

6. (*a*) State whether the capacitance of a capacitor is increased or decreased if:
   (i) its plates are brought closer together.
   (ii) the area of the plates is increased.
   (*b*) Two capacitors of 6 μF and 3 μF capacitance are arranged in series in an electric circuit, the p.d. of which is 50 V. Calculate EACH of the following:
   (i) the total capacitance of the circuit.
   (ii) the charge on the 6 μF capacitor.
   (SCOTVEC July 1993).
7. With regard to the circuit printed below, find:
   (*a*) the total capacitance of the circuit.
   (*b*) the charge on each capacitor.
   (*c*) the voltage drop across each capacitor.



   (SCOTVEC December 1991).
8. Two capacitors, of 10 microfarads and 20 microfarads capacitance, are arranged in series in an electric circuit. If the p.d. across the circuit is 100 volts, calculate:
   (i) the total capacitance of the circuit.
   (ii) the charge on the 10 microfarad capacitor.
   (SCOTVEC July 1991).

9. Three capacitors, of values 2, 5 and 10 microfarads, are connected in series to a 100 volt d.c. supply. Calculate:

(i) the total capacitance.

(ii) the charge stored on each capacitor.

(iii) the voltage across each capacitor.

(SCOTVEC December 1989).

CHAPTER 22

ELECTRICITY

D.C. circuits. Heating, magnetic and chemical effects

22.1 RELATION BETWEEN CURRENT FLOWING THROUGH A CONDUCTOR (OR PART OF A CONDUCTOR) AND THE DIFFERENCE OF POTENTIAL BETWEEN ITS ENDS

We have already seen (Chapter 21) that before the electrons can be set moving along the conductor, thus causing what we call a "current of electricity" there must be an e.m.f. available at one end of the conductor — a sort of "pressure", measured in **volts**.

Of course, if both ends of the conductor were at the same "pressure", or **voltage**, then no current would flow.

In fact, for a current to flow, one end of the conductor must have more volts available than the other — it must be at a **higher voltage** — and the difference between the two voltages is called the Difference of Potential, or **Potential Difference**, or p.d.

N.B. For a current to flow in a conductor, or part of a conductor, there must be a p.d. between the ends of the conductor (or part of) being considered I

For example, a "2 volt battery" just means that there is a total e.m.f. (voltage) of 2 volts available to drive current through the whole circuit. Some of this would be required to drive current through the battery itself thus completing the circuit, so that the volts available to drive current through the external circuit would be a bit less than this. This slightly less quantity would usually be called the p.d.

Thus we usually speak of "e.m.f." (volts) when we are thinking of the whole force available for the whole circuit.

We usually speak of "p.d." (volts) when we are thinking of the force available between any two points in the circuit. This will be made clearer in later examples.

Further, for any given conductor, it seems obvious that if we double the p.d., we will double the amount of current flowing, and this is the case.

N.B. Current flowing in a given conductor is directly proportional to the p.d. between its ends II

22.2 RESISTANCE — THE OHM

Another factor enters into it however.

Every conductor offers a **resistance** to the passage of a current, and here again, it seems clear that if we increase the resistance of a conductor, we will decrease the amount of current flowing in it. In fact, if we double the resistance, we halve the current.

N.B. Current flowing in a given conductor is inversely proportional to its resistance III

22.3 PRACTICAL UNITS — DEFINITIONS

The **ohm** has already been defined in Chapter 21.

The **ampere — Definition:**

1 ampere is the current which, if flowing in two straight parallel conductors of infinite length, placed 1 metre apart in a vacuum, will produce on each of the conductors a force of 2×10^{-7} newton per metre length IV

This is the international definition of the ampere. As will be seen, it is a definition drawn from the realms of electromagnetism, and reference has been made earlier to the field or force in the vicinity of a conductor carrying a current.

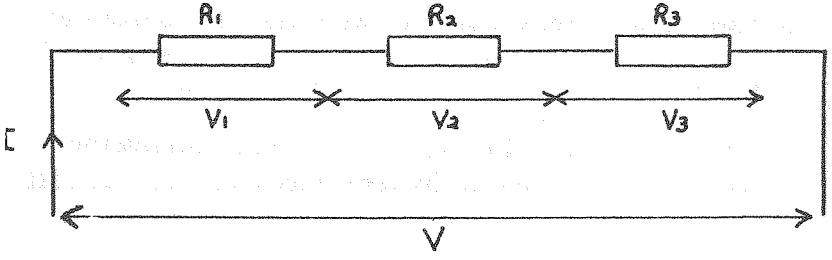
The **volt — Definition:**

Two points are at a potential difference of 1 volt if 1 joule of work is done in passing 1 coulomb between the two points ..... **V**

The relationship between these has already been summed up in Ohm's Law, namely,

$$\mathbf{V = IR} \quad \text{.....} \quad \mathbf{VI}$$

22.4 COMBINATION OF RESISTORS IN SERIES AND IN PARALLEL



Resistors connected in a line like this are said to be **in series.**

They will all have the SAME CURRENT, I, flowing through them.

Note also that the total p.d. V is the sum of the individual p.d.s. across R_1, R_2 and R_3 i.e.

$$V = V_1 + V_2 + V_3$$

Let R be the combined resistance of R_1, R_2 and R_3.

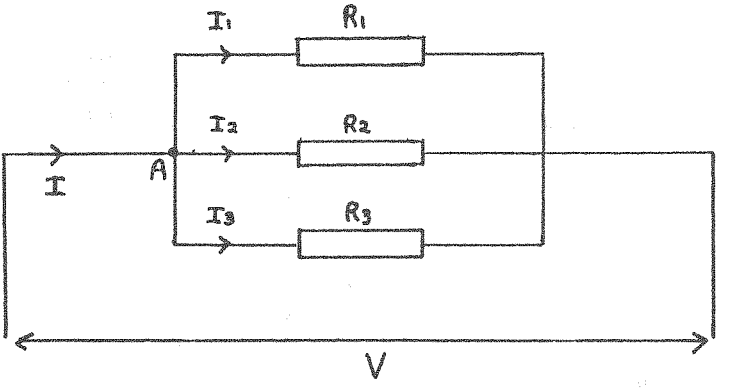
Thus using Ohm's law in the above equation we have

$$IR = IR_1 + IR_2 + IR_3$$

Dividing both sides by I we have

$$\mathbf{R = R_1 + R_2 + R_3} \quad \text{.....} \quad \mathbf{VII}$$

Resistors connected as shown below are said to be **in parallel.**



This time the resistors have the same p.d., V, across them but the supply current I will split into three at point A.

$$I = I_1 + I_2 + I_3$$

From Ohm's Law

$$I = \frac{V}{R} \text{ and applying this in the above equation}$$

We have

$$\frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

Dividing both sides by V we have

$$\frac{\mathbf{1}}{\mathbf{R}} = \frac{\mathbf{1}}{\mathbf{R_1}} + \frac{\mathbf{1}}{\mathbf{R_2}} + \frac{\mathbf{1}}{\mathbf{R_3}} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad \textbf{VIII}$$

22.5 SUMMARY FOR COMBINATIONS OF RESISTORS (AND CAPACITORS)

	Resistors	Capacitors
Basic Formula	$V = IR$	$Q = CV$
Series	$R = R_1 + R_2 + R_3$	$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$
Parallel	$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$	$C = C_1 + C_2 + C_3$
Series	Same I	Same Q
Parallel	Same V	Same V

WORKED EXAMPLE 1

Resistors of 2, 3 and 4 ohms are connected, *(a)* in series, *(b)* in parallel.

Find their combined resistance in each case.

(a) In series: $R = R_1 + R_2 + R_3$

$= 2 + 3 + 4\ \Omega$

$= 9\ \Omega$

Note that the symbol Ω is used for ohm.

(b) In parallel: $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$

$= \frac{1}{2} + \frac{1}{3} + \frac{1}{4}$

$R = \frac{12}{13}\ \Omega \qquad (0{\cdot}9231\ \Omega)$

WORKED EXAMPLE 2

For each of the circuits shown below calculate

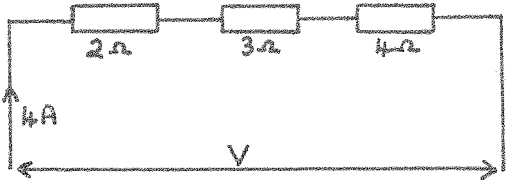
(a) a total resistance

(b) the total p.d. V

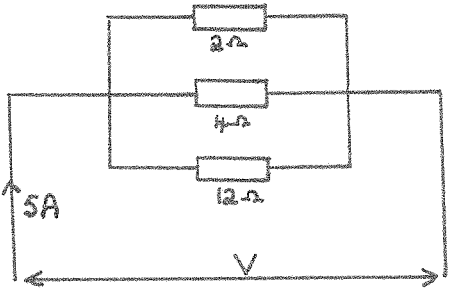
(c) the p.d. across each resistor

(d) the current in each resistor

CIRCUIT I



CIRCUIT II



For circuit I

(a) $R = R_1 + R_2 + R_3$

$= 2 + 3 + 4\ \Omega$

$= 9\ \Omega$

(b) For the circuit as a whole

$V = IR$

$= 4 \times 9\ V$

$= 36\ V$

(c) Using Ohm's Law for each resistor

$2\ \Omega$	$3\ \Omega$	$4\ \Omega$
$V = IR$	$V = IR$	$V = IR$
$V = 4 \times 2\ V$	$= 4 \times 3\ V$	$= 4 \times 4\ V$
$= 8\ V$	$= 12\ V$	$= 16\ V$

Note: adding these p.d.s gives 36 V — the answer to part *(b)*.

(d) Since the resistors are in series they will have the same current, 4A, flowing through them, a fact used to answer part *(c)*.

For circuit II

(a) $$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$
$$= \frac{1}{2} + \frac{1}{4} + \frac{1}{12}$$
$$R = 1{\cdot}2\ \Omega$$

(b) For the circuit as a whole
$$V = IR$$
$$= 5 \times 1{\cdot}2\ V$$
$$= 6\ V$$

(c) Since the resistors are in parallel they will have the same p.d. across them. This p.d. is the 6 V we calculated in part *(b)*

(d) Using Ohm's Law for each resistor

$2\ \Omega$	$4\ \Omega$	$12\ \Omega$
$V = IR$	$V = IR$	$V = IR$
$6 = \mathrm{I} \times 2$	$6 = \mathrm{I} \times 4$	$6 = \mathrm{I} \times 12$
$\mathrm{I} = 3\ A$	$\mathrm{I} = 1{\cdot}5\ A$	$\mathrm{I} = 0{\cdot}5\ A$

Note: the sum of these currents is 5 A, as we would expect. Also the current in the 2 Ω resistor is twice that in the 4 Ω resistor and six times that in the 12 Ω resistor. The current in each resistor is **inversely** proportional to its resistance.

In this particular problem this means the currents are in the ratios $\frac{1}{2} : \frac{1}{4} : \frac{1}{12}$. Multiplying by 12 we have 6 : 3 : 1. (This does not affect the ratio).

$6 + 3 + 1 = 10$.

Thus given the total current flowing is $5A$ we have

current in the 2 Ω resistor $= \frac{6}{10} \times 5\,A = 3\,A$

current in the 4 Ω resistor $= \frac{3}{10} \times 5\,A = 1{\cdot}5\,A$

current in the 12 Ω resistor $= \frac{1}{10} \times 5\,A = 0{\cdot}5\,A$

Thus we can obtain the currents by a consideration of ratios and is an alternative method to using Ohm's Law.

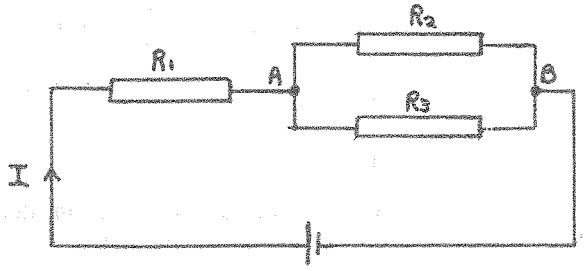
EXERCISE A

1. Three resistors, 22 kΩ, 47 kΩ and 4·7 kΩ, are connected in series across a 250 V supply. What will be:
   (a) the total resistance of the circuit?
   (b) the current drawn from the supply?
   (c) the p.d. across the 47 kΩ resistor?
2. Three resistors of 2Ω, 4Ω and 8Ω are connected in parallel. What is the total resistance of the combination?
3. A resistor of 47 kΩ is connected in parallel with one of 33 kΩ.
   Calculate: *(a)* their total resistance.
   (b) the total current flowing if they are connected across 250 V source.
   (c) the current flowing in each resistor.
4. A circuit consists of 2 resistors in series and is required to pass a current of 20 mA when the applied p.d. is 250 V.
   Calculate: *(a)* the required total resistance.
   (b) the value of one resistance if the other has a value of 5 kΩ.

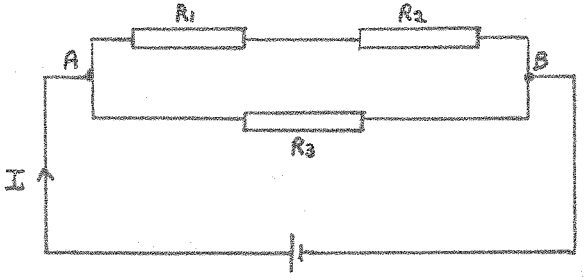
22.6 RESISTORS IN SERIES AND PARALLEL

We shall now consider circuits in which resistors are in series and parallel.

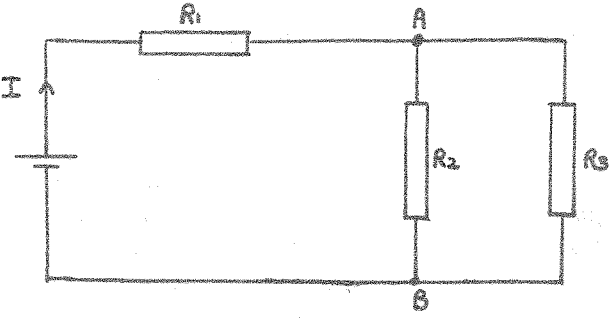
CIRCUIT I



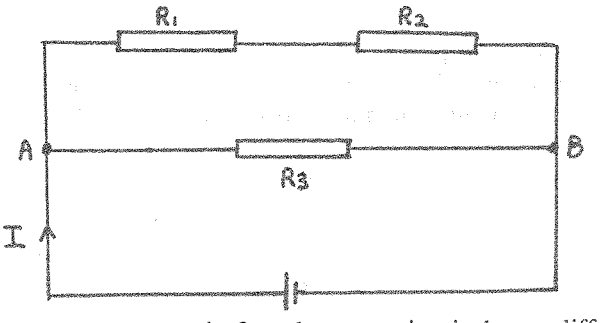
CIRCUIT II



CIRCUIT III



CIRCUIT IV



Circuits I and III are in fact the same circuit drawn differently. To see why they are the same consider what happens to the current as it flows around the circuit.

In both circuits the supply current I flows through R_1. On arriving at junction A the current splits so that some of it flows through R_2 and some through R_3. Finally at junction B the currents flowing through R_2 and R_3 unite to form I again which flows back to the supply.

For circuits I and III R_2 and R_3 are in parallel with each other and their combined resistance is in series with R_1.

Using similar reasoning to the above we can see that circuits II and IV are identical. In both circuits R_1 and R_2 are in series with each other and their combined resistance is in parallel with R_3.

WORKED EXAMPLE 3

Suppose in circuit I above $R_1 = 2\,\Omega$, $R_2 = 4\Omega$, $R_3 = 12\,\Omega$ and the supply current $I = 6A$. Calculate:

(a) the total resistance of the circuit

(b) the supply p.d.

(c) the p.d. across R_1

(d) the currents in R_2 and R_3

(a) For R_2 and R_3 $\quad \dfrac{1}{R} = \dfrac{1}{4} + \dfrac{1}{12}$

$R = 3\,\Omega$

Total resistance of circuit $= 2\,\Omega + 3\,\Omega = 5\,\Omega$

(b) For the whole circuit $V = IR$

$= 6 \times 5\ V = 30\ V$

(c) For R_1 $V = IR$
$= 6 \times 2\ V \quad = 12\ V$

(d) Since, from part *(c)*, 12 V are dropped across R_1 then this means $30 - 12 = 18$ V are dropped across the parallel arrangement. The p.d. across each of R_2 and R_3 is 18 V. Using Ohm's Law for each of R_2 and R_3 we have

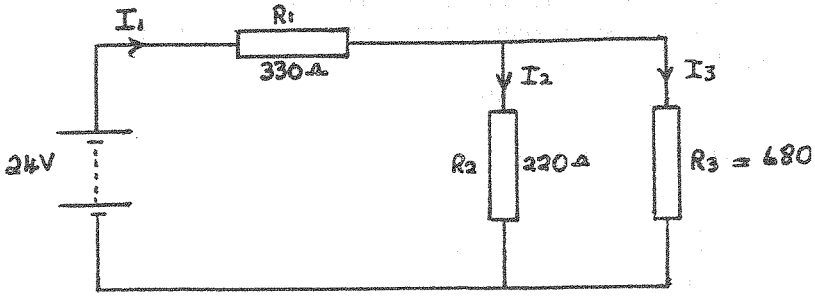
For R_2 $V = IR$
$18 = I \times 4$
$I = 4{\cdot}5\ A$

For R_3 $V = IR$
$18 = I \times 12$
$I = 1{\cdot}5\ A$

Note these two currents add up to the supply current of 6 A

EXERCISE B

1. Consider the following circuit:



Calculate:

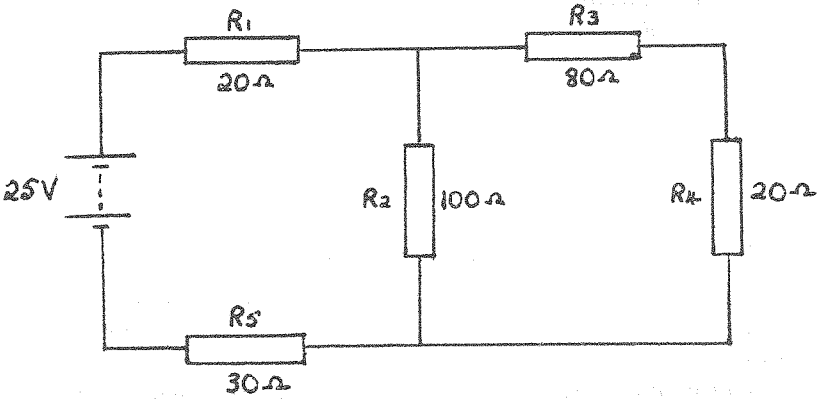
(a) the total resistance of the circuit.

(b) I_1, I_2 and I_3.

(c) the p.d.s. across R_1, R_2 and R_3.

2. Rework question 1 given that the supply voltage is 48 V, $R_1 = 150\ \Omega$, $R_2 = 270\ \Omega$ and $R_3 = 820\ \Omega$.

3. Consider the following circuit.



(a) the total resistance of the circuit
(b) the current flowing in each resistor.
(c) the p.d. across each resistor.

WORKED EXAMPLE 4

Suppose in circuit II above $R_1 = 2\,\Omega$, $R_2 = 4\,\Omega$, $R_3 = 12\,\Omega$ and the supply current $I = 6\ A$. Calculate:

(a) the total resistance of the circuit.
(b) the supply p.d.
(c) the current through R_3
(d) the p.d.s across R_1 and R_2

(a) R_1 and R_2 are in series.
Therefore their combined resistance is $2\,\Omega + 4\,\Omega = 6\,\Omega$
Total resistance of the circuit is given by $\frac{1}{R} = \frac{1}{6} + \frac{1}{12}$
$R = 4\,\Omega$

(b) For the whole circuit $V = IR$
$= 6 \times 4\ V \quad = 24\ V$

For R_3 $V = IR$
$24 = I \times 12$
$I = 2\ A$

(d) The supply current is 6 *A* and 2 *A* of it flows through R_3. This means 4 *A* flows through R_1 and R_2.

For R_1 $V = IR$
$= 4 \times 2\ V = 8\ V$

For R_2 $V = IR$
$= 4 \times 4\ V = 16\ V$

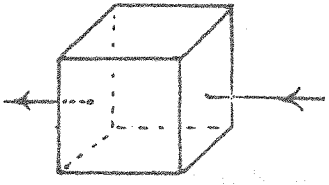
Note these p.d.s. add up to give the supply p.d. of 24 *V*.

22.7 SPECIFIC RESISTANCE, OR RESISTIVITY

It is important to be able to compare the resistance of one conducting material with another and this is done by means of its **resistivity** (symbol ρ(rho)).

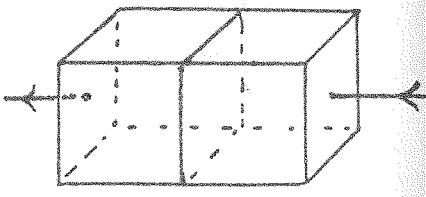
Definition: the resistivity, or specific resistance, of a material is the resistance between opposite faces of a 1 metre cube of that material.

The S.I. unit is the ohm metre (Ω m). Since its value can be very small, the sub unit Ω cm is often used.



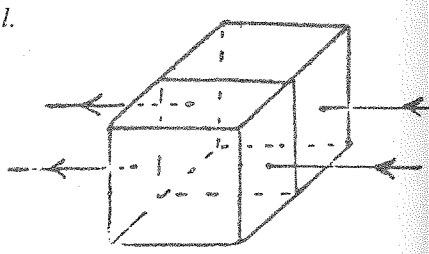
1 m cube.

It is now clear that if we have two such cubes end-on, the resistance of the conductor will be doubled.



R is proportional to length *(l)* i.e. $R \alpha l$.

Alternatively, if we have two side by side the resistance will be halved, i.e.,



R is inversely proportional to cross section area *(A)* $R \alpha \frac{1}{A}$

Combining these two relationships we have

$$R \propto \frac{l}{A}$$

$$R = \text{constant} \times \frac{l}{A}$$

The constant is the resistivity ρ

Thus

$$\mathbf{R = \frac{\rho l}{A}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad \mathbf{IX}$$

The value of the resistivity for various conductors is usually quoted as "at 0° C". The resistance of all metals **increases slightly with rise in temperature.**

The resistivity of copper is 0·000000017 Ωm ($1{\cdot}7 \times 10^{-8}$ Ωm) or more conveniently 0·017 μΩm.

Some other typical values are:

silver	$1{\cdot}5 \times 10^{-8}$ Ωm	or 0·015 μΩm
lead	20×10^{-8} Ωm	or 0·200 μΩm
iron	$9{\cdot}1 \times 10^{-8}$ Ωm	or 0·091 μΩm
constantan	57×10^{-8} Ωm	or 0·570 μΩm

WORKED EXAMPLE 5

Find the resistance of a kilometre of copper wire whose cross-sectional area is 0·05 cm² given that its resistivity is 0·017 μΩ m

$$R = \frac{pl}{A}$$

$p = 0{\cdot}017\ \mu\Omega\text{m} = 0{\cdot}017 \times 10^{-6}\ \Omega\text{m}$

$l = 1\ \text{km} = 1000\ \text{m}$

$A = 0{\cdot}05\ \text{cm}^2 = 0{\cdot}05 \times 10^{-4}\ \text{m}^2$

$$R = \frac{0{\cdot}017 \times 10^{-6} \times 1000}{0{\cdot}05 \times 10^{-4}}\ \Omega$$

$$R = 3{\cdot}4\ \Omega$$

EXERCISE C

1. Calculate the resistance of a silver wire of length 1·2 m, cross-sectional area 2 mm² and resistivity 0·015 μΩm.

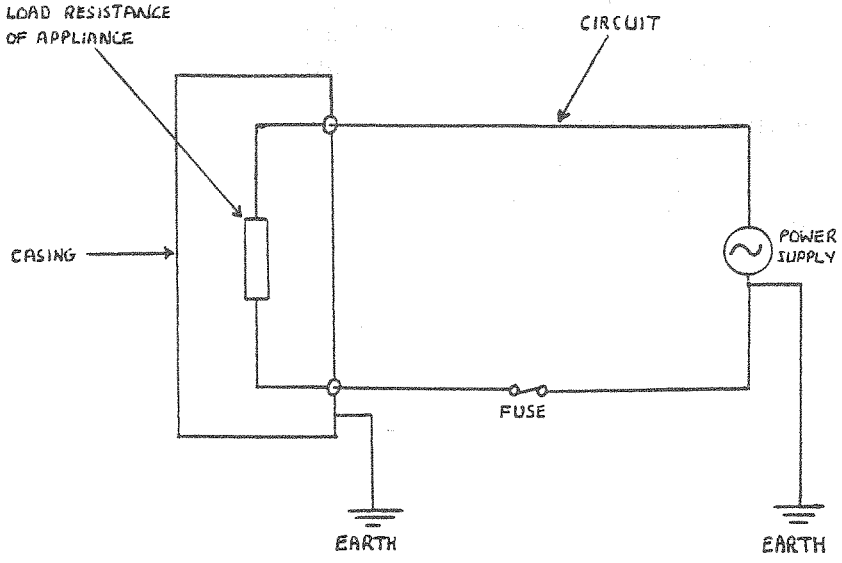
2. A copper wire, resistivity 0·017 $\mu\Omega$m, has a cross-sectional area of 6 mm^2 and a resistance of 5Ω. Calculate its length.
3. A constantan wire of length 18 m has a resistance of 2·2Ω. Calculate its cross-sectional area given that its resistivity is 0·57 $\mu\Omega$m.

22.8 INSULATION RESISTANCE—EARTHING

(a) In an electric circuit, the flow of current is confined to some predetermined path by covering its conductor with an insulating material. Also, as a safety precaution, most electrical installations are connected to the earth at one point, usually near the source of supply, or generator.

It is also within common knowledge that an appliance in a circuit should be "earthed"—that is, should have a wire leading from the casing of the appliance, to earth.

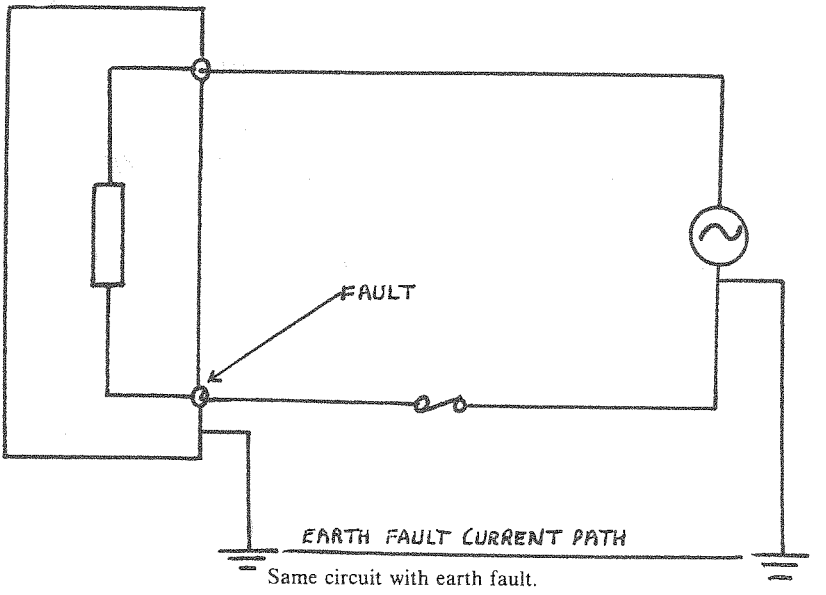
This situation is shown in the diagram below.



Fully insulated circuit, with earth near generator, and on appliance.

Note that the circuit is completely insulated, and no current is flowing in the earth, since there is no circuit for it.

In the diagram below, an earth fault (or "fault to earth") has developed — perhaps by the insulating material becoming worn so that the wire carrying current is in contact with the case of the appliance, or with some other part of the main structure, providing a path to earth.



Same circuit with earth fault.

A leakage current will now flow, and since the resistance of the earth is small, such a current may be large and may cause damage. The normal result anticipated would be for the circuit fuse to blow.

Note the safety aspect of the earth wire. Without it, the case of the appliance could become live without anyone being aware of it until he accidentally provided a path to earth through his body, often with fatal results.

(*b*) **Testing for earth fault.** It is clear from the foregoing that electrical installations must be tested periodically in order to discover any fault in the insulation. The test consists in measuring the resistance, in megohms, between the conductors; in the circuit and the earth. A good average value for the insulation resistance would be about 1 megohm.

(*c*) **Measurement of insulation resistance.** On circuits rendered "dead" an instrument called the **megger** is used. Although the

construction of this instrument is beyond the scope of this book, the method of using it is comparatively simple and the reader should take the first opportunity of seeing one in use.

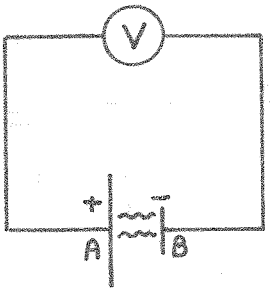
On two-wire circuits which must always have the supply of power on, measurement of insulation resistance is done by **high resistance voltmeter** of value (say) R ohms. Connect the + and − mains alternatively to earth through the voltmeter, the voltages e_1 and e_2 indicated on the voltmeter being recorded.

Then if e is the mains voltage, it can be shown that

$$\mathbf{R_1 = insulation\ resistance\ of + main} = R\left\{\frac{e-(e_1+e_2)}{e_2}\right\}$$

$$\mathbf{R_2 = insulation\ resistance\ of - main} = R\left\{\frac{e-(e_1+e_2)}{e_1}\right\}$$

22.9 THE INTERNAL RESISTANCE OF AN ELECTRIC CELL



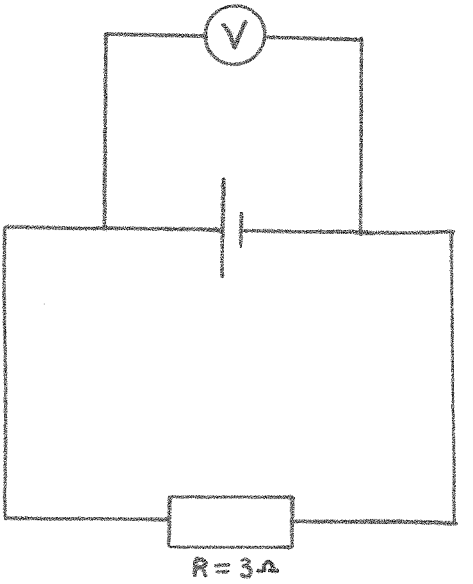
Consider a cell, or accumulator, whose positive and negative terminals are A and B. Nothing is happening at the moment. There is no current flowing — it is not "connected up to anything". Of course, current could flow through the cell itself (called the **Internal Circuit**) as shown by the wavy lines, only there is no continuous path for the current, as yet, so nothing is happening.

A voltmeter has a very large resistance and allows practically no current to flow through it — it is almost a dead stop for current. When joined as shown across any part of a circuit it is just a sort of "pressure gauge", i.e., it measures the difference in potential (volts) between the two points.

In this case, suppose it reads 2 volts.

This means that the whole e.m.f. available for driving current through the whole circuit, internal as well as external, is 2 volts.

Now join up the **external circuit** of resistance 3 ohms as shown here, and suppose the voltmeter now reads 1·6 volts.



The voltmeter is now giving us the P.D. available to drive current through the external circuit. And it follows that 0·4 volts are needed to drive the current through the internal circuit.

Let the resistance of the internal circuit (i.e., the resistance of the cell) be r, and the current flowing I.

Remember that the current is the same all the way round the circuit, comprising the cell and the 3 ohm resistance.

Applying Ohm's Law to the external circuit we have

$$1{\cdot}6 = I \times 3$$
$$I = 0{\cdot}53 \text{ A}$$

Applying Ohm's Law to the internal resistance of the cell we have

$$0{\cdot}4 = 0{\cdot}53 \times r$$
$$r = 0{\cdot}75\Omega$$

Note that we have considered the cell's internal resistance to be in series with the external resistance.

If the reader understands this example he will have no difficulty with this type of problem.

WORKED EXAMPLE 6

The P.D. at the terminals of a cell is found to change by 0·36 volts where the cell is connected to a resistance of 5 ohms through which it sends a current of 0·72 amps. Find the e.m.f. and internal resistance of the cell.

Let the e.m.f. be E and the internal resistance be r. If the p.d. at the terminals of the cell changes by 0·36 V then it is because 0·36 V are dropped across the internal resistance of the cell when the circuit is completed.

This leaves $E - 0{\cdot}36$ V available for the external circuit.

Applying Ohm's Law to the 5Ω resistor we have

$$E - 0{\cdot}36 = 0{\cdot}72 \times 5$$
$$E - 0{\cdot}36 = 3{\cdot}6$$
$$E = 3{\cdot}96 \text{ V}$$

Applying Ohm's Law to the internal resistance we have

$$0{\cdot}36 = 0{\cdot}72 \times r$$
$$r = 0{\cdot}5\Omega$$

EXERCISE D

1. A voltmeter when connected across the terminals of a battery shows 6 volts. When the terminals are connected up to an external resistance, an ammeter in the circuit reads the current flowing to be 0·6 amps and the reading on the voltmeter falls to 5·1 volts. Find the external and internal resistances.
2. A battery has an e.m.f. of 40 volts and an internal resistance of 3·2 ohms. Find the p.d. available to drive a current through an external circuit of resistance 24 ohms, and the current flowing in the circuit.
3. A battery of e.m.f. 12 volts and an internal resistance of $1^6/_7$ ohms is in circuit with resistors of 2, 4, 8 ohms, all in parallel. Find (*a*) the current flowing in the whole circuit.
   (*b*) the current flowing in each of the resistors.
   What do you notice about these currents?

22.10 HEATING EFFECT OF A CURRENT

(*a*) Heaters

Whenever a current flows in a conductor it causes the temperature of the conductor to rise. If the conductor has a **low** resistance, e.g., a good thick copper wire such as is used in ordinary electric circuits, then the heating effect is hardly noticeable.

If the resistance is **high**, such as the very thin filaments used in an electric lamp, fire, kettle or iron heating element, then the heating is so great that the conductor becomes red hot, a result with which everyone is familiar.

(*b*) Fuses

A fuse is just a short piece of **thin** wire of **low melting point**, forming part of an electric circuit.

If by any chance a circuit without a fuse were carrying too much current, it would start to get overheated. If the whole circuit got so hot that finally it broke at one point (i.e., at some point or other the wire **fused** — melted) firstly we should never know where that point was; secondly, even when located it might be in some inaccessible spot; thirdly, there would be serious risk of fire starting where the break occurred. It is to prevent this that a fuse is **always** included in a circuit and is always enclosed.

The **fuse**, on account of its low melting point, always goes first, thus breaking the circuit. Moreover, all the fuses for the different circuits can be located in one place, so that we always know exactly where to look.

(*c*) Filament Electric Lamp

An electric lamp consists of a filament of a rare metal (tungsten) mounted in a glass bulb from which all oxygen has been withdrawn and replaced by nitrogen. The filament is so designed that when the correct pressure (voltage) is applied to it, the current passing through is just sufficient to raise it to the temperature at which it will give out a bright light.

If placed in a circuit of lower voltage it will not burn so brightly, and if in a higher voltage, so much current may be forced through as to cause it to burn out. Hence filament lamps are always rated in voltage, together with the watts (power) the lamp is using at that voltage.

(*d*) **Arc Lamp**

If an air break is made in a circuit carrying a large current, the current may leap across the gap, and if the voltage is kept up, then the current will continue to flow across the gap, forming an **arc**.

A lightning flash is just an instance of this, millions of volts building up in a cloud until finally there is a discharge to the nearest point on the ground. In this case, however (perhaps fortunately) the discharge reduces the voltage, so that the discharge is not continuous.

In the **arc lamp**, the two poles between which the arc is formed are called **electrodes**, and are usually of carbon. D.C. is almost invariably used for the production of light in this way. Arc lamps, are not now much used for general illumination, but they have been employed in searchlights, where the light from the arc is focused by a parabolic mirror so as to project a powerful beam.

(*e*) **Welding**

This is a method of joining two pieces of metal together by bringing them into contact and then raising their temperature to fusing point. Welding is thus a very important application of electricity to the production of heat.

In **arc welding**, the necessary heat is provided by an arc formed between one electrode, held by the operator, and the work to be welded. The work becomes molten and joins, and, in addition, molten material from the electrode is deposited on the weld. This forms a protective covering to the weld, as well as strengthening it.

There are other methods of welding, but all depend on the heat which can be generated by means of an electric current.

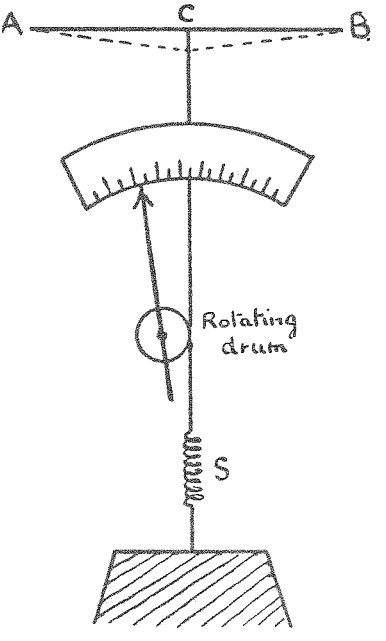
(*f*) **Change of Resistance**

The resistance of metal conductors increases with rise of temperature. This fact is ignored in simple circuits such as we shall be dealing with, but it has to be allowed for in instruments, for example, where delicate measurements are to be made.

(*g*) **Hot Wire Measuring Instruments, e.g., Ammeter**

The principle is illustrated in the diagram below. The wire AB is connected to the two terminals of the instrument. Another wire or thread, of non-conducting material is joined to AB at C and kept taut by the spring S. When the current passes through AB it becomes heated and expands, and is pulled by the spring into the

position shown by the dotted line. Thus the pointer is made to rotate and indicates the value of the current on a suitable scale.



Principle of hot-wire measuring instrument.

Since the heating is the same whichever way the current is flowing, hot-wire instruments can be used to measure either D.C. or A.C. However, they are not very robust and are only employed in delicate work, or for measuring current at very high frequencies such as are found in radio circuits.

22.11 MAGNETIC EFFECT OF A CURRENT

(*a*) On the Compass

It will be remembered from magnetism that every conductor carrying D.C. is surrounded by a magnetic field.

For this reason **single** wires should never pass near a magnetic compass. **Any electric leads which have to go to the compass, or near it, should always be in pairs**, current flowing is opposite

directions in each, so that the two magnetic fields neutralize each others.

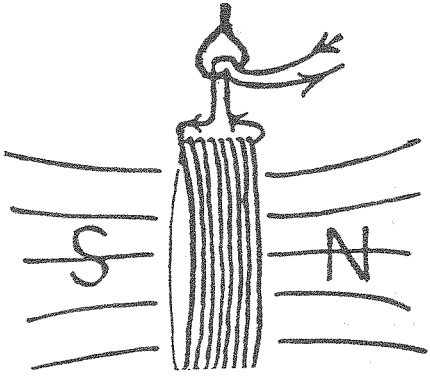
The compass should be kept well away from the powerful magnetic fields in the vicinity of dynamos, motors, or W/T installations.

(*b*) **In Making Magnets**

Again it will be recalled (from Chapter 18) that a bar of iron placed near to a conductor carrying D.C. will become magnetised by the strong magnetic field surrounding the conductor. In chapter 18 it was shown how magnets could be made by placing a bar of iron inside a coil in which a strong direct current was flowing.

(*c*) **The Construction of Galvanometers**

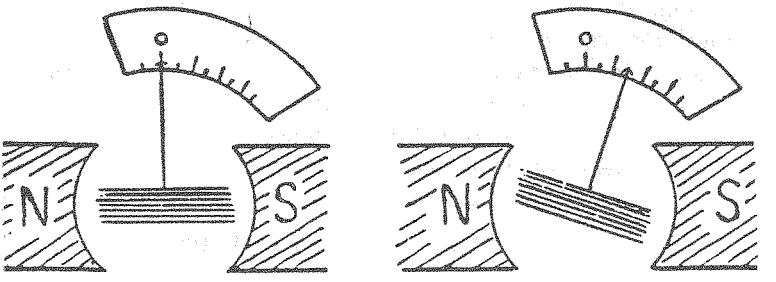
The term "galvanometer" is applied to instruments designed to detect currents by magnetic means. If calibrated to read in amps they are called **ammeters**. They can also be calibrated in volts, to detect pressure differences, in which case they are called **voltmeters**.



Coil freely suspended.

It will be appreciated that if such a coil, consisting of many hundreds of turns of insulated wire were freely suspended as shown, then, **when the current is passing**, the arrangement becomes a strong magnet with poles at opposite faces of the coil.

The fact is utilized in the construction of some **ammeters** and **voltmeters** of the type which for obvious reasons are called **moving coil meters.**



(a) (b)

Principle of the moving coil meter.

The diagram above shows the action of the instrument. The coil is mounted in the instrument between the poles of a powerful horseshoe magnet.

When no current is flowing through the coil, it will come to rest with its plane parallel to the lines of force, as in (*a*).

When current flows, the coil becomes a magnet and, working against a spring, will rotate as shown in (*b*). This is because the coil's magnetic field interacts with that of the horseshoe magnet. The extent of the rotation depends on the strength of the poles, that is, on the current in the coil. The rotation of the coil can be indicated by a pointer on a suitable scale, graduated to read volts or amps, as desired.

These instruments will be dealt with in greater detail later.

22.12 CHEMICAL EFFECT OF CURRENT

(*a*) **Electro-chemical series**

Every metal has its own electrical potential which can be tabulated in what is termed the **electro-chemical series**. The main metals of practical interest are listed below in their correct order:—

Magnesium
Zinc
Lead
Aluminium
Steel
Copper

If a metal at the upper end of the series is placed in contact with one lower down the upper metal tends to corrode. The greater the separation in the table the greater the rate of corrosion. A knowledge of this series is important when dealing with the prevention of corrosion as will be explained later.

(*b*) **Chemistry of corrosion**

This is a very complex process but to put it in its simplest form

Rust = Iron + Oxygen + Water

In other words corrosion will not occur unless oxygen and moisture are present. In addition, as corrosion is an electro-chemical process there is also a flow of current between the affected parts, one of them, the anode, will corrode away and the other, the cathode, will be relatively unaffected. These terms anode and cathode are relative; to go back to the electro-chemical series the metal which is higher in the series will always be the anode. E.g., if steel and copper are in contact the steel is the anode, but if steel and aluminium are in contact it is the aluminium which is the anode and will thus corrode.

(*c*) **Galvanic Action**

(From Galvani, 1739-98, one of the pioneers in electrical research).

Briefly, if two different substances (e.g., two dissimilar metal electrodes) are placed close together in certain liquids, there will be a flow of current from one substance through the surrounding liquid (the **electrolyte**) to the other substance, and one of the two substances will be corroded. For example, in a modern style steel frame building, those portions of the steel girders embedded in the earth (moist, of course) begin to corrode by galvanic action from the moment they are placed there. Such currents are extremely small and the process is very slow indeed.

On board ship, there is always galvanic corrosion taking place on the underwater portions, sea water being quite a good electrolyte. It is not necessary to have dissimilar metals for corrosion to occur. Any steel plate will have many minute areas which are of differing potential and this can give rise to a general rusting to about the same degree all over. This, however, is not so serious as the **pitting** resulting from dissimilar metals.

The process is hastened where we have two different metals close together, e.g., in a ship with a **bronze propeller**. In this case, to

reduce corrosion of the steel plating, **zinc plates** are fitted round the propeller aperture. **Current flows through the zinc rather than through the steel plating.** The zinc is corroded rapidly, but of course, is easily renewed.

Similar action may occur in smaller vessels of the pilot cutter type, often copper sheathed over a wood plank skin, with a steel propeller. In this case, the propeller will be corroded.

In fact, any mixture of metals is to be avoided on the underwater portions. Reputable yacht builders are very careful to avoid any mixture of metals that might lead to corrosion from this case. For example, if planking is secured with bronze bolts, they should **all** be bronze. A steel keel should be secured with steel bolts, and so on.

Lastly, the use of light metal alloys for superstructures, funnel, etc., may give rise to trouble where the alloy butts against a steel deck.

(*d*) **Primary Cell**

The construction and method of operation of these cells has already been referred to. They are called **primary cells** because they can produce a current themselves, unlike an accumulator, which has to be charged first, and is therefore called a **secondary cell.**

In its essentials, a primary cell consists of two **electrodes** and an **electrolyte**. When connected through an external circuit, current will flow, and the anode may be corroded.

Thus, this is just another case of galvanic action, chemical energy being converted into electrical energy. The details of the chemical action is beyond the scope of this book, but the construction of the cells themselves, and especially the means taken to **depolarise** them, should be carefully noted.

(*e*) **Secondary Cell**

The commonest type, in general use, is the **lead-acid** accumulator, i.e., one in which the electrolyte is **dilute sulphuric acid**, and the electrodes consist of **lead grids** in which is held the active material in the form of a thick paste. This is a compound of lead oxides though not the same compound in the positive as in the negative plates.

Again, we cannot go into the chemical explanations of what occurs. It must suffice to say that when D.C. is passed through the cell (a process called charging the cell) chemical reactions between the dissimilar materials in the plates, through the electrolyte, causes

one plate to become electrically positive to the other, i.e., one is at a **higher potential** than the other.

Thus, when connected through an external conductor, it will act as the "pump" which we have seen is always necessary, and will cause a current to flow.

In short, chemical energy stored up in the accumulator during charging is delivered back to us as electrical energy during discharge — though not the *whole* of the chemical energy is returned. There is some wastage, and actually we get back from 75 per cent to 80 per cent of it. Thus, as machines go, the accumulator may be considered very efficient.

(f) **Electrolysis**

When a current is forced to pass through a conducting liquid, **it causes chemical decomposition**. This decomposition is called **electrolysis**.

For example, if platinum electrodes are immersed in a weak solution of sulphuric acid and water, and pressure (voltage) is applied to the electrodes so that current is conducted through the electrolyte, then the water of it will be split up into its two constituents. The chemical term for water is H_2O, meaning two parts hydrogen to one part oxygen, and if the gases formed by the electrolysis of water are collected they will be found to be hydrogen and oxygen in the ration of 2:1, hydrogen being liberated at the cathode, and oxygen at the anode.

We have already referred to this indirectly, in connection with the polarising of a cell by the liberation of hydrogen molecules which collect on the cathode.

Numerous other instances might be given but,

To sum up:

(1) Chemical action (electrolysis) always takes place when electricity passes between two metallic electrodes through a liquid composed of water with some acid or salt in solution.

(2) The actual decomposition resulting and the actual substance deposited on the electrodes, will depend upon the choice of electrolyte and electrodes.

(3) If the electrolyte is a solution of some salt of the metal of the electrodes, then the electrolyte remains unaffected, but metal is transferred from one electrode to the other.

Electrolysis has an important application in **electro-plating**, where the object to be plated is made one of the electrodes.

For example:

Nickel Plating. The article is placed as the cathode, with a nickel anode in a solution of nickel ammonium sulphate.

Silver Plating. The article is placed as the cathode, with a silver anode in a solution of potassium and silver cyanide.

22.13 CATHODIC PROTECTION

Once we know the causes of corrosion we can use that knowledge to control corrosion. One method of doing this is cathodic protection where we tackle the problem from the electrical aspect. There are two methods of cathodic protection.

1. Active or Sacrificial Anodes

This is used in ballast tanks and also to protect the underwater portion of the vessel. Magnesium or zinc anodes are attached to the steelwork and these, being the anode, corrode away and thus protect the steelwork which is the cathode. The anodes are then replaced when necessary.

2. Passive or Impressed Current Method

This is normally used to protect the underwater part of the ship both when underway or laid up. Steel anodes (sometimes coated with platinum) are strategically placed along the ship's bottom and are then connected to a generator. The generator is earthed to the ship's hull and when the current flows the water surrounding the ship forms ône part of the circuit between anode and generator. The current which is impressed on the steelwork prevents the formation of galvanic action currents and thus prevents the corrosion from taking place.

22.14 ELECTROCHEMICAL EQUIVALENT

The electrochemical equivalent (e.c.e.) of a substance is defined as the mass of the substance liberated during electrolysis by the passage of 1 coulomb of electricity X

It is expressed in kilogrammes per coulomb (kg/C) in SI units.

So if z is the e.c.e. of a substance, this means that $z$kg of it would be liberated (deposited) by 1 coulomb.

Since 1 coulomb is 1 amp for 1 second, then if the current is I amps, in t seconds the mass (m) deposited will be given by

$$\mathbf{m = zIt\ kg} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad \mathbf{XI}$$

From what has been said earlier, the correct electrolyte must be used in every case. Discussion of this is more appropriate to a book on chemistry.

WORKED EXAMPLE 7

A copper cathode is weighed and then suspended in an electrolyte of copper sulphate with a copper anode. A current of 5 A is passed for 30 minutes, after which the increase in weight of the cathode is found to be 2·97 g. Calculate the e.c.e. of copper.

Since $m = zIt$, i.e. $z = \dfrac{m}{It}$, we have,

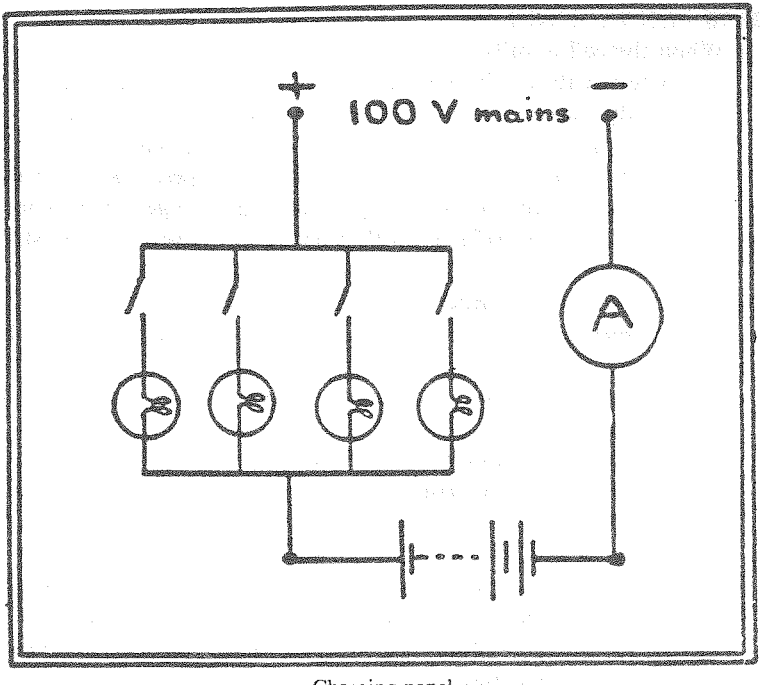
$z = \dfrac{2{\cdot}97 \times 10^{-3}}{5 \times 30 \times 60}$ which reduces to $3{\cdot}3 \times 10^{-7}$ kg/C = e.c.e. of copper.

22.15 CHARGING BATTERIES

This is usually done by connecting the accumulator (a number of accumulators connected together is called a **battery**) to a **charging board**, which is just a convenient arrangement drawing its current from the ship's D.C. supply.

The positive terminal on the board is connected to the positive on the battery, and the negative on the board to the negative on the battery. Thus, when the switch is closed, current flows through the battery in the reverse direction to that in which it flows when the battery is in normal use, connected to an external circuit, i.e., it now flows in at the positive terminal.

Of course, we cannot connect the cell or battery direct to the mains. To do so would cause a very large current to flow through the cell which would very quickly be irretrievably ruined.



Charging panel.

A convenient (though wasteful) arrangement is shown in the diagram above.

The resistance is provided by carbon filament lamps. An ammeter indicates the current passing. Or instead of some of the lamps we might use a variable resistance, though at least one lamp must be kept in as an indicator.

For example, with 100 V mains, a lamp of 100 ohms resistance would allow 1 amp to pass. If all four lamp switches were closed, then 4 amps would pass, which would normally be the maximum permissible — in fact 1 or 2 amps would be about an average rate of charge. However, this varies and **when charging and discharging, the makers' instructions must be complied with, and the stipulated rates must not be exceeded.**

There is no objection to charging as a battery, providing **all the cells are in the same condition.** A fairly safe rule is: charge as a battery if it is used in service as a battery; if cells are used singly, they should be charged singly.

22.16 ON CHARGE

When the cell is fully charged.

(*a*) A test with a voltmeter across the terminals should show a steady maximum voltage. As long as charging continues, this e.m.f. may rise to as much as 2·65 V, but it drops to 2·1 V almost immediately charging is stopped, and remains at 2 V for almost half the period of discharge. Even when "fully discharged" (i.e., within permitted limit) it would still show about 1·8 V.

Thus, it is impossible to distinguish with a voltmeter between a cell which is fully charged and one which is half discharged.

(*b*) By testing with a hydrometer, the electrolyte should show a steady maximum density.

This is a much more reliable guide, as the relative density of the electrolyte falls at a steady rate throughout the whole period of discharge. The R.D. is about 1·25 when fully charged, and about 1·18 when discharged.

(*c*) If charging is continued too long, i.e., beyond the point where the cell is fully charged, the cell will start "gassing". This just means that electrolysis is taking place, dividing the water into hydrogen and oxygen, which are given off at the electrodes.

Thus, gassing indicates that charging is complete — has in fact, gone on too long (see (*b*) below).

(*d*) If the cell is in good condition, when charged, the positive plate should be brown in colour, the negative plate grey.

22.17 CARE OF ACCUMULATORS

(*a*) The rates of charge and discharge as laid down by the makers must be adhered to. If the cell is made to give, or take, too big a current for the size of its plates, the plates may buckle as a result of overheating, the paste fall out, and the cell become useless.

(*b*) Do not overcharge. If the gassing persists too long, some of the paste is blown off the positive grid and falls to the bottom of the container. If this accumulates, it may short-circuit the cell. An accumulator which is persistently overcharged grows old before its time.

(*c*) Do not discharge too long. In a bad case of over-discharge the cell may be unable to recover, even when put on charge again.

For the same reason, do not allow a cell to stand for a long period in the discharge condition.

(*d*) The electrolyte must be kept covering the plates. The acid does not escape except by spilling or leakage — only the water needs replacement, and this should be done by topping up with **distilled water**.

(*e*) If a cell is not being worked regularly, it should be checked for e.m.f. and relative density at least once a week. Better still, try to give it a little work, and put it on charge, if only for a short period. Like a bank account, it is best to keep it working!

(*f*) Finally, keep the terminals clean and dry. Do not allow water to lie about on the cover of the cells.

22.18 RATING OF ACCUMULATORS

Cells are rated in ampere-hours, usually quoted at the 10 hr rate unless otherwise stated. Thus, "1 Ah means $^1/_{10}$ amp for 10 hrs". If discharged at less than the maximum rate it will usually yield rather more ampere-hours.

The capacity of a cell is the quantity of electricity in ampere-hours it will give, when discharged at the authorized rate, or less.

For example, a cell might be rated at "40 Ah at the 20 hour rate". Now the "20 hour rate" is the rate at which the cell would be fully discharged in 20 hours, i.e., 2 amps. So in short, this cell, from the fully charged state, will yield 2 amps for 20 hours, i.e., 40 Ah altogether.

If discharged at a different rate, the actual number of ampere-hours obtained might differ slightly from 40, but for most practical purposes we may say that the same cell would give 1 amp for 40 hours, 2 amps for 20 hours, ½ amp for 80 hours, and so on. Remember that the maker's maximum rate of discharge must never be exceeded.

Cells may be grouped in a battery of course, so as to give quite a large capacity in ampere-hours. A 60 Ah battery would be expected

to give 3 amps for about 20 hours, 2 amps for about 30 hours, or 1 amp, for about 60 hours.

22.19 ADVANTAGE OF AN ACCUMULATOR

The accumulator has one big advantage over the primary cell — its internal **resistance is small**. For an accumulator in good condition, the internal resistance is possibly as little as 0·03 ohm.

22.20 GROUPING OF PRIMARY CELLS

(*a*) **In Series**

Three cells as shown in the diagram below are said to be connected **in series.**

Note particularly the method of connecting up, viz., the positive of one cell to the negative of the next, and so on.



Cells in series.

In this case, the total e.m.f. is the sum of the e.m.f.s, and the total internal resistance is the sum of the internal resistances.

i.e., If there are n cells, each of e.m.f. E and internal resistance r, then

$$\left.\begin{aligned}\text{Total e.m.f. of the battery} &= nE\\ \text{Total resistance of the battery} &= nr\end{aligned}\right\} \quad \ldots\ldots \quad \textbf{XII}$$

WORKED EXAMPLE 8

Find the current passing through a lamp of resistance 4 ohms, connected across the terminals of a battery of 4 cells in series, each of e.m.f. 1·5 volts and internal resistance 0·4 ohms.

Total e.m.f. $= 4 \times 1{\cdot}5 = 6$ volts.

Total internal resistance $= 4 \times 0{\cdot}4 = 1{\cdot}6\ \Omega$

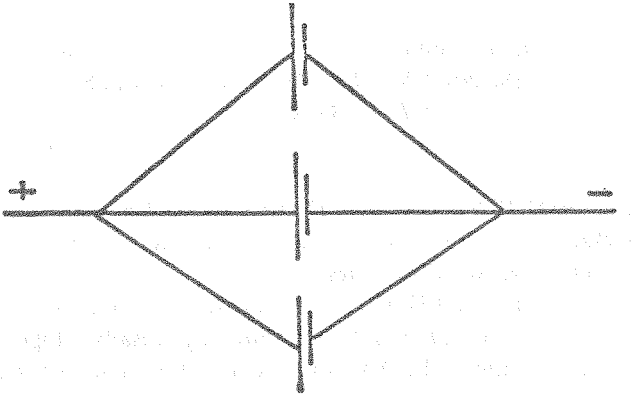
For the whole circuit,

$$I = \frac{6}{1{\cdot}6 + 4} = \frac{6}{5{\cdot}6} = 1{\cdot}07 \text{ amps.}$$

(*b*) **In Parallel**

Three cells as shown in the diagram below are said to be connected **in parallel.**

Note particularly the method of connecting up, viz., all the positives together, all the negatives together.



Cells in parallel.

In this case, the total e.m.f. is just that of one cell. This seems odd at first sight, but in effect, joining them as shown is really just increasing the *area* of the plates, which does not increase their potential difference. Thus, the combined e.m.f. is just *E*.

On the other hand, there are now three routes for the current to take, so the combined resistance is only one third of the individual resistance.

i.e., if there are *n* cells as before, then,

$$\left.\begin{aligned}\text{Total e.m.f. of the battery} &= E\\ \text{Total resistance of the battery} &= \frac{r}{n}\end{aligned}\right\} \ldots\ldots \text{XIII}$$

WORKED EXAMPLE 9

Repeat the calculation of Worked Example 8 but with the cells in parallel.

Total e.m.f. $= 1{\cdot}5$ volts.

Total internal resistance $= \frac{0{\cdot}4}{4} = 0{\cdot}1\ \Omega$

For the whole circuit,

Using Ohm's Law $1{\cdot}5 = I \times (4 + 0{\cdot}1)$ from which

$$I = 0{\cdot}37\ \text{A}$$

22.21 GROUPING OF SECONDARY CELLS

Accumulators, because of their small internal resistance are nearly always grouped **in series.**

Consideration of **XII** will show that with accumulators in series, we get the increased e.m.f. **without the disadvantage of the increased resistance.** The Ah would remain the same as for a single cell.

On the other hand, accumulators in parallel (as we can see by considering **XIII**) would only give the same e.m.f. as a single accumulator, once again, with no resistance. The only point in this sort of grouping would be that the battery would give the same current as a single one, **but for n times as long,** i.e., with four accumulators in parallel, we should get **four times the Ah of a single one.** Sometimes this is desirable.

In general, however, for big e.m.f.s accumulators should be grouped in series.

To sum up:

n accumulators in series, e.m.f. = nE volts
(Ah as for a single accumulator)
n accumulators in parallel, e.m.f. = E volts
(Ah n times that of a single accumulator) ..... **XIV**

And in both cases there is negligible internal resistance.

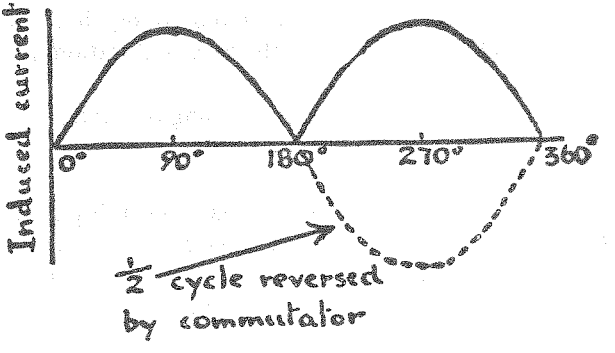
22.22 PRODUCTION OF DIRECT CURRENT — RECTIFICATION

From what has been said earlier, it will be appreciated that all current produced by a rotating coil type generator is in fact alternating current. Before leaving this chapter therefore, it might be appropriate just to consider how D.C. is obtained from A.C., a process known as **rectification**.

(*a*) **By commutator.** We have already seen how, by means of the commutator, an alternating voltage in the rotating coil can be made to give a unidirectional current in the external circuit.

The commutator causes the current to flow in the same direction in the external circuit throughout the complete revolution of the coil.

The curve of the alternating current would then appear as follows.

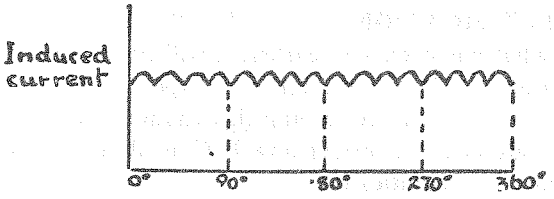


Uni-directional current from dynamo with commutator.

This kind of current is in one direction but is very uneven and would be of little practical use. The coil is only producing enough current to be useful around 90° and 270°, and twice in each cycle the current actually falls away to zero.

However, this is easily overcome by increasing the number of coils.

With eight equally spaced revolving coils we should get a current output like this



Current from eight coils with commutator.

Modern generators have many coils evenly spaced on the surface of the armature and the commutator has a correspondingly large number of sectors. This smooths out the pulsations of voltage, but even so, there is still a slight residual pulsation called **commutator ripple.**

Although so far we have spoken only of the armature rotating, there is no difference in principle if the armature is kept stationary and the field magnets rotated. To avoid ambiguity, the inner rotating part is called the **rotor**, and the external, stationary part, is called the **stator.**

Thus, we might have the windings forming the stator. The rotor would consist of magnets, in pairs of poles, mounted on a shaft which rotated at high speed within the stator.

This actually is the commonest arrangement in big generators.

(*b*) **By rectifier.** A rectifier is a device which possesses the property of being able to pass current in one direction only. This is achieved in a variety of ways. For example the thermionic diode valve has been frequently used for this purpose but more recently has been superseded by the semiconductor diode rectifier (see Chapter 26). Where large current carrying capacity is required metal rectifiers have often been used. The details of operation of these devices will not be dealt with here.

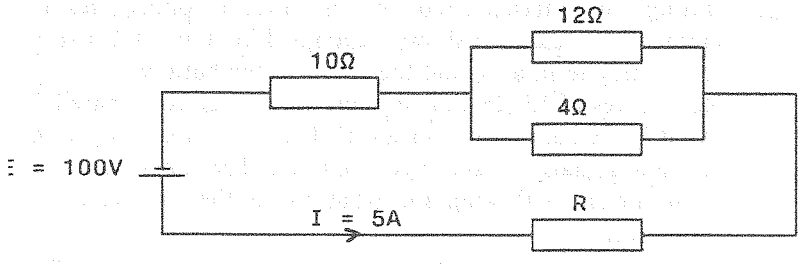
Rectification is considered again in Chapter 26.

TEST PAPER 22
D.C. Circuits

1. A Voltmeter across the terminals of a battery shows 6·6 volts, but when an external circuit of resistance 10 ohms is connected up the reading falls to 5·5 volts. Find the current flowing in the circuit and the internal resistance of the battery.
2. A battery of internal resistance 4 ohms is connected up through an external circuit of resistance 16 ohms, and an ammeter in the circuit shows 2·2 amps. Find the p.d. between the battery terminals, and the e.m.f. of the battery.
3. Resistances of 15, 25 and 30 ohms are connected in parallel. What is the combined resistance? If they are are now joined to the terminals of a 2-volt cell, of internal resistance 2 ohms, what current is flowing and what p.d. is there between the terminals?
4. What resistance must be placed in parallel with a 4 ohm resistance so as to reduce the resistance to 3 ohms? If the combination is now connected to the terminals of a 6 volt battery and an ammeter shows 1·5 amps, find the internal resistance of the battery, the p.d. between the terminals, and the current in each resistance.
5. Find the current passing through each of two lamps in parallel, resistances 2 and 4 ohms, joined to the terminals of 3 cells in series, each of e.m.f. 2 volts and internal resistance 0·5 ohms.
6. Four cells of e.m.f. 1·6 volts each, and internal resistance of 0·4 ohms each, are connected in parallel, and resistances of 4, 4, and 6 ohms, in series, are connected to the terminals. Find the current flowing in the circuit and the p.d. between the terminals.
7. A small metal article is placed in a solution of potassium and silver cyanide, together with a silver anode, and a current of 5 A is passed for 3 hours. Calculate the mass of silver deposited on the article (e.c.e. of silver $= 1{\cdot}12 \times 10^{-6}$kg/C).
8. Taking the specific resistance of copper to be $1{\cdot}59 \times 10^{-8}\Omega$m, calculate the resistance of 10 km of copper wire of radius 1 mm.

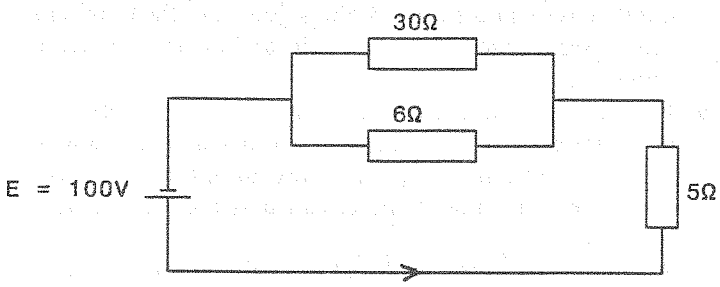
9. A length of 20 m of silver wire 0·1 mm in radius was found to have a resistance of 9·8 ohm. Calculate the resistivity of silver.

10. With regard to the circuit printed below, find:
(a) the value of the resistor 'R'.
(b) the power dissipated in the 10 ohm resistor.



(SCOTVEC December 1990).

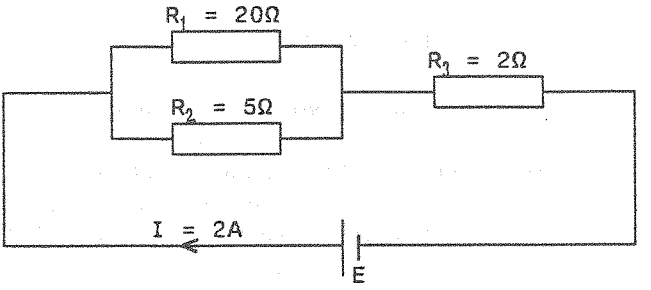
11. In the electric circuit printed below, calculate:
(a) the current flowing through each of the resistors.
(b) the potential difference across each resistor.



(SCOTVEC March 1991).

12. In the circuit printed below, find:
(a) the E.M.F.
(b) the voltage drop across each resistor.

(c) the current through each resistor.
(d) the power dissipated in the 20Ω resistor.



(SCOTVEC March 1992).

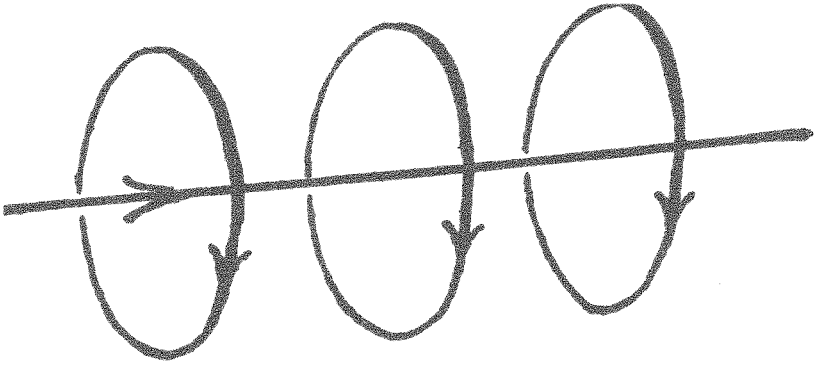
CHAPTER 23

ELECTRICITY

Measuring instruments. Ammeter and voltmeter

23.1 MAGNETIC FIELD ROUND A STRAIGHT WIRE

We have already seen (in Chapter 18) that when a direct current is flowing in a wire, the wire is surrounded by a magnetic field. The direction of this field may be determined by the RIGHT-HAND GRIP rule which requires us to imagine that the wire is gripped in the right-hand so that the thumb is pointing along the wire in the direction of the current. The direction of the fingers will give the direction of the magnetic field.



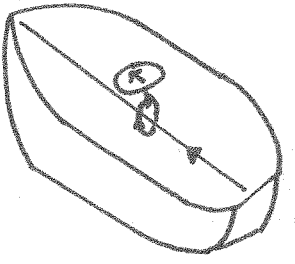
Field due to a straight wire carrying a current.

Note that only D.C. will have these effects — there is no resultant magnetic field surrounding a wire carrying A.C., since the direction of the field, like the current, changes direction many times a second.

WORKED EXAMPLE 1

A ship heading North has a single wire lying fore and aft under the compass. What deviation would be caused when current is passing through the wire from aft to forward?

The diagram below shows the arrangement.



A little thought (and the application of the right-hand grip rule) will show that UNDER the wire the magnetic field is from starboard to port, but ABOVE the wire, where the compass is placed, the field is from port to starboard. This means the north end of the needle would be urged to starboard. In this case we have an EASTERLY deviation of the compass.

EXERCISE A

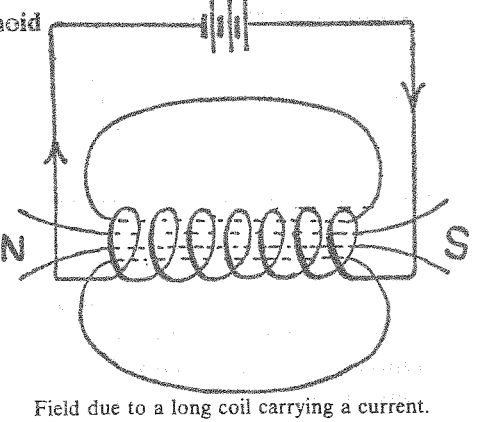
Determine the deviation of the compass in Worked Example 1 if the ship is heading (*a*) South (*b*) West (*c*) East.

23.2 MAGNETIC FIELD DUE TO A COIL

(*a*) The long coil, or solenoid

This has also been referred to in Chapter 18, where we saw that there is a powerful field inside and surrounding a solenoid, which could be utilized in making strong permanent magnets.

However, the field is there with or



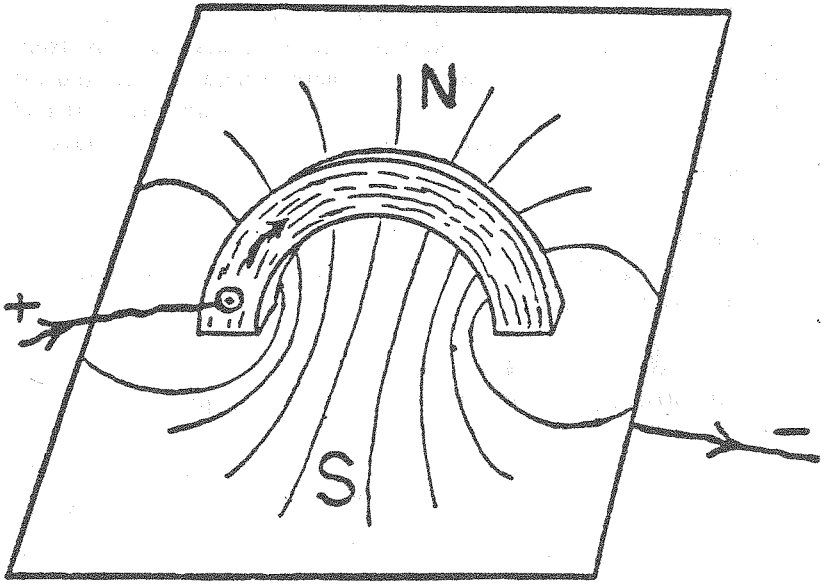
Field due to a long coil carrying a current.

without the iron core (see the diagram above). Remember that this is only a diagram. To be effective, many hundreds of turns of insulated wire are needed.

Such a coil would behave in many ways like a magnet. It would have N. and S. poles as shown and if near a compass needle it would cause deflections in accordance with the laws of attraction and repulsion. If freely suspended it would align itself north and south.

(*b*) **The short coil**

A "short" coil consists of a large number of turns of insulated wire (possible hundreds of turns) wound in the form shown in the diagram below, where the coil is shown passing through a sheet of paper on which iron filings have been sprinkled.



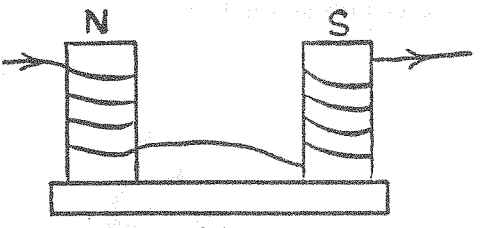
Field due to a short coil carrying a current.

If the current is flowing in the direction indicated by the arrow, then the field would be as shown.

The field at the centre of a short coil is fairly uniform over an appreciable range. This fact is of some importance in the construction of **galvanometers.**

23.3 THE ELECTRO-MAGNET

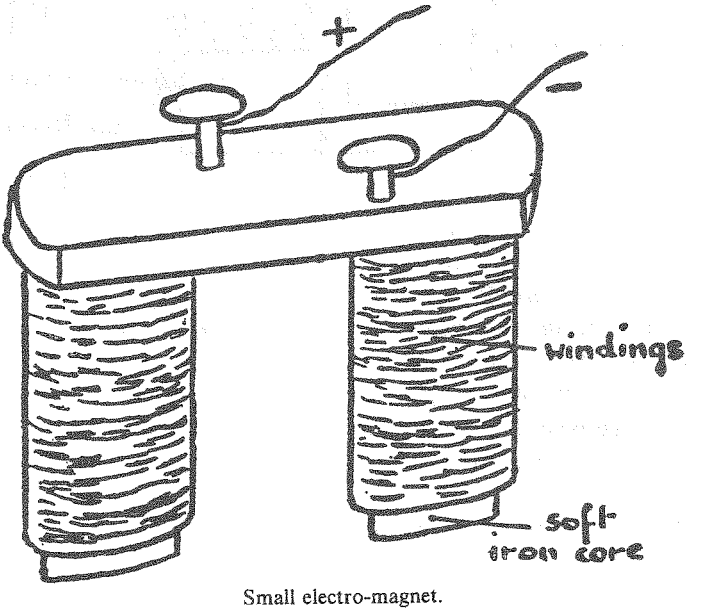
This sketch shows the principle of the electro-magnet. It consists of a soft iron core, shaped roughly like a horseshoe, with windings as shown.



Principle of electro-magnet.

Soft iron is used since it becomes magnetised the instant the current is switched on, and loses its magnetism when the current is switched off. It also greatly increases the intensity of the magnetic field inside the coil.

Note particularly the direction of the windings, and the polarity in the two sides of the horse-shoe magnet.

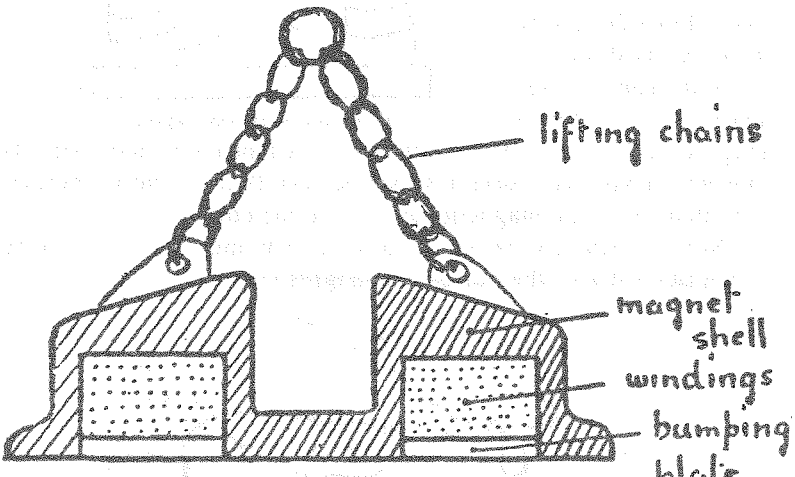


Small electro-magnet.

A small electro-magnet such as might be used in an electric bell or buzzer is illustrated in the diagram above. It is of the horseshoe type, with a soft iron core. The windings consist of hundreds of turns of insulated copper wire.

23.4 CIRCULAR LIFTING MAGNET

The diagram below shows a section through a heavy-duty lifting magnet such as would be used for handling steel ingots or scrap iron.



Circular lifting magnet.

The majority of lifting magnets have windings of flat strip, this being more durable than fine wire of circular section. Aluminium is sometimes preferred to copper, since it weighs much less. It is of course important to keep the weight of the magnet as low as practicable since this is just a useless additional load to be moved every time the crane is operated.

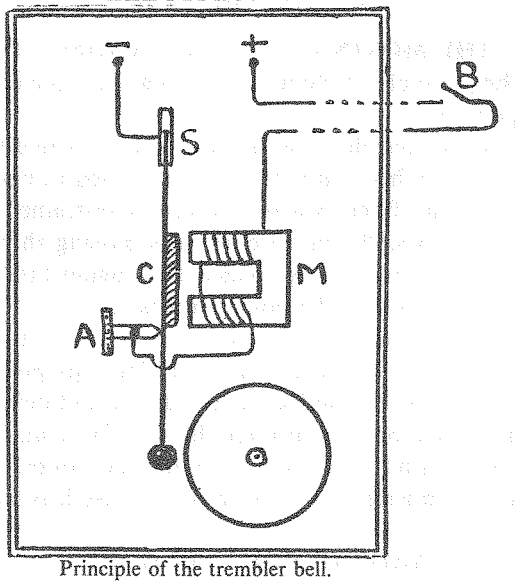
A large type of magnet such as the one illustrated might weigh as much as 2 tonnes and be capable of exerting a pull of 10 tonnes.

23.5 THE ELECTRIC BELL

(*a*) The trembler bell

The sketch illustrates the principle.

When the bell push *B* is pressed, the circuit is completed, viz., through the coils of the electro-magnet *M* to the adjusting screw at *A*, through the spring *S* upon which the hammer is mounted and so to the negative terminal.



Principle of the trembler bell.

However, the instant the circuit is made, the electro-magnet attracts the soft iron piece *C*, breaking the circuit. The moment this happens, *M* ceases to be a magnet and the spring jumps back and remakes the circuit, starting the whole process over again.

This happens many times a second, resulting in a continuous sound from the bell.

The electric buzzer works on exactly the same principle. The only difference lies in the type of "gong" employed. This is usually just a diaphragm (a flat metal sheet) so as to give a dull buzzing sound instead of the ringing note of the bell.

This type of circuit is sometimes referred to as a "make-and-break" circuit.

(*b*) **The single stroke bell**

This type of bell or gong has only a limited usefulness. It is sometimes employed to transmit orders in accordance with some predetermined code. In principle, it is simply a trembler bell with the make-and-break omitted. Thus in the diagram above, there is no adjusting screw *A*, the circuit going direct from *M* to the negative terminal. Each time the button is pressed, the hammer will be attracted by the electro-magnet and will make one stroke on the gong.

23.6 THE MOVING COIL GALVANOMETER

The principle of the moving coil galvanometer was explained in Chapter 22.

It will be recalled that a short coil is pivoted between the poles of a powerful horseshoe magnet, well aged to ensure constancy.

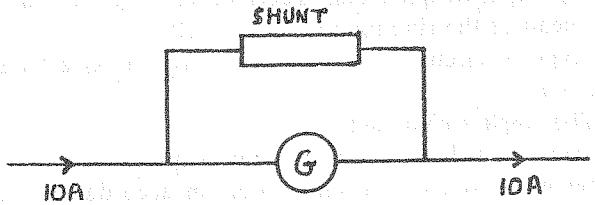
Rotation of the coil on its axis is restrained by a hair spring round the axis. When no current is passing through the coil, the tension in the hair spring holds the coil against the mechanical stops and in this position the pointer reads zero.

When the galvanometer is placed in a circuit so that the current passes through it, the coil at once develops poles on its opposite faces and so will be deflected. The amount of deflection will depend on the current passing through the coil. Thus, an instrument such as this can be graduated so as to read either amps, in which case it is called an **ammeter**, or volts, in which case it is called a **voltmeter**.

23.7 THE MOVING COIL AMMETER

(*a*) Principle of Operation

The ammeter or milliammeter is a moving coil galvanometer calibrated to measure current in amps or milliamps. It is always connected **in series** with the circuit, so that the whole current in the circuit passes through the ammeter, though not necessarily through the moving coil itself. Thus in order not to alter the current it is attempting to measure the ammeter should have a low resistance. The right degree of sensitivity in the moving part is obtained by providing a **shunt** in parallel with the coil.



Use of shunt.

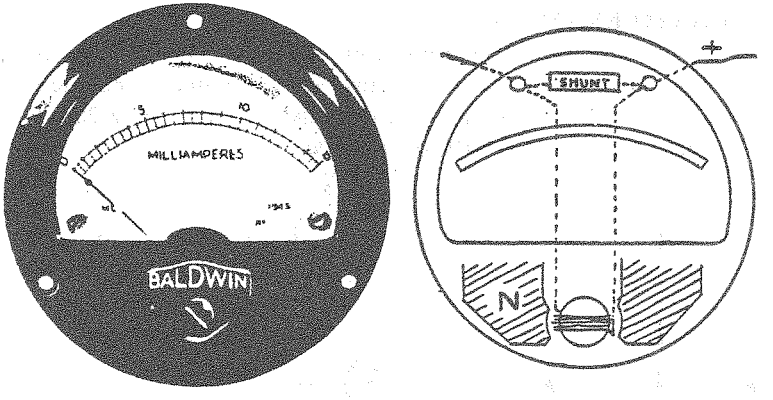
As we can see from the above diagram a shunt is simply a resistor placed in parallel with the meter. It will have a low resistance enabling some of the current to be diverted through it. This means the meter can measure currents greater than those for which it was designed.

For instance, if the resistances, shunt and coil, are such that with 10 amp in the circuit, 9 amp pass through the shunt and 1 amp through the coil then the needle would give a certain deflection. With 20 amps in the circuit, 18 amp will pass through the shunt and 2 amp through the coil, and the deflection will be doubled.

Thus, in reading the scale of such an ammeter we need not concern ourselves unduly with how much current is actually passing through the coil. Whenever the needle reaches the first point, the current (in the circuit) is 10 amp, at the second point, 20 amp, and so on.

(*b*) **Construction of moving coil ammeter**

The combination, shunt and moving coil in parallel, constitute the instrument called an "ammeter". The actual instrument is illustrated in the diagram below which, as may be seen, is graduated to read milliamps. There are also microammeters for very sensitive work, reading millionths of an amp. The principle in all cases is the same.



Principle of moving coil ammeter (shunt in parallel with coil).

To outward appearance it is a black bakelite case into which is set a graduated scale. A pointer moves across the scale, and the only other outward fittings are the two terminals, which are marked positive and negative. These are on the back of the instrument shown.

It is more important however to remember what is inside the case and an attempt has been made to show this in the sketch, in a diagrammatic way. Current entering at the positive terminal

immediately divides, most of it going through the shunt, a small fraction of it through the coil, which is shown located between the poles of the permanent magnet. This current passing through the coil causes it to rotate on its pivot, as already explained.

The actual details of construction are beyond the scope of this book and in any case, vary considerably between different makes of instrument, depending on the purpose for which it is intended. The broad principles given above however, should be thoroughly understood.

(*c*) **Calculations**

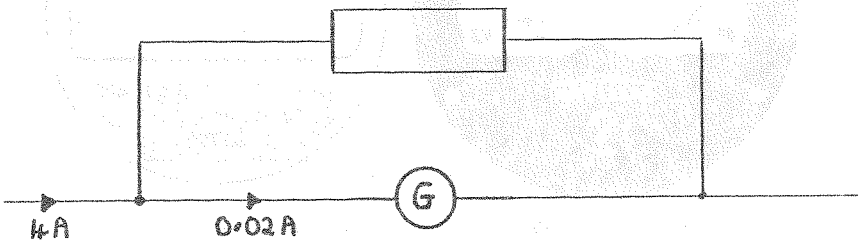
One of the purposes of a shunt in an ammeter is to adjust the sensitivity of the instrument to the various currents it will be called upon to measure.

Of course it is unlikely that we will ever be called upon to adjust the shunt. However, one or two typical calculations will help to fix ideas.

WORKED EXAMPLE 2

Given a sensitive galvonometer can only take 0·02 A as its maximum current, what shunt must be used with it to enable it to read up to 4 A, the resistance of the galvonometer being 20 Ω?

The arrangement is as shown below:



Clearly if only 0·02 A are allowed to pass through *G* then 3·98 A must pass through the shunt.

Using Ohm's Law on *G* $V = 0{\cdot}02 \times 20$ V

$V = 0{\cdot}4$ V

Since the shunt is in parallel with *G* it too will have a p.d. of 0·4 V across it.

Using Ohm's Law on the shunt $0{\cdot}4 = 3{\cdot}98 \times R$

$R = 0{\cdot}1005\ \Omega$

WORKED EXAMPLE 3

The galvonometer in Worked Example 2 is now shunted with 0·5 Ω. What current would cause maximum deflection?

G still only can take a current of 0·02 A and thus the p.d. across it will still be 0·4 V.

This means the shunt also will have a p.d. of 0·4 V across it.

Applying Ohm's Law to the shunt we have $0{\cdot}4 = I \times 0{\cdot}5$

$$I = 0{\cdot}8 \text{ A}$$

Thus the total current $= 0{\cdot}8 + 0{\cdot}02 = 0{\cdot}82$ A.

EXERCISE B

1. A galvanometer of resistance 40 ohms gives maximum deflection with a current of 0·2 amp passing through the coil. You wish to convert it to take just that amount out of a main current of 20 amp. What shunt must be put in parallel with the coil?
2. A galvanometer of resistance 250 ohms is shunted so that only 1/10th of the main current flows in it. What is the resistance of the shunt alone and the shunt and the galvanometer combined?

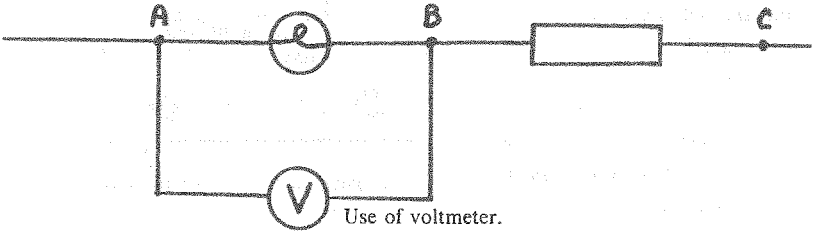
23.8 THE MOVING COIL VOLTMETER

(*a*) Principle of operation

A voltmeter is simply a galvanometer which is calibrated to measure the difference in pressure (p.d.) between two points in a circuit, in volts.

It is always connected **in parallel** with the circuit.

For example, if connected to the circuit at points *A* and *B* as shown, it would give the difference of pressure between *A* and *B*, that is, the pressure (or p.d.) in volts available to drive current through the lamp. If connected *A* and *C*, it would give us the volts available to drive current through lamp and resistance.



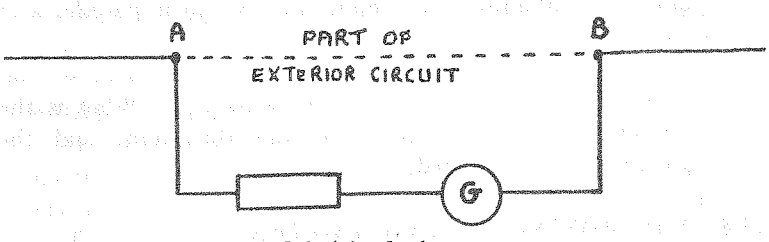
Use of voltmeter.

Clearly, the resistance of the voltmeter itself must be very large, or it would reduce the current in the part of the circuit being tested — in short, too much current would pass through the voltmeter.

Some current must pass through it, of course, in order to magnetise the coil. This current is kept very low however, by putting a high resistance **in series with the galvanomter coil.**

The resistance, as in the ammeter, is built into the instrument, which is then graduated accordingly. This just means that when we connect our voltmeter terminals to the circuit, as in the diagram below, a very small current passes in at *A*, through the resistance, through the moving coil and out at *B* — just enough current to give the coil the necessary deflection.

Needless to say, if the p.d. between *A* and *B* were twice as great, then, small as it is, twice as much current would pass through the coil and the deflection also would be doubled.



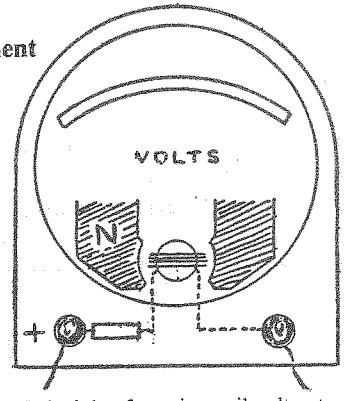
Principle of voltmeter.

Thus the deflection of the needle is proportional to the p.d. between *A* and *B* and with the scale graduated in volts the p.d. may be read off.

(*b*) **Construction of instrument**

The combination, resistance and moving coil in series, constitute the instrument known as a "voltmeter". The essential features of the voltmeter are illustrated in the diagram.

Like the ammeter, it consists of a black case, with inset scale and two terminals marked positive and negative.



Principle of moving coil voltmeter
(High resistance in series with coil).

The high resistance in series with the moving coil allows only a very small current to enter at the positive terminal. This current passes first through the resistance, then through the coil and out again at the negative terminal.

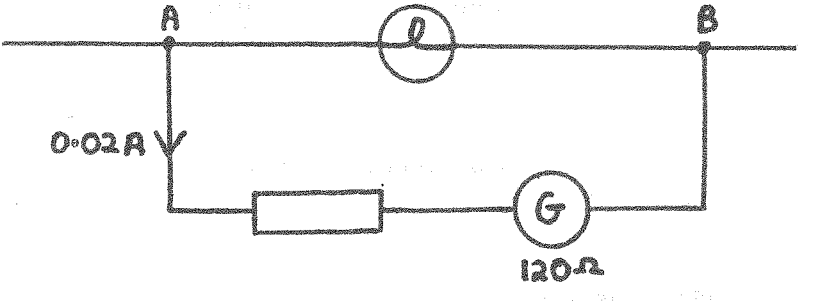
This brief outline is sufficient for our purpose and constructional details will not be gone into. It should be borne in mind, however, that there are many types of voltmeters and ammeters. Here, we have only considered those operated on the "moving coil" principle.

(*c*) **Calculations**

Some calculations will help to fix ideas.

WORKED EXAMPLE 4

The maximum current taken by a sensitive galvanometer is 0·02 amp, and its resistance is 120 ohms. What series resistance must be added to make it into a voltmeter to read up to 60 volts?



When it is connected to two points in a circuit the p.d. between its terminals is to be 60 V, and current passing is to be 0·02 amp (i.e., when the maximum permissible current is passing, the deflection must be a maximum, and this must correspond to 60 V).

Thus, by Ohm's Law.

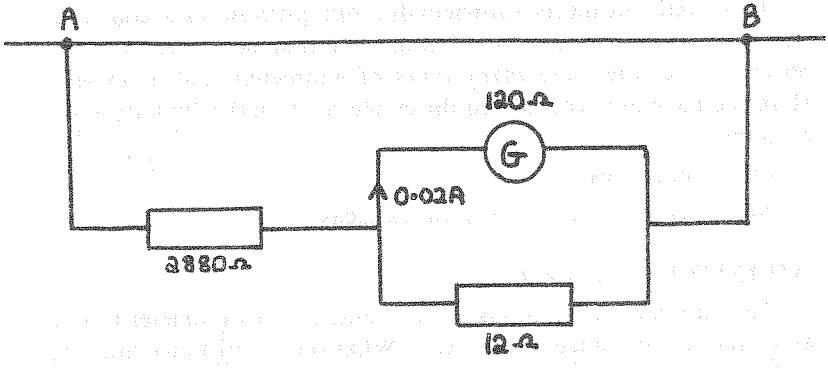
$$60 = 0{\cdot}02 \times R$$
$$R = 3000\,\Omega$$

This is the *total* resistance of the voltmeter.

Therefore series resistance needed = (3000 − 120) ohms
= 2880 ohms

WORKED EXAMPLE 5

The galvanometer coil in Worked Example 4 is shunted with a resistance of 12 ohms. What voltage is now required to cause full-scale deflection?



At full-scale deflection G must have a current of 0·02 A through it.

Using Ohm's Law on G we have $V = 0{\cdot}02 \times 120$ V
$= 2{\cdot}4$ V

Using Ohm's Law on the 12 Ω resistor $2{\cdot}4 = I \times 12$
$I = 0{\cdot}2$ A

Therefore G and the shunt together carry $0{\cdot}2 + 0{\cdot}02 = 0{\cdot}22$ A.

This is the current passing through the 2880 Ω resistor and using Ohm's Law on it we have

$V = 0{\cdot}22 \times 2880$ V
$= 633{\cdot}6$ V

Therefore total p.d. between A and B $= 633{\cdot}6 + 2{\cdot}4 = 636$ V.

EXERCISE C

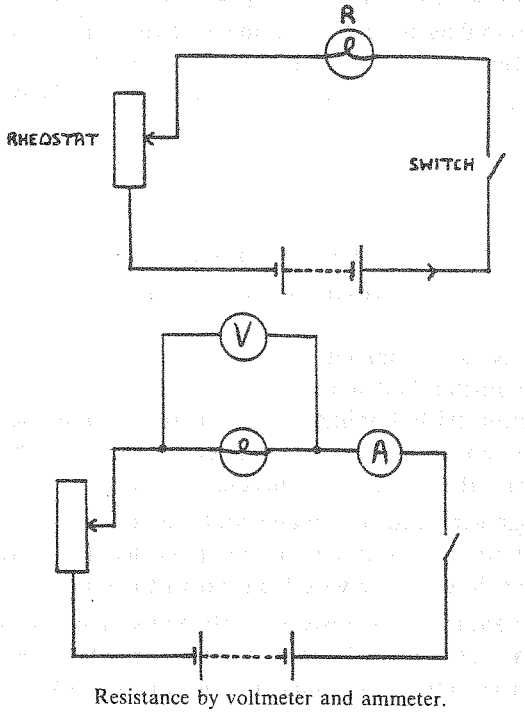
1. A galvanometer of resistance 100 ohms gives full deflection with a current of 4 milliamp (0·004 amp). What series resistance must be included to enable it to read up to 120 volts?
2. A galvanometer has a resistance of 10 ohms and when a current of 1 milliamp (0·001 amp) flows through it the needle is deflected 1 scale division. What series resistance must be included so that a deflection of 1 scale division will indicate a p.d. of 1 volt between its extreme terminals?

3. An instrument of resistance 150 ohms takes a maximum current of 0·03 amp. How would you convert it into a voltmeter to read up to 210 V.
4. A galvanometer coil of resistance 80 ohms is in series (in a voltmeter) with a resistance of 2000 ohms and gives maximum deflection when 0·025 amp is passing through the coil. The coil is now shunted with a 20 ohm resistance. What voltage is required now to cause full scale deflection?

23.9 RESISTANCE BY AMMETER AND VOLTMETER

In circuitry, the most direct method of measuring resistance is to use a voltmeter and ammeter as shown in the diagram. The voltmeter will be of very high resistance and the ammeter of very low resistance.

A variable resistor (rheostat) is included in the circuit as a safety measure.



Resistance by voltmeter and ammeter.

For example, to find the resistance of a lamp.

When the circuit is made, the ammeter will record the current (I amps) passing through the unknown resistance R (plus what is passing through the voltmeter, which we must assume is negligible).

The voltmeter will record the p.d. (V volts) across the lamp.

Then by Ohm's Law,

$$R = \frac{V \text{ (volts)}}{I \text{ (amps)}}$$

which will give the resistance of the lamp in ohms.

In practice, there are many inaccuracies in this method and a series of readings should be taken and the results averaged.

23.10 SYMBOLS USED IN CIRCUIT DIAGRAMS

We have previously emphasized the need for using correct symbols at all times, and many of these symbols have already been illustrated in the various circuit diagrams in this book.

On the following page is a summary of the symbols commonly employed. Get into the habit of *drawing the circuit* in all appropriate problems — it will be found that this helps with the solution.

TEST PAPER 23

Measuring instruments

1. How is an ammeter placed in an electric circuit? A galvanometer had a resistance of 200 ohms and gave full deflection with 1 milliamp (0·001 amp). Describe how you would convert it into an ammeter to read up to 5 amp, and calculate the value of the necessary resistance.
2. A single wire leads north and south under the compass. What would be the effect on the compass needle when a steady current flows in the wire from north to south?
3. A galvanometer of resistance 80 ohms gives full deflection with a current of 0·02 amp through the coil. How may it be converted into a voltmeter to read up to 110 V?

CELL



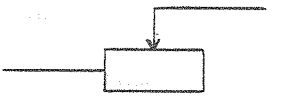
BATTERY



RESISTANCE



VARIABLE RESISTANCE (RHEOSTAT)



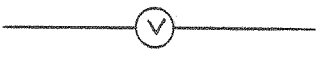
SWITCH



LAMP



VOLTMETER



AMMETER



GALVONOMETER



OR



FUSE



OR



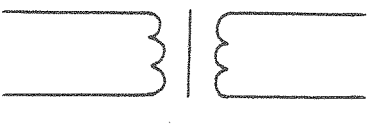
A.C. SUPPLY



CAPACITOR



TRANSFORMER



INDUCTOR



4. State how a voltmeter is connected in an electric circuit and explain your answer with the aid of a sketch.

   In a voltmeter, the series resistance is 4000 ohms and the moving coil has a resistance of 200 ohms. What current would cause maximum deflection, the scale being graduated up to 220 volts maximum?
5. A single wire lies fore and aft under the compass. A steady current flows from aft to forward in the wire. What deviations would be caused on N., E., S. and W.?
6. In a milliammeter, the resistance of the moving coil is 12 ohms and 0·004 amp gives maximum deflection. It is shunted with a resistance of 0·4 ohms. What main current would cause maximum deflection?
7. In a voltmeter, the series resistance is 8000 ohms and the coil has a resistance of 260 ohms. The scale is graduated up to 250 volt. What current will cause maximum deflection?
8. A galvanometer has a resistance of 150 ohms and gives full deflection with 0·002 amp through the coil. Find the resistance of the shunt necessary in order to convert it into an ammeter to read up to 5 amp.

CHAPTER 24

ELECTRICITY

Electromagnetic induction, transformers, the motor principle

24.1 INDUCTION

We have already discussed induction in connection with our study of magnetism. For example, we saw that if a soft iron bar were held in the Earth's field it became magnetised. The Earth's flux lines were attracted into and through the bar which as a result developed magnetic poles. This was an example of purely magnetic induction — magnetism was "induced" in the bar.

Reference has also been made to the fact (Chapter 21) that if a conductor is moved so as to cut the lines of magnetic flux, an electric current will flow in the conductor. Conversely, it is also true to say that if a direct current is flowing in a conductor, the conductor will give rise to a magnetic flux in its vicinity.

The process of obtaining electric current from a magnetic field is called **electromagnetic induction**.

24.2 INDUCTION AND INDUCTANCE

The usage of these two terms sometimes causes a little difficulty. We shall try to use "induction" as the process by which something-or-other, some quality, is transferred. We do not know what this "something" is; it can only be observed by its results, as for example, when something, some quality, passes from the flux, and becomes electricity in the wire.

"Inductance" will be used as indicating how much of that quality a given coil or appliance possesses.

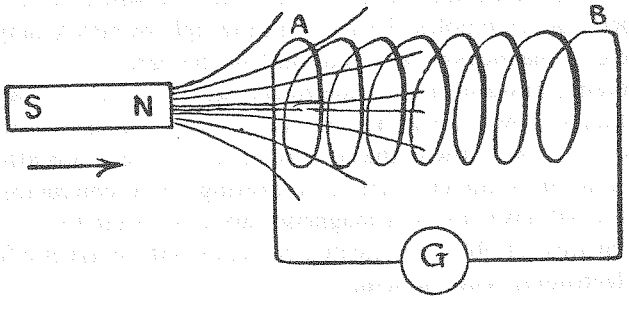
This is rather analogous to our discussion on capacitors. "Capacity" was the quality an appliance possessed (of storing electricity) and "capacitance" was how much of that quality the appliance possessed — its ability, quantitively expressed. Induction and inductance are very similar in concept.

To carry the analogy one step further, we should perhaps have the appliance itself called an "inductor" (like the capacitor).

24.3 E.M.F. INDUCED IN A CLOSED LOOP IN A CHANGING MAGNETIC FIELD

The following results are variously attributed to different scientists working independently, early in the 19th Century. Probably no one person can lay claim to having discovered what is one of the most fundamental laws of electromagnetism. It is certain however, that much if the development work is due to Faraday (1791–1867) who is credited with discovering in 1831 how to produce an e.m.f. by the movement of a conductor in a magnetic field.

Consider the arrangement shown, in which the two terminals of a coil of wire AB are connected to a galvanometer.



E.m.f. induced in a long coil.

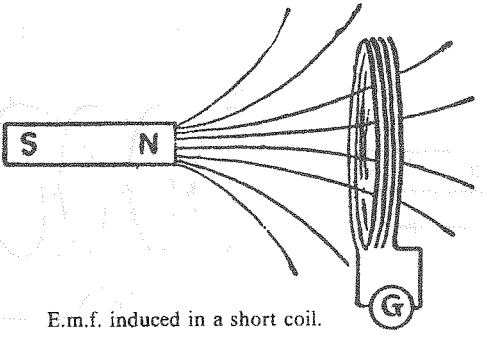
On moving the magnet up towards the coil (and for as long as it is kept in motion), the galvanometer needle will be found to be deflected, indicating an e.m.f. in the coil. As soon as the magnet comes to rest the needle returns to zero. As the magnet is withdrawn from the coil, similar results are obtained but this time, the galvanometer needle is deflected the opposite way.

In this experiment, it is clear that the movement of the magnet causes a change in the number of lines of force which are actually threaded through the solenoid at any instant.

Whenever the magnetic flux threading through a loop changes, an e.m.f. is induced in the loop.

The magnitude of the e.m.f. can easily be seen to depend on the speed at which the magnet is moved, that is, the rate at which the flux is changing. **The greater the speed at which the magnet is moved, the greater will be the induced e.m.f.**

Precisely the same results will be observed if we use a short coil, as in the diagram below. In fact, with a coil like this, in which the turns are so close together that we may assume that the flux change acts through all turns alike, then it seems reasonable to assume that the greater the number of turns, the greater will be the e.m.f. and this can be shown to be the case.



E.m.f. induced in a short coil.

To sum up:

Whenever the magnetic flux threading a circuit changes, an e.m.f. is induced in the circuit.

The induced e.m.f. is directly proportional to the rate at which the flux is changing.

If the circuit is a short coil, the induced e.m.f. is also directly proportional to the number of turns in the coil.

24.4 E.M.F. INDUCED IN A MOVING LOOP

Clearly, exactly the same results will be observed if we keep the magnet still and move the coil towards or away from it.

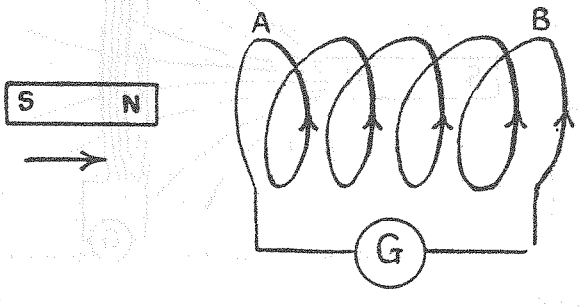
No change in principle is involved and the effect would be identical in every respect. It so happens however, as we shall see later when we consider dynamos and motors, that purely from considerations of design, or efficiency, it is sometimes desirable to construct a machine in which we keep the coil(s) still and move the magnet(s); at other times, to keep the magnet(s) still and move the coil(s).

Electromagnetically speaking however, there is no difference between the two cases.

24.5 DIRECTION OF THE E.M.F. — LENZ'S LAW

So far, all we have noted is that there is an e.m.f. in the coil while the flux is changing, and that the reversal of the movement of the magnet results in a reversal of the e.m.f.

Lenz's Law is a general statement which enables us to deduce the direction of the induced e.m.f. in any given case.



Direction of induced current — Lenz's Law.

LENZ'S LAW. The induced current always flows in such a direction that its effect is to oppose the motion producing it.

Let us consider what this means.

Now fundamentally, the cause of the induced current is a change of flux. Lenz's Law therefore means, that the induced current will always flow in such a direction in the circuit that the flux produced by this current will try to neutralize, or oppose, the movement causing the flux change; that is, the movement of the magnet.

In the diagram above, therefore, as the magnet moves towards the coil the current will flow as shown (so as to produce a N. pole at *A* which exerts a repulsive force on the advancing N. pole of the magnet).

As the magnet is withdrawn from the coil we should have a S. pole at *A* (again so as to oppose the movement of the magnet) and therefore the current would flow the opposite way in the circuit.

24.6 FLUX LINKAGE — MAGNITUDE OF E.M.F. INDUCED

We have been referring to the "flux threading through a loop", a phrase which explains exactly what is meant. In practice however, it is more customary to speak simply of the "flux linkage".

Units of flux (the weber), flux density (B) and total flux (Φ), have already been referred to. And we have now seen that for an e.m.f., we must have the flux linkage changing, and that the e.m.f. induced is proportional to the rate at which the flux linkage is changing.

In the notation of the calculus, the "rate of change of Φ with respect to t" is simply $d\Phi/dt$. Hence, if the e.m.f. is E, we have that $E \alpha d\Phi/dt$. and with Φ in webers, t in seconds and E in volts, this becomes

$$\mathbf{E = -\frac{d\Phi}{dt}} \quad \text{I}$$

The minus sign is in accordance with Lenz's Law, namely, that the e.m.f. is always in opposition to the change in flux linkage.

We shall not pursue this equation further at present, but from it, we see confirmation of the definition of a weber already given (Chapter 19). For if the R.H.S., the rate of change, is 1 weber in 1 second, then clearly the e.m.f. is 1 volt.

Unit of inductance

This also might be the place to remind ourselves of the unit of inductance, the henry, which is given Chapter 19. We shall be referring to this in more detail below.

24.7 SELF-INDUCTANCE — BACK E.M.F.

Let us consider the coil in the diagram above. As we see there, a current is flowing in the coil so long as the flux linkage is changing. But this current itself will develop flux lines through the coil, and moreover, by Lenz's Law these act in opposition to the inducing flux.

This property, by which the coil develops its own system of flux lines *within* the main system, is called **Self-Inductance**. It causes an e.m.f. to act in the coil contrary to the induced e.m.f. and so is called "**Back e.m.f.**". It opposes the growth of current in the coil.

Hence there is something rather resembling "inertia" about self-inductance, since it always acts so as to oppose its own changes. Thus, when the e.m.f. is applied, self-inductance causes the current to lag behind. As the back e.m.f. is overcome, the current reaches its peak value.

By definition:

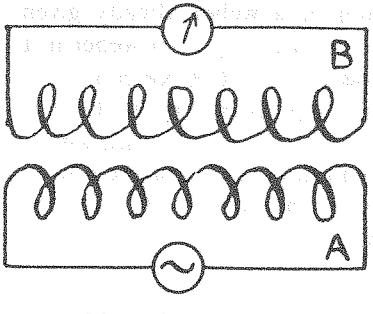
A coil has a self inductance of 1 henry when a rate of change of current of 1 amp per second through it causes an induced back e.m.f. of 1 volt.

The symbol for self-inductance is L, sometimes called the "coefficient of self-inductance".

From the above definition, since "rate of change of current" is dI/dt, if a coil has an inductance of L henrys, then the back e.m.f., E (back), will be given by

$$\mathbf{E\ (back) = L\frac{dI}{dt}} \quad \text{II}$$

24.8 MUTUAL INDUCTANCE



Mutual inductance.

Consider two coils A and B.

If a variable or alternating current is supplied to A, then A will be surrounded by a flux pattern which is building up and collapsing with the same frequency as the impressed current in A.

This flux will link with B, and consequently will cause an alternating e.m.f. in B of exactly the same frequency as the e.m.f. in A.

This phenomenon, by which an e.m.f. is caused in one coil simply by being linked by a changing flux with another coil, is called **Mutual Inductance.**

Note that it is completely mutual, that is, there is a full interchange of flux from A over to B and from B back to A, just as frequently as the impressed e.m.f. changes.

The unit of mutual inductance is the henry, and:

By definition:

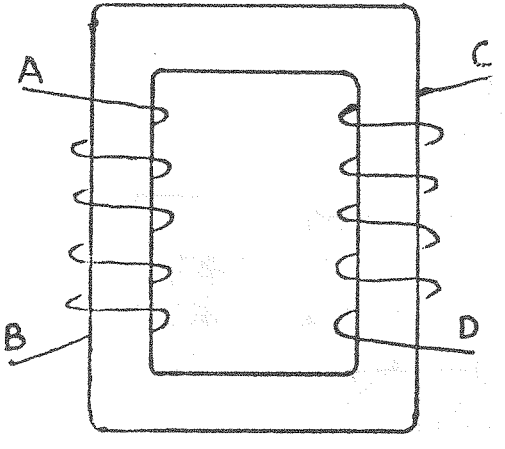
The mutual inductance of two coils is 1 henry when a rate of change of current of 1 amp per second in one causes an e.m.f. of 1 volt in the other.

The symbol for mutual inductance is M (the coefficient of mutual inductance). And from what has been said above, it will be clear that

$$\text{E (in B)} = \text{M}\frac{\text{dI}}{\text{dt}}\text{ (in A)} \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{III}$$

24.9 THE TRANSFORMER

Consider a coil AB which has been arranged as shown around a soft iron core. The purpose of the latter is simply to intensify the flux and also to link this changing flux with a second coil CD.



Principle of transformer.

This illustrates the principle of the transformer. It is an important application of **mutual inductance**.

If A.C. is supplied to the coil AB, called in this case the **primary coil**, the effect will be that the flux in the soft iron core will be changing with the same frequency as the A.C.; this in turn, being linked with coil CD, will cause an alternating e.m.f. in CD, called in this case the **secondary coil**.

Hence, current can be made to flow in the secondary coil merely by inductance from the primary.

There would be no particular point in this device were it not for the fact that by its use, merely by varying the number of turns in the coils, it is possible to make the values of the e.m.f.s in the two circuits different — in fact, the voltages are directly proportional to the number of turns in the coils linked with the flux. That is,

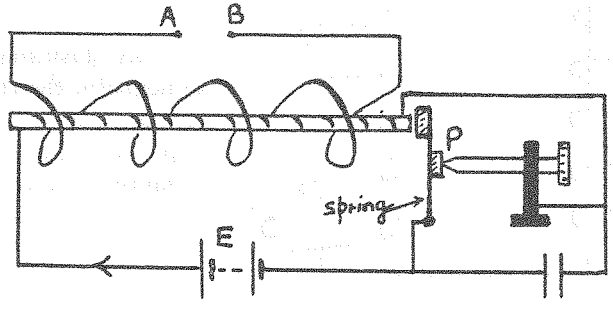
$$\frac{\text{primary voltage}}{\text{secondary voltage}} = \frac{\text{No. of turns in primary}}{\text{No. of turns in secondary}} \quad \ldots . \quad \textbf{N.B.}$$

This is the principle of all transformers. By its use, A.C. may be generated at (say) 6000 volts, stepped up to 132,000 volts for transmission using a **step-up** transformer, that is, one changing the voltage from low to high.

Conversely, at the point at which it was desired to utilise the electrical energy, a **step-down** transformer would be used to change the voltage from high to low.

24.10 THE INDUCTION COIL

The induction coil is a device used to produce very high voltages with negligible current.



Principle of induction coil.

In its essentials, it consists of a stout inner primary coil of relatively few turns wound round a core of closely packed soft iron rods. Outside of it is a larger secondary coil consisting of a great number of coils of thin wire.

The battery *E* sends a current through the primary coil via the points, *P*. The iron core thereupon becomes a magnet, attracts the tapper and so breaks the primary circuit, whereupon the core ceases

to be a magnet and the tapper returns under the action of its spring to make contact again with *P*. This make and break action can be made to continue as desired.

At the same time, the flux in the core is linked with the secondary coil and so whenever there is a change in the primary current, at make or break, an induced e.m.f. appears in the secondary. This e.m.f. can be strong enough to break down the air resistance between *A* and *B* and a spark will leap across.

This normally occurs at break, rather than at make; this is because the *growth of current on make is slower than the decay of current on break*, and since the e.m.f. is dependent on "rate of change", it will be greater on break.

A capacitor is included, as shown. Its purpose is to prevent unwanted sparking at the points *P* which would otherwise be burnt away.

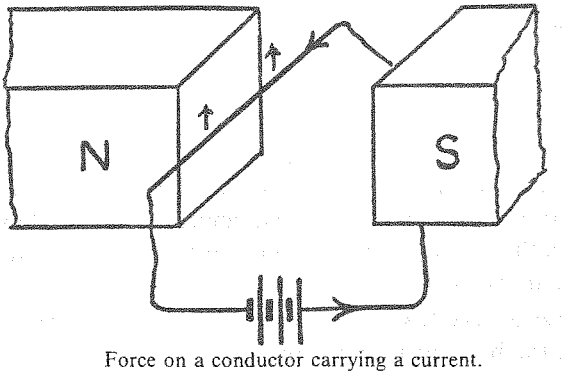
This type of coil is much used in the electrical system of the internal combustion engine. It is clearly an important application of mutual induction.

24.11 FORCE ON A CONDUCTOR CARRYING A CURRENT IN A MAGNETIC FIELD

We have seen that when a conductor is moved so as to cut the lines of magnetic force, a current will flow in the conductor.

Conversely, if we pass current through a conductor lying across a magnetic field, then **there will be a force on the conductor tending to move it.**

This is illustrated in the diagram below where current is being passed through the conductor in the direction shown.

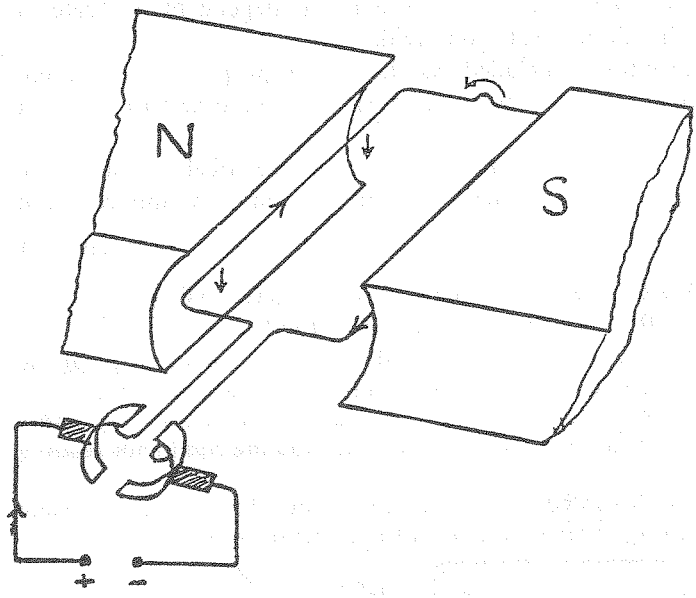


Force on a conductor carrying a current.

Then the force on the conductor is in the direction of the arrows, that is, vertically upwards in this case. This is explained in more detail below.

24.12 PRINCIPLE OF THE D.C. MOTOR

If the single wire of the diagram above were bent into a coil as shown in the diagram, suitably mounted so that it is free to rotate, then clearly the force on the coil will tend to rotate it.



Force on a coil carrying a current — principle of the D.C. motor.

This is the principle underlying all D.C. motors. Current is fed to the motor through the commutator brushes and so through the commutator itself, which ensures that current is always flowing the same way in the coil and thus the force on the coil is one which will cause continuous rotation.

Of course, a large D.C. motor bears little resemblance to the coil shown, but the basic principle is the same.

24.13 DIRECTION OF ROTATION

The direction of rotation of the coil is given by **Fleming's Left Hand Rule** ("Motors keep to the left" — Left Hand rule for motors).

Thus:

With the thumb, forefinger and second finger of the **Left Hand** mutually at right angles, then,

Forefinger indicates direction of **Magnetic Field**.

Second finger indicates direction of **current**.

Thumb indicates direction of resulting **Motion**.

The reader should apply this rule for practice to the diagrams in the preceding two sections.

To sum up:

In the motor:

we provide the magnetic field,

we provide the current,

we obtain motion (of the coil) and the direction of this motion is given by Fleming's Left Hand Rule.

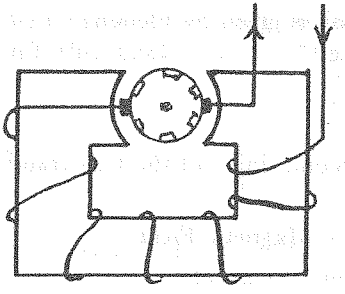
24.14 ESSENTIAL FEATURES OF D.C. MOTORS

The inner (rotating) part is called the armature. The D.C. motor is, in effect, a dynamo working backwards. The rotation of the armature within the magnetic field causes a back e.m.f. to be induced in it by the fact that the armature coils are cutting the lines of flux. This back e.m.f. is an essential requirement, since it prevents the armature from becoming over-heated or even "burnt out" as a result of too much current passing through it.

Hence every D.C. motor needs a starting resistance, so arranged that it can be progressively reduced as the motor picks up speed. Without such a device, at the moment of switching on too large a current would rush through the armature coils before the back e.m.f. became effective.

There are two main types of D.C. motors.

1. Series wound motor



Series wound.

The term derives from the fact that the field windings which create the electromagnet within which the armature rotates, are in series with the armature coils. The same current therefore passes through both.

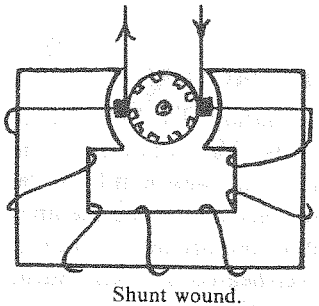
Such a motor

(*a*) gets a powerful starting torque, and

(*b*) the effect of a sudden increase in load is to slow the motor down considerably, rather than to increase the current taken.

This motor is therefore useful for traction such as in a crane, but useless for driving a constantly varying load at steady speed.

2. Shunt wound motor



Shunt wound.

The term derives from the fact that the field windings are in parallel with the armature coils. Hence the field current is constant and the torque depends on the armature current. Thus, if the motor is slowed down by an increase in load, the back e.m.f. will also fall off and so the *current through the armature will increase.*

In other words, the effect of an increase of load is almost entirely to increase the current through the armature and so *increase the torque.*

Such a motor is particularly useful for driving factory machinery where constant speed is important.

TEST PAPER 24
Questions for Self Examination

1. Describe with the aid of a sketch what is meant by self-inductance. Define the unit of self-inductance and state an equation that will give the instantaneous value of the e.m.f. in a coil of self-inductance L, explaining any terms used.
2. With the aid of a sketch define the following terms: armature windings, brushes, commutator, field magnet, slip ring. From your sketch describe the principle of a D.C. motor.
3. State Lenz's Law. Draw three different cases, depicting in each sketch a solenoid with a magnet moving towards or away from it, and show by arrows the direction of the induced current in the solenoid.
4. State Fleming's Left Hand Law as applied to motors. Give a simple sketch illustrating the principle of the motor, and showing the direction of rotation that would result from the input current depicted in your sketch.
5. Give the laws governing the production of an e.m.f. in a circuit which is cutting across the lines of magnetic flux.
6. Describe with the aid of a circuit diagram how you would connect up two coils and any other equipment you need to a 12 V battery in order to produce a high tension spark.
7. What is mutual inductance? Define the unit of mutual inductance and state the differential equation that will give the value of the e.m.f. in a coil of mutual inductance M, explaining any terms used.
8. Describe with the aid of a sketch the principle of the transformer. Current is fed into the primary coil, of 6000 turns, at 11,000 volts. If there are 30,000 turns in the secondary coil, what is the output voltage?

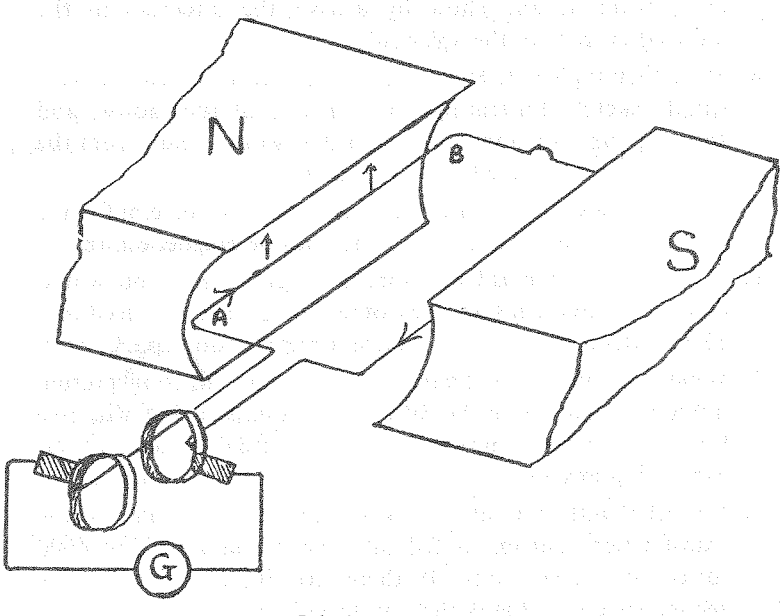
CHAPTER 25

ELECTRICITY

Alternating Current. L, C and R in A.C. Circuits

25.1 PRINCIPLE OF THE A.C. GENERATOR

This has already been referred to in Chapter 21, where we saw that when a coil is rotated rapidly in a magnetic field, an induced current is caused to flow in the coil.

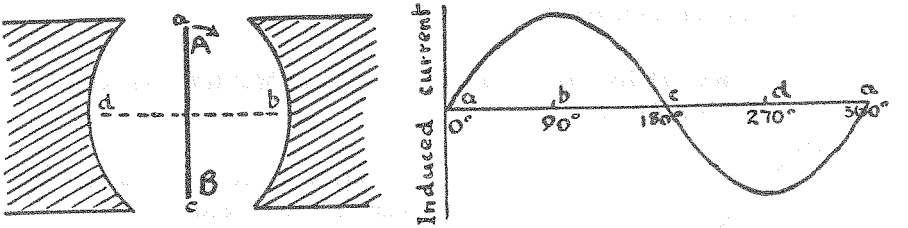


Principle of dynamo.

The reason for the induced current should now be clearly understood in the light of the preceding chapters.

It will be recalled that as the coil rotates the current in the external circuit changes direction every half revolution, and for that reason is called **alternating current**.

The sort of current we should get from a single coil like this is shown in the following curve, which illustrates one **cycle** of the alternating voltage.



Alternating current from a single coil.

Consider the side of the coil marked *AB*.

At *a*, the coil is momentarily moving parallel to the lines of flux, that is, the rate of cutting the lines is zero and therefore the current is zero. This is the point marked *a* on the curve.

When the coil has turned through 90°, to *b*, it is now cutting the lines of flux at the maximum rate and so the current has its maximum value.

As the coil continues to rotate however, the rate of cutting the lines diminishes until at *c*, when it has turned through 180°, the current is again zero.

At *d*, when the coil has turned through 270°, current is again a maximum. It then decreases to zero again as the coil completes one revolution of 360°.

A coil such as this is the simplest form of alternating-current dynamo. It is generally referred to as an **alternator**, or yet again, as an **A.C. generator**, and though a big generator in a power station or a ship's engine-room bears little resemblance to the coil shown, the basic principle is the same.

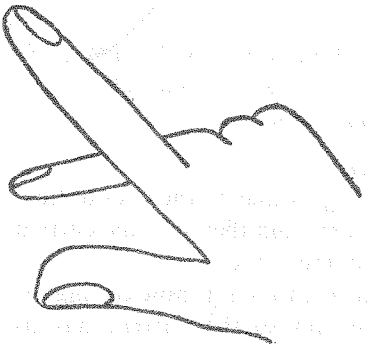
In the case we have been considering, the voltage went through one complete cycle in one revolution of the coil. By increasing the number of magnetic poles, or by increasing the r.p.m. of the coil, the **frequency** of the voltage, or **number of cycles per second**, can be increased considerably. For example, a normal supply frequency may be 50 cycles per second.

In S.I. the unit of frequency is the **Hertz (Hz)**, and 1 Hz simply means "1 cycle per second". So for instance, to borrow from the

previous paragraph, we might have said that "a normal supply frequency is 50 Hz".

By increasing the windings to many hundreds of turns, as is done on the drum armature of a large generator, a greatly increased power output is obtained.

25.2 DIRECTION OF CURRENT — FLEMING'S RIGHT HAND RULE



Fleming's Right Hand Rule.

So far, however, we have not considered the direction of the induced current in the coil. The easiest rule for determining this is known as **Fleming's Right Hand Rule** (after Dr. Fleming).

This is as follows:

Place the thumb, forefinger and second finger of the right hand mutually at right angles.

Then,

Forefinger indicates direction of Magnetic **Field**
(Forefinger—field—all F's).

Thumb indicates direction of **Motion** of coil.
(ThuMb—Motion—M's).

Second finger indicates direction of **current**.

If this rule is applied to the first diagram in this chapter (remembering that the "direction of the field" is from N. towards S.) then it is easy to see that the current will be flowing in the coil as shown by the arrows.

The reader should practise applying this rule by drawing a few coils for himself. It only requires a little care — and imagination.

To sum up:

In the dynamo or generator:

we provide the magnetic field,

we provide the motion (of the coil).

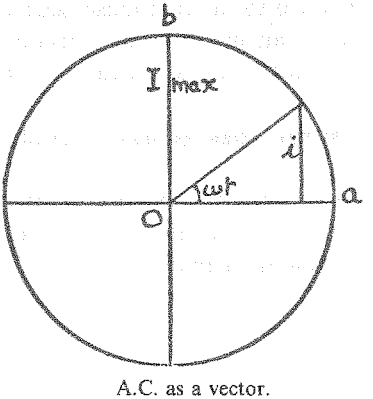
we obtain current, and the direction of this current is given by Fleming's Right Hand Rule.

25.3 THE SINE WAVE — INSTANTANEOUS VALUES OF E AND I

Most readers will have little difficulty in recognising the curve of the induced current in the dynamo coil, drawn in section 25.1, as being a SINE curve.

The sine wave is taken as the standard wave form for alternating current. All A.C. is assumed to be sinusoidal in form and for that reason is sometimes referred to as **sinusoidal current**.

It would be difficult to plot a sine curve each time we wished to deduce something about an alternating current, and a much easier method is to represent the alternating quantities (currents and e.m.f.s) by a rotating vector.



A.C. as a vector.

Let the coil be rotating at constant angular velocity ω rad/s. Then if the frequency of the rotation is f, i.e., f cycles per second, since there are 2π radians in one revolution, it will sweep out $2\pi f$ radians per second. Thus $\omega = 2\pi f$. When it is used in this fashion that is, in lieu of $2\pi f$, ω is given the term **angular frequency**. Now consider the diagram opposite.

Remember that we started our curve from a point a, when the coil was at the commencement of its revolution. This is shown in the diagram above at a.

The "angle" used in the sine curve is simply the angle through which the coil has rotated. So, when the coil has turned through 90°, it is at b and the current and voltage, are at a maximum.

Hence Ob, the radius vector represents the maximum value of current or voltage, which we will call I max. and E max. respectively. They are also frequently referred to as the **peak values**.

If the coil in the simple alternator is rotating at constant angular velocity ω radians per second, it is clear that after any time t, the angle swept out will be ωt radians and the value of the current and voltage at that instant will be given respectively by

$$i = I(max) \sin \omega t$$
$$e = E(max) \sin \omega t \quad \text{..... I}$$

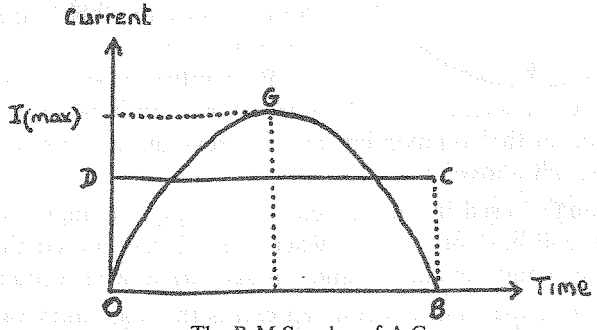
Thus without any need for drawing the sine curve, the **instantaneous values** of current and voltage may be written down.

25.4 EFFECTIVE, OR ROOT-MEAN-SQUARE, VALUE OF A.C.

We have already used several formulae in terms of D.C. When the current is constant, as in D.C., this is quite straightforward. In considering A.C., we are up against a difficulty, in that the current and voltage are continually changing, and the question arises — which value are we to use? At first glance, since during the first half cycle the current is in one direction, during the second half cycle in the opposite direction, we might be inclined to say that these cancel out — or the average current is zero — an incorrect conclusion, since we all know that A.C. not only exists but can in fact, does work.

It is by the output of work, or energy, that we have a ready means of comparing A.C. and D.C.

Consider the graphs in the diagram in which the horizontal line DC represents the value of a D.C. (constant, and therefore parallel to the x axis) and the curve is the sinusoidal current.



The R.M.S. value of A.C.

It is necessary first to state that if a force acts for a given time, and a graph be plotted of the the values of this force over a given interval, then **the work done (or output of energy) is given by the area under the curve.** This is true whether the force is a constant one (as from D.C.) or a variable one (as from A.C.).

It is now clear that the energy output of an A.C. during the interval represented by *OB* will be given by the area under the curve *OBG*, and the effective value of the A.C. will be that value of D.C. that makes the area of the rectangle *OBCD* equal to the area under the curve *OBG*.

It is shown in more advanced works that this value is given by

$$\text{Effective value of A.C.} = \sqrt{\text{mean value of the square of the current}} \quad \ldots\ldots\ldots \text{II}$$

and hence the effective value is often called the Root Mean-Square (or R.M.S.) value. That is, in full, from the expression on the right hand side of the above equation, "the square *root* of the *mean* of all the *squares* of the values of the current during the cycle". Students of the calculus will recognise the quantity on the R.H.S. as an important integral. Here we can only state that it reduces to

$$\frac{\text{maximum value}}{\sqrt{2}}$$

The R.M.S. or effective value of an A.C. is

$$\frac{I\,(max)}{\sqrt{2}} \text{ or } \frac{E\,(max)}{\sqrt{2}}, \text{ or since } \frac{1}{\sqrt{2}} = 0{\cdot}7071$$

as these are more usually expressed:

$$\left.\begin{aligned} E\,(R.M.S.) &= 0{\cdot}7071\ E\,(max) \\ I\ (R.M.S.) &= 0{\cdot}7071\ I\,(max) \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \text{III}$$

The significance of the R.M.S. value of an alternating voltage or current is that it is the value of a D.C. voltage or current that will convert electrical energy in a given resistance at the same rate as the A.C. When dealing with A.C. it is quite common to assume that voltages and currents quoted are R.M.S. values unless stated otherwise.

25.5 DETERMINATION OF E.M.F. IN A ROTATING COIL

We have been using terms such as "*E* (max)", "R.M.S. value", "instantaneous value of e.m.f.", and so on. How are these, or any one of them, determined? For of course, if we know any one of them we can determine the others.

One way would be to use a measuring instrument, but even these have to be calibrated initially. We now have to give some

thought to the manner in which a given e.m.f. may be calculated from first principles, using the basic idea that an e.m.f. is caused by the rate at which lines of magnetic flux are being cut by a conductor in motion.

Consider a coil of area A m^2 being rotated in a magnetic field of flux density B tesla. It can be shown that the e.m.f. E being generated at any particular instant is given by

$$E = B.A.\omega. \sin \theta.$$

where ω is the angular velocity and

θ is the angle of the normal to the coil to the field.

If there are N such coils we have

$$\mathbf{E = B.A.N.\ \omega.\ \sin \theta} \quad \ldots\ldots\ldots\ldots\ldots\ldots \mathbf{IV}$$

Also we have seen $\omega = 2\pi f$.

This gives

$$E = 2\pi f\, BAN \sin \theta.$$

Now E will be a maximum when $\theta = 90°$,

i.e., when $\sin \theta = 1$,

Thus we have

$$\mathbf{E\,(max) = 2\pi f\, BAN} \quad \ldots\ldots\ldots\ldots\ldots\ldots \mathbf{V}$$

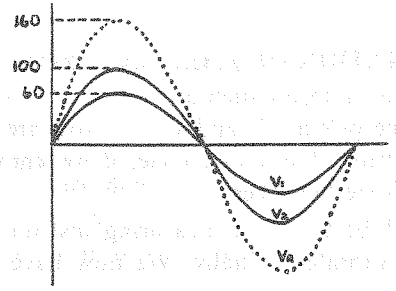
25.6 ADDITION OF ALTERNATING QUANTITIES

If two alternating voltages or currents of the same frequency are exactly in phase then this means they are both zero at the same time, both reach their maximum at the same time and so on.

Their resultant voltage or current is simply the sum of the two.

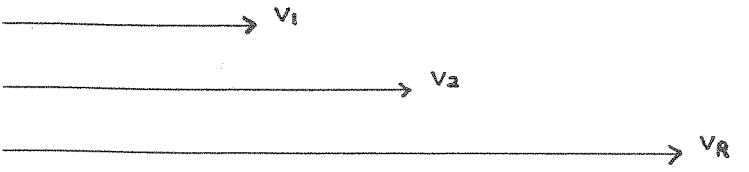
Consider, for example, two alternating voltages with peak values of 60 V and 100 V. The peak value of their resultant will be 160 V.

The following diagram illustrates the situation.

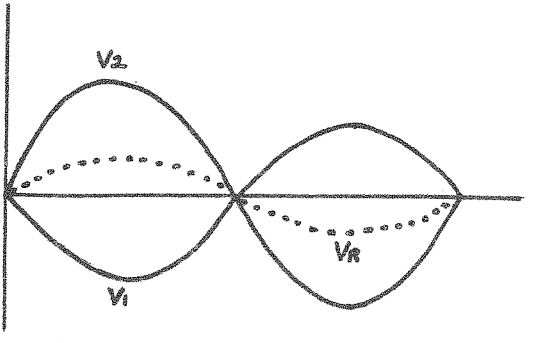


If the two voltages v_1 and v_2 are represented by $v_1 = 60 \sin \omega t$ and $v_2 = 100 \sin \omega t$ then their resultant is given by $v_R = 160 \sin \omega t$.

In terms of vectors we could represent the above as follows:



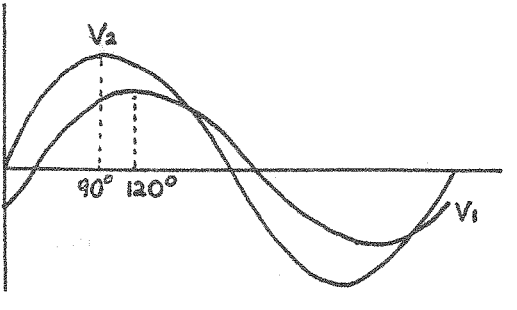
If v_1 and v_2 above were completely out of phase then the situation would be as shown below:



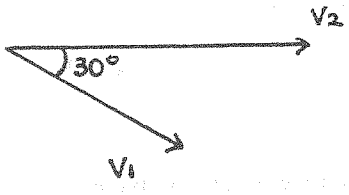
In this case v_1 is 180° (i.e. completely out of phase) out of phase with v_2. This time $v_R = 40 \sin \omega t$ and in terms of vectors we have



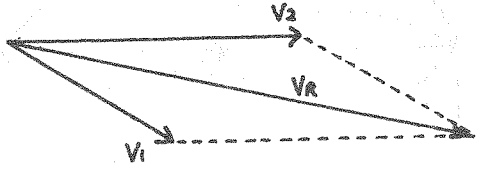
Consider now the following situation



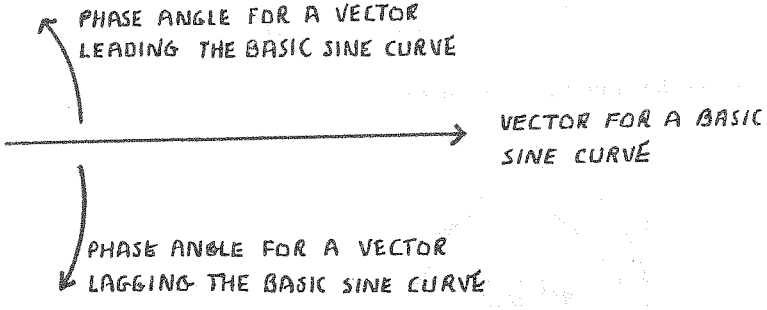
In this case v_2 LEADS v_1 by 30° (or v_1 LAGS v_2 by 30°).
v_2 leads because it reaches its maximum value first.
In terms of vectors we now have



To find v_R, the result of adding v_1 and v_2, we use vector addition as described in Chapter 3.
Thus we have



We can see v_R lags v_2 but leads v_1.
The following diagram summarises phase angle:



Note that the work of this section only applies for alternating quantities which have the SAME FREQUENCY.

25.7 THREE PHASE ALTERNATING CURRENTS

Modern marine power systems employ three phase generators; these produce three separate sinusoidal voltages from their stator windings, identical in magnitude and frequency, but differing in phase by one-third of a cycle, or 120°.

The advantages of such a three-phase system of generation are:

(*a*) Less copper is required for distribution.

(*b*) A choice of two supply voltages is available, "phase" volts for lighting and domestic purposes (usually 240 V) and "line" volts, the vector sum of two phase voltages (usually 415 V, i.e., if the phase voltage is 240 V) for power purposes.

(*c*) Such a system of three-phase voltages applied to three identical field windings spaced 120° apart produce a magnetic field which is constant in magnitude but which rotates in space at a constant angular velocity.

25.8 THE INDUCTION MOTOR

The rotating magnetic field mentioned above makes possible a simple, robust, A.C. motor having no slip rings or commutator or brush gear — the induction or "squirrel cage" motor.

The field system is a cylindrical, slotted stator of iron laminations, whilst the rotor resembles a D.C. armature with the windings replaced by heavy copper bars short circuited together at each end of the rotor. The rotating flux cuts the rotor conductors, inducing in them alternating e.m.f.s and the resulting rotor currents interact with the rotating flux to produce a rotor torque. As a result, the rotor turns in the same direction as the rotating flux, but never achieves the latter's "synchronous speed", for if it did, the cutting action of the flux would cease and the torque would disappear. Rotor speed is about 96% of the synchronous speed and could be of the order of 2880 revs/min.

The superior qualities of a motor having no connections to its rotor, but relying instead on induction, are obvious. Moreover sophisticated induction motors can be designed which require no starting arrangements but start "direct on line"; that is, power to the motor may be supplied direct from the mains, unlike D.C. motors, which require a starting resistance to prevent too large a current going through the armature coils at the instant of switching on.

25.9 A.C. CIRCUITS

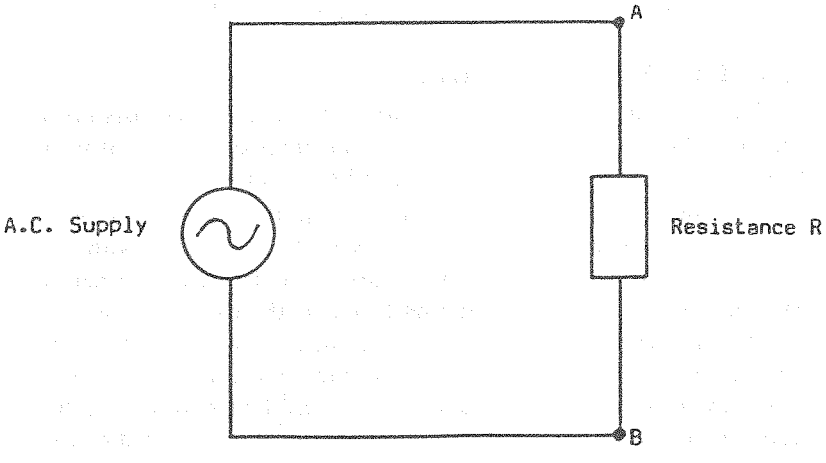
In the remainder of this chapter we shall consider what happens when an alternating voltage is applied to a circuit which contains, in the first instance, a pure resistance; secondly a pure capacitance and thirdly a pure inductance.

Combinations of resistors, capacitors and inductors are then considered.

It should be noted that this work relies heavily on the use of vectors.

25.10 RESISTIVE CIRCUIT

First we shall consider an alternating voltage connected across a resistor.



Suppose that the alternating voltage applied across AB is given by

$$v = V_{max} \cdot \text{Sin wt} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad \text{VI}$$

where v is the potential difference between A and B at a given instant.

The current at the same instant will be given by Ohm's law:

$$V = IR$$

$$\therefore \quad I = \frac{V}{R}$$

Substituting from (1)

$$i = \frac{V_{max} \cdot \text{Sin wt}}{R} \quad \text{VII}$$

Similarly

$$I_{max} = \frac{V_{max}}{R} \quad \text{VIII}$$

Therefore by substituting from (VIII) into (VII) we obtain

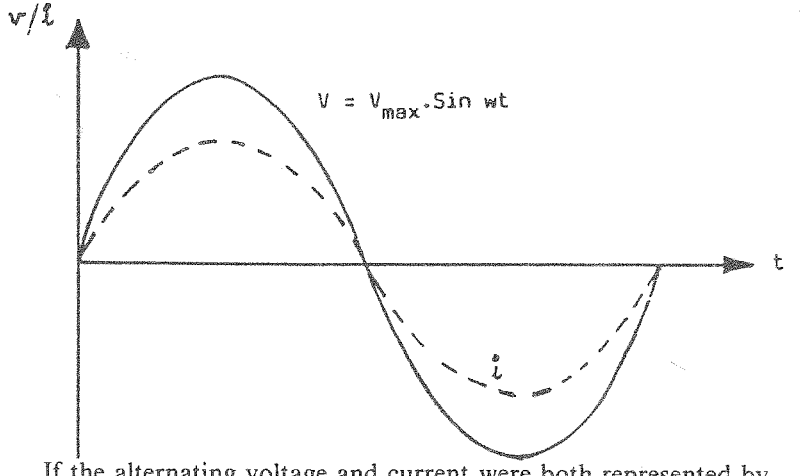
$$i = I_{max} \cdot \text{Sin wt} \quad \text{IX}$$

We started by saying

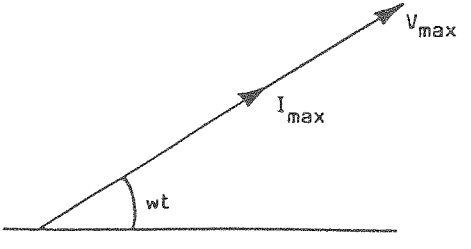
$$v = V_{max} \cdot \text{Sin wt}$$

Thus in a purely resistive circuit the voltage and current vary with Sin wt.

Therefore, the voltage and current are **IN PHASE**. This is illustrated below.

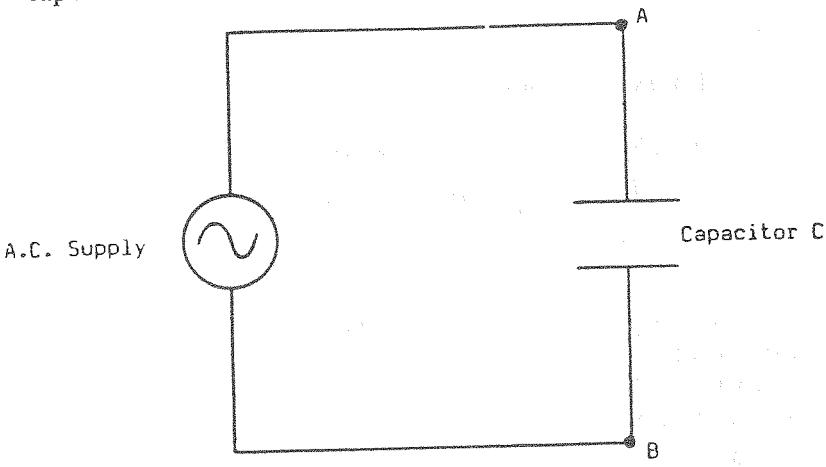


If the alternating voltage and current were both represented by vectors on the same diagram we obtain the following:



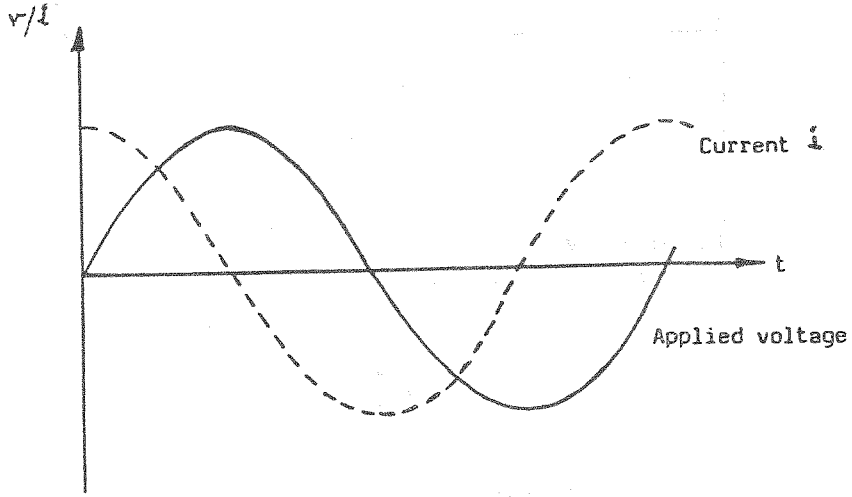
25.11 CAPACITIVE CIRCUIT

We shall now consider an alternating voltage connected across a capacitor.



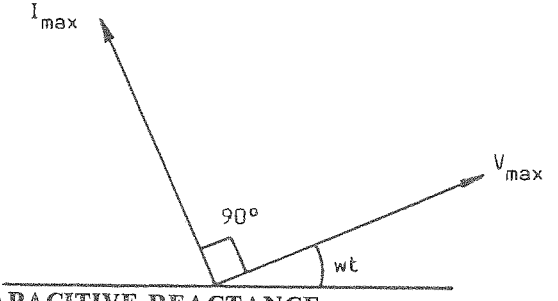
When an alternating voltage $v = V_{max}.\text{Sin wt}$ is applied to the capacitor, its charge Q will rise and fall in unison with the voltage. However a **PHASE DIFFERENCE** occurs between the applied voltage and the current. This is shown below.

Voltage and current out of phase



Because the current reaches its maximum value before the voltage it is said that the current **LEADS** the voltage. In fact the two waves are 90° out of phase.

Thus the **current leads the voltage by 90°** and the vector diagram is as follows.



25.12 CAPACITIVE REACTANCE

An alternating current can, in effect, pass through a capacitor. However the capacitor does offer considerable opposition to the passage of the alternating current. This oppoosition is called **CAPACITIVE REACTANCE.**

Since capacitive reactance resists or impedes the flow of current, it is measured in ohms and is denoted by the symbol X_C.

The value of X_C varies accordingly to the frequency f of the alternating voltage supply. In fact as the frequency is increased so the value of X_C decreases. Also, as one might suspect, the value of X_C depends on the value of the capacitance C. Again the greater the value of C the smaller is X_C.

Thus we have

$$X_C \alpha \frac{1}{f} \qquad X_C \alpha \frac{1}{C}$$

Combining these we have

$$X_C \alpha \frac{1}{fC}$$

$$X_C = \text{constant} \times \frac{1}{fC}$$

The value of the constant is in fact $\frac{1}{2\pi}$

$$X_C = \frac{1}{2\pi fC} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots X$$

WORKED EXAMPLE 1

Find the capacitive reactance and applied voltage across a capacitance of 50μF when it is connected to an alternating supply of frequency 40 Hz and an R.M.S. current of 3 A is obtained.

$f = 40$ Hz $\quad C = 50 \times 10^{-6}$C $\quad I = 3$ A

$$X_C = \frac{1}{2\pi fC}$$

$$= \frac{1}{2\pi \times 40 \times 50 \times 10^{-6}} \ \Omega$$

$$X_C = 79{\cdot}6\ \Omega$$

Now the equivalent of Ohm's law can be applied, namely

Voltage = current $\times$ capacitive reactance

(capacitive reactance: instead of resistance)

$$V = I.X_C$$

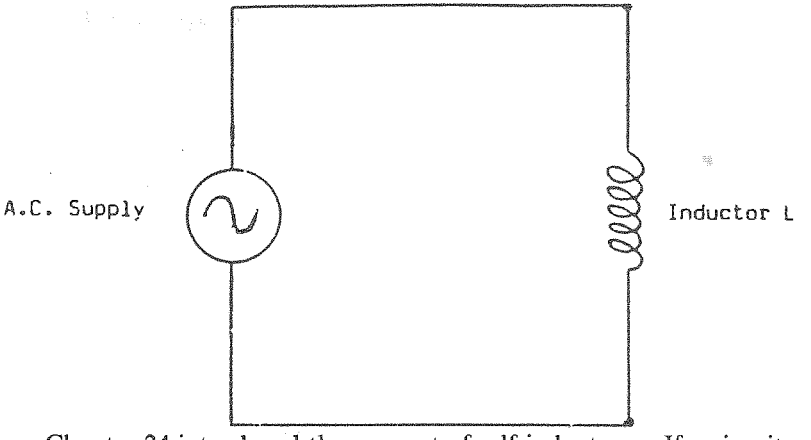
$$\therefore \quad V = 3 \times 79{\cdot}6 \text{ V}$$

$$V = 239 \text{ V} \quad \text{(to 3 sig. figs.)}.$$

EXERCISE A

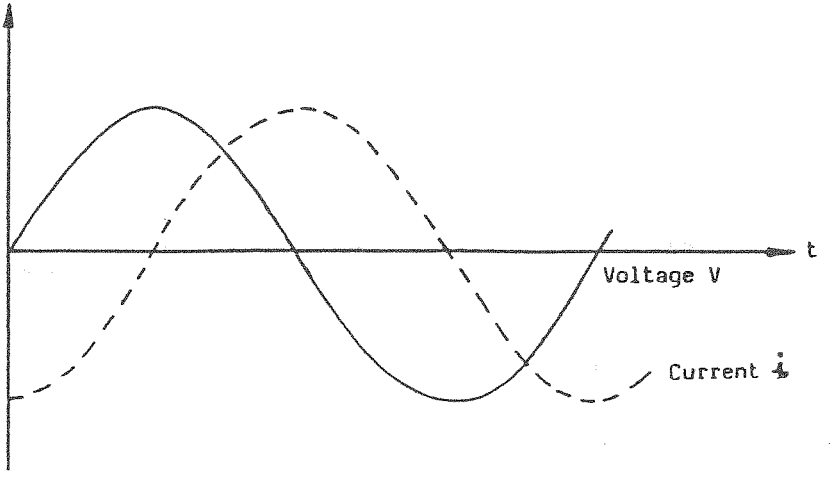
1. Find the capacitive reactance when a 60μF capacitor is connected to a 240 V (R.M.S.), 50 Hz supply. Determine the R.M.S. current.
2. A 40 μF capacitor has a reactance of 70Ω when connected to an A.C. supply. Find the frequency of this supply.
3. When a capacitor is connected to an A.C. supply of frequency 50 Hz and R.M.S. voltage 250 V, an R.M.S. current of 4 A flows. Determine the value of the capacitance.

25.13 INDUCTIVE CIRCUIT

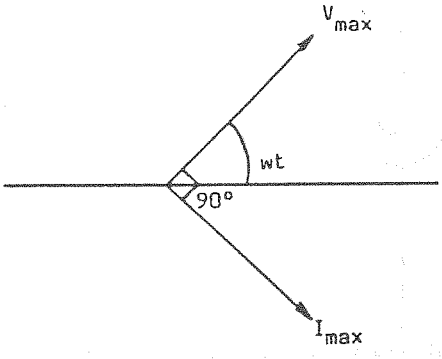


Chapter 24 introduced the concept of self-inductance. If a circuit produces a back e.m.f. or induced voltage, when there is a change of magnetic flux in the circuit there is said to be INDUCTANCE. An inductor is any component that is deliberately used because it possesses this property of inductance. In practice an inductor is a coil; hence the representation in the diagram above.

As in the case of a capacitor, when an alternating voltage $v = V_{max}$. Sin wt is applied to an inductor a PHASE DIFFERENCE occurs between the voltage and the current. This is illustrated below.



Unlike the capacitor case, however, this time the voltage reaches its maximum value first. Thus this time the voltage LEADS the current by 90°.



25.14 INDUCTIVE REACTANCE

We have already seen that a capacitive circuit offers opposition to, or impedes, the current. Similarly an inductive circuit offers opposition. This INDUCTIVE REACTANCE is denoted by X_L.

Once again the value of X_L varies accordingly to the frequency but this time as the frequency increases so does the value of X_L. Also the value X_L depends on the value L of the inductance. Again the greater the value of L the greater is X_L. Thus we have

$X_L \propto f$ $\quad$ $X_L \propto L$

$X_L \propto fL$

$X_L = \text{constant} \times fL$

The constant has a value of 2π

$$X_L = 2\pi fL \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad \text{XI}$$

WORKED EXAMPLE 2

An alternating voltage of 240 V and frequency 50 Hz is applied to an inductance of 5H. Find its inductive reactance. Hence find the current.

$V = 240$ V $\quad$ $f = 50$ Hz $\quad$ $L = 5$ H

$X_L = 2\pi fL$

$= 2\pi \times 50 \times 5\ \Omega$

$= 1571\ \Omega$ (to 3 sig. figs.).

Now again the equivalent of Ohm's law can be applied, namely
voltage = current × inductive reactance

$$V = I.X_L$$
$$240 = I \times 1571$$
$$I = 0{\cdot}153 \text{ A } (153 \text{ mA})$$

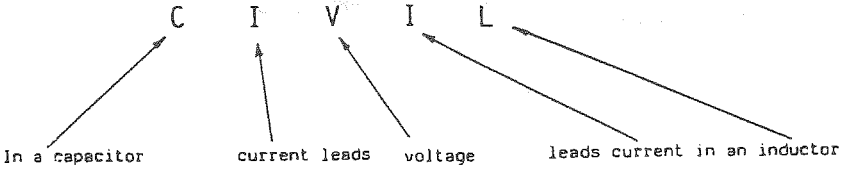
NOTE: No information is given about the applied voltage being a peak or R.M.S. value. If, however, we assume it is an R.M.S. value then so also will be the current value calculated.

EXERCISE B

1. Find the inductive reactance when a 6 H inductor is connected to a 240 V (R.M.S.), 50 Hz supply. Determine the R.M.S. current.
2. A 5 H inductor has a reactance of 1200 Ω when connected to an A.C. supply. Find the frequency of this supply.
3. When an inductor is connected to an A.C. supply of frequency 50 Hz and R.M.S. voltage 250 V, a current of 0·5 A flows.
   Determine the value of the inductance.

25.15 REMEMBERING

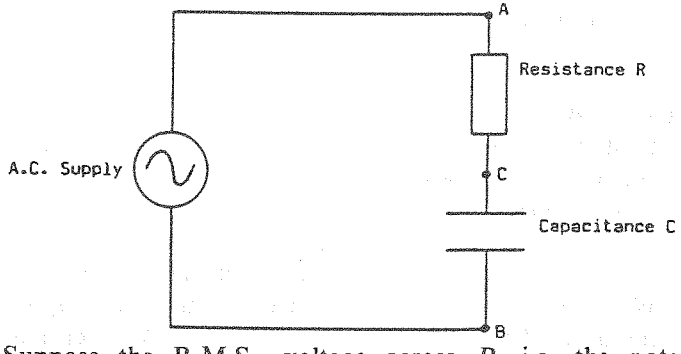
To remember whether voltage or current leads the others in a capacitive or inductive circuit, some people use the following mnemonic (aid to memory).



25.16 SERIES CIRCUITS

So far we have considered what happens when an alternating voltage is applied to either a pure resistance or a capacitance or an inductance. The next step is to consider series combinations of these, since any circuit containing a capacitance or an inductance must also have a certain amount of resistance. The current will be affected by both this resistance and the capacitance or inductance.

25.17 C-R CIRCUIT



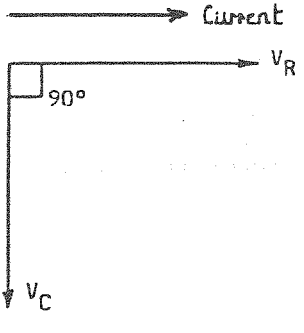
Suppose the R.M.S., voltage across R, i.e. the potential difference (p.d.) between A and C is denoted by V_R whilst that across C (p.d. between C and B) is denoted by V_C.

We have seen in section 25.10 that the voltage and current for a resistor are in phase. We have also seen in section 25.11 that the current in a capacitor leads the voltage by 90°.

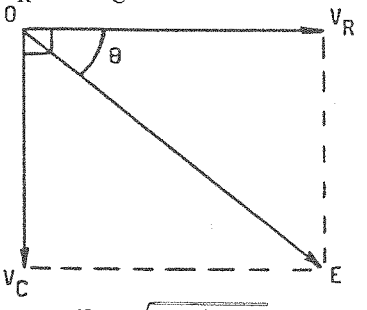
Now the current through the resistor and capacitor will be the same.

Thus the voltage in the resistor must lead that in the capacitor by 90°.

Thus we have



We can now do our normal vector addition to find the sum of V_R and V_C.



OE = Resultant Voltage (E)

$$E = \sqrt{V_R^2 + V_C^2}$$ (E will be an R.M.S. value).

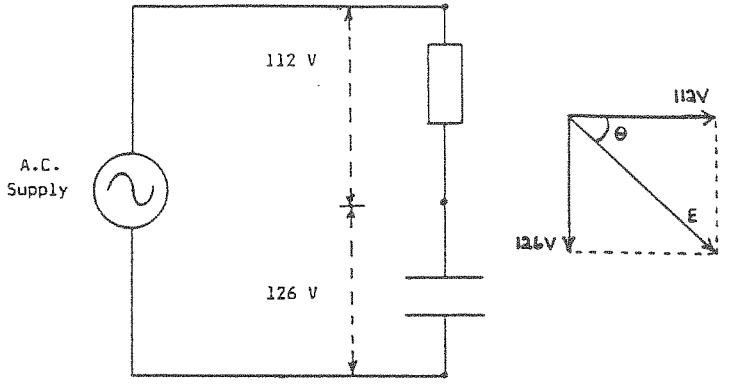
and also

$$\tan\theta = \frac{V_C}{V_R}$$

Thus we can find the resultant voltage E and its phase angle θ.

WORKED EXAMPLE 3

Find the resultant voltage and its phase angle in the following circuit.



$$E = \sqrt{112^2 + 126^2}$$

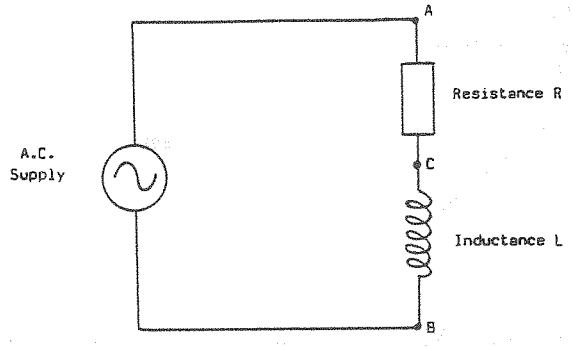
$E = 169$ V (to 3 sig. figs.).

$$\tan\theta = \frac{126}{112}$$

$\theta = 48° 22'$

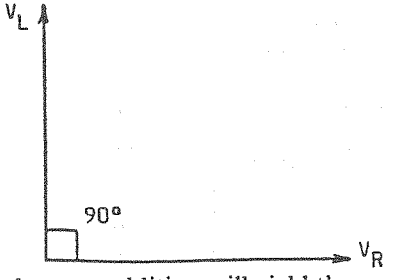
Voltage LAGS current by 48° 22'.

25.18 L-R CIRCUIT

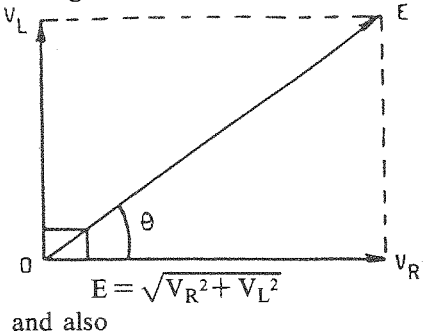


Let voltage A to C be V_R.
and voltage C to B be V_L.

Again for the resistor the current and voltage are in phase whilst for the inductance the voltage leads the current by 90°. The current through R and L will be the same. Therefore V_L must lead V_R by 90°.



Again the usual vector addition will yield the sum of V_L and V_R.



OE = resultant voltage (E)

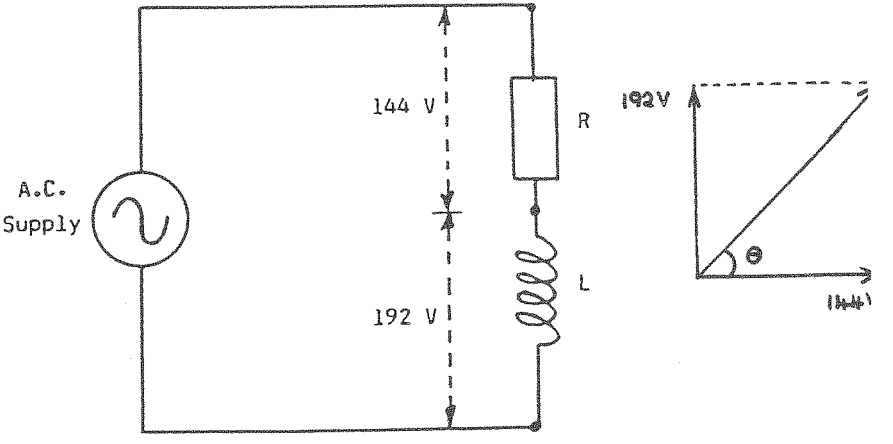
$$E = \sqrt{V_R^2 + V_L^2}$$

and also

$$\tan\theta = \frac{V_L}{V_R}$$

WORKED EXAMPLE 4

Find the resultant voltage and phase angle in the following circuit.



$$E = \sqrt{144^2 + 192^2}$$

$$E = 240 \text{ V}$$

$$\tan \theta = \frac{192}{144}$$

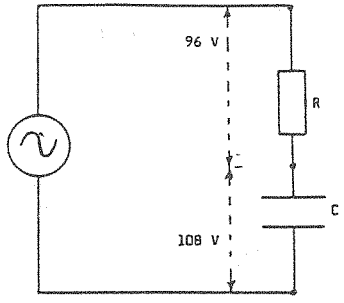
$$\theta = 53° 08'$$

The voltage LEADS the current by 53° 08′.

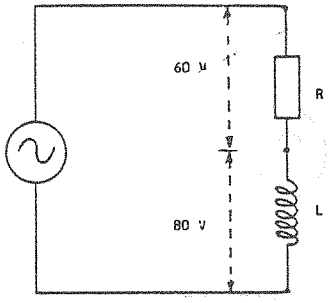
EXERCISE C

Find the resultant voltage and its phase angle in each of the following circuits.

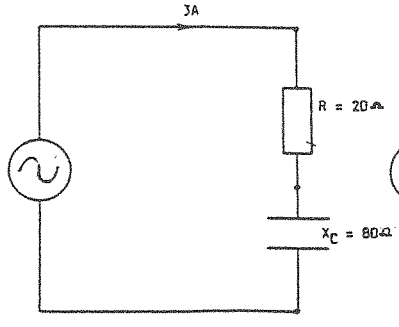
1



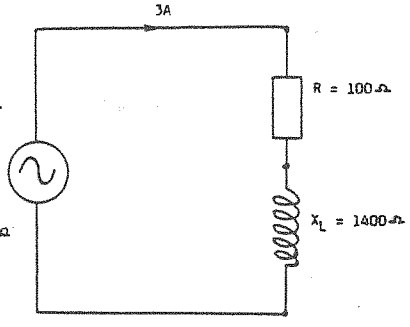
2



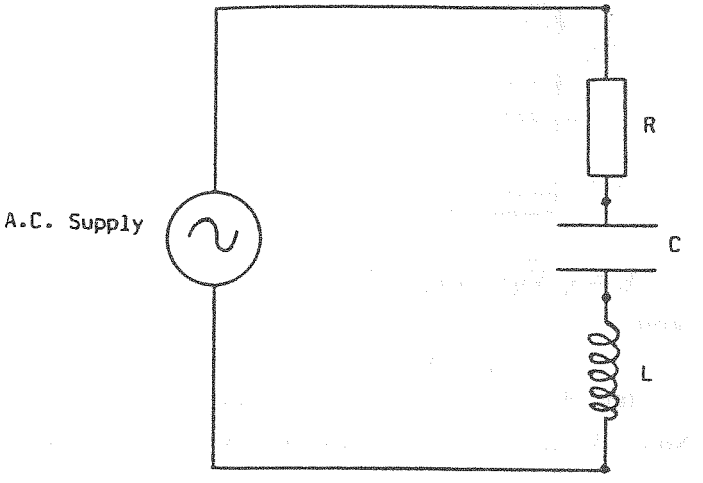
3



4



25.19 L-C-R CIRCUIT



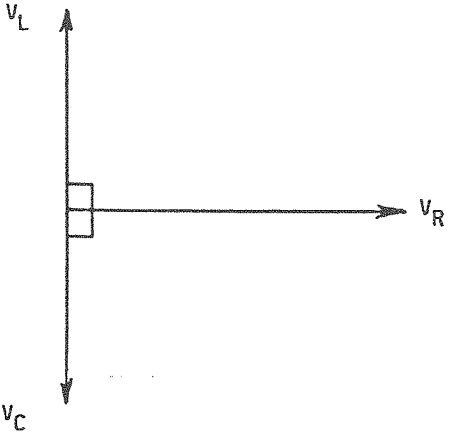
Exactly the same reasoning applies as in the previous series circuits.

The voltage V_R across the resistor is in phase with the current.

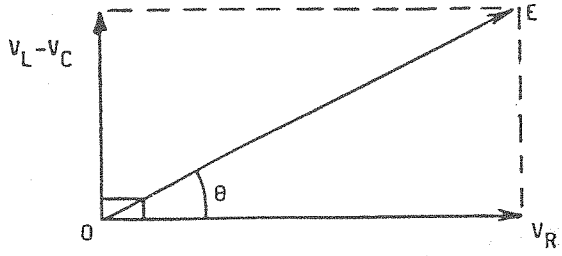
The voltage V_C across the capacitor lags behind the current by 90°.

The voltage V_L across the induction leads the current by 90°.

Thus we have:



Suppose for example, V_L is larger than V_C, then the diagram above can be reduced to:



$$\mathbf{E = \sqrt{V_R^2 + (V_L - V_C)^2}} \quad \ldots\ldots\ldots\ldots\ldots \textbf{XII}$$

and also

$$\mathbf{\tan\ \theta = \frac{V_L - V_C}{V_R}} \quad \ldots\ldots\ldots\ldots\ldots\ldots \textbf{XIII}$$

Now if V_C had been bigger than V_L we would have simply had

$$E = \sqrt{V_R^2 + (V_C - V_L)^2}$$

and

$$\tan\ \theta = \frac{V_C - V_L}{V_R}$$

25.20 IMPEDANCE IN AN L-C-R CIRCUIT

The resistor, capacitor and inductor present opposition to the flow of current in the L-C-R series circuit. This opposition is called IMPEDANCE.

Any impedance is given by the equivalent of Ohm's law

potential difference = current × impedance

Thus for our L-C-R circuit we have

$E = I \times Z$ where Z is the impedance of the circuit

But

$$E = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$\therefore \quad IZ = \sqrt{V_R^2 + (V_L - V_C)^2}$$

But

$$V_R = IR, \qquad V_C = IX_C, \qquad \text{and} \qquad V_L = IX_L$$

$$\therefore \quad IZ = \sqrt{(IR)^2 + (IX_L - IX_C)^2}$$

$$IZ = \sqrt{I^2R^2 + (I(X_L - X_C))^2}$$

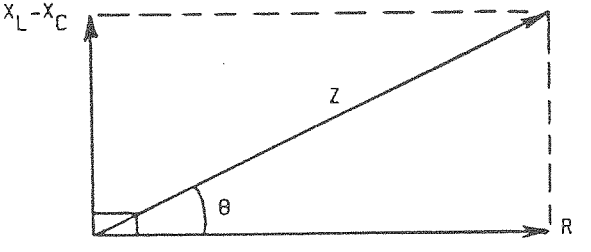
$$= \sqrt{I^2R^2 + I^2(X_L - X_C)^2}$$

$$= \sqrt{I^2(R^2 + (X_L - X_C))^2}$$

$$= I\sqrt{R^2 + (X_L - X_C)^2}$$

$$\mathbf{Z = \sqrt{R^2 + (X_L - X_C)^2}} \quad \ldots\ldots\ldots\ldots\ldots \quad \mathbf{XIV}$$

NOTE This is a Pythagoras-type relationship and can be represented by a vector type diagram.

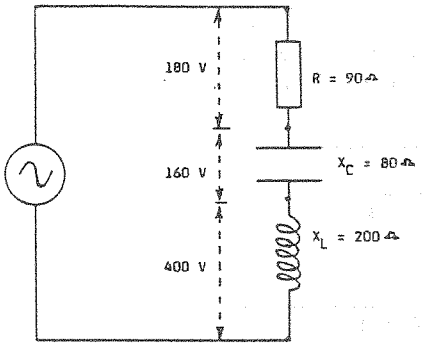


$$\tan \theta = \frac{X_L - X_C}{R}$$

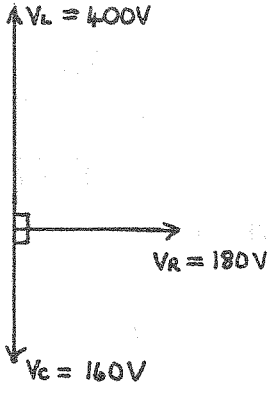
where θ is the phase angle between the resistance and the total impedance of the circuit.

WORKED EXAMPLE 5

What is the applied e.m.f. (which is the resultant voltage), the impedance, the phase angle and the current flowing in the following circuit?



$$E = \sqrt{V_R^2 + (V_L - V_C)^2}$$
$$= \sqrt{180^2 + (400 - 160)^2}$$
$$E = 300 \text{ V}$$



$$Z = \sqrt{V_R^2 + (X_L - X_C)^2}$$
$$= \sqrt{90^2 + (200 - 80)^2}$$
$$Z = 150\ \Omega$$
$$\tan\ \theta = \frac{X_L - X_C}{R}$$
$$= \frac{120}{90}$$
$$\theta = 53°\ 08'$$

For the circuit as a whole

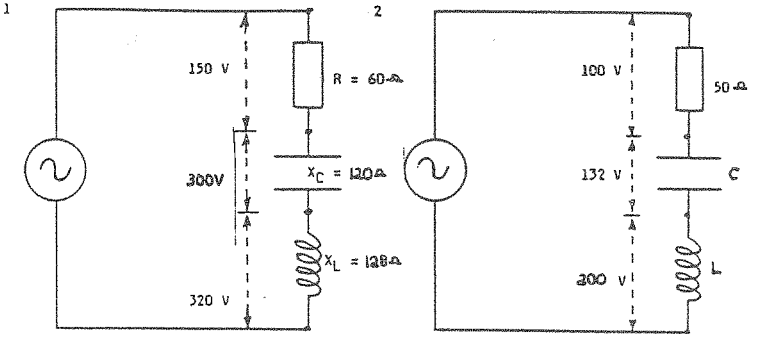
$$E = IZ$$
$$I = \frac{E}{Z}$$
$$= \frac{300}{150}\ A$$
$$I = 2\ A$$

Note that we could have obtained the current by applying Ohm's law to any of the three components R, C or L.

EXERCISE D

In each of the following circuits find the applied e.m.f., the impedance, the phase angle and the current flowing.

For question 2 find also the values of C and L.



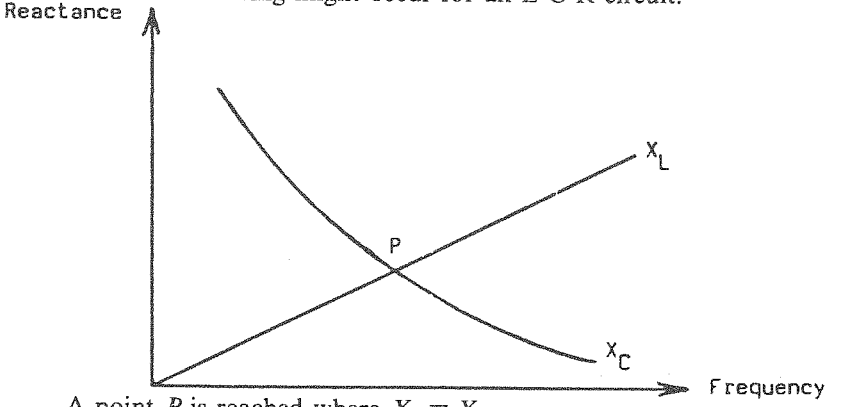
Supply frequency = 50 Hz.

25.21 RESONANCE

It was stated in section 25.12 that the reactance X_C of a capacitor decreases with frequency increase.

It was stated in section 25.14 that the reactance X_L of an inductor increases with frequency increase.

Thus the following might occur for an L-C-R circuit:



A point P is reached where $X_L = X_C$.

At this point the net reactance of the circuit is zero and the current in the circuit attains a maximum value. This condition is known as RESONANCE.

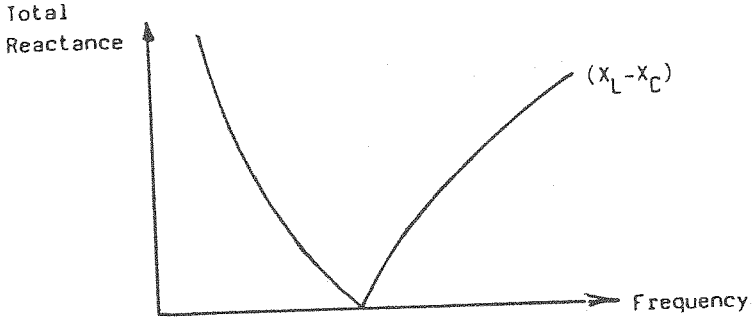
For an L-C-R series circuit $Z = \sqrt{R^2 + (X_L - X_C)^2}$

but for resonance $X_L = X_C$ i.e. $X_L - X_C = 0$

$$\therefore \quad Z = \sqrt{R^2}$$

$$Z = R$$

NOTE When resonance occurs the impedance in the circuit is purely resistive and at a minimum.



$$X_C = \frac{1}{2\pi fC} \qquad \text{(section 25.12)}$$

$$X_L = 2\pi fL \qquad \text{(section 25.14)}$$

At resonance $X_C = X_L$ and let $f = f_r$

$$\therefore \frac{1}{2\pi f_r C} = 2\pi f_r L$$

$$1 = 4\pi^2 f_r^2 CL$$

$$\frac{1}{4\pi^2 CL} = f_r^2$$

$$\therefore \quad \textbf{Resonant frequency } \mathbf{f_r} = \frac{1}{2\pi\sqrt{LC}} \cdots\cdots \textbf{XV}$$

The phase angle is given by $\tan\theta = \frac{X_L - X_C}{R} = \frac{0}{R} = 0$

$$\therefore \quad \theta = 0°$$

There is no phase difference between the voltage and the current as one would expect in a circuit which has become purely resistive.

To sum up: at the resonant frequency the circuit behaves as though it contained resistance only, even though there is an inductance and capacitance present.

Such a circuit can be used in radio reception because it resonates to an e.m.f. set up in the receiving aerial by the incoming signal.

WORKED EXAMPLE 6

Calculate the resonant frequency for a circuit which contains a 3 mH inductance in series with a $2\mu F$ capacitor.

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

$$= \frac{1}{2\pi\sqrt{3\times 10^{-3}\times 2\times 10^{-6}}}$$

$$= 2050 \text{ Hz} \qquad \text{(to 3 sig. figs.).}$$

25.22 POWER IN A.C. CIRCUITS

$$\text{Electrical Power} = \text{Current} \times \text{Voltage}$$

$$P = IV$$

$$\text{or instantaneous power } p = iv$$

In a pure resistor the current and voltage are in phase, thus at a given instant they are either both positive or both negative. Therefore the power consumed is always positive $[(+i)\times(+v)]$ or $[(-i)\times(-v)]$.

However the same is not true for the power in a pure capacitor or inductor. In either case the current and voltage are 90° out of phase with each other. Thus there will be times when we have positive current and a negative voltage or vice-versa. The product of current and voltage will then be negative

$$\text{i.e. } p = (+i)\times(-v)$$

$$\text{or } p = (-i)\times(+v)$$

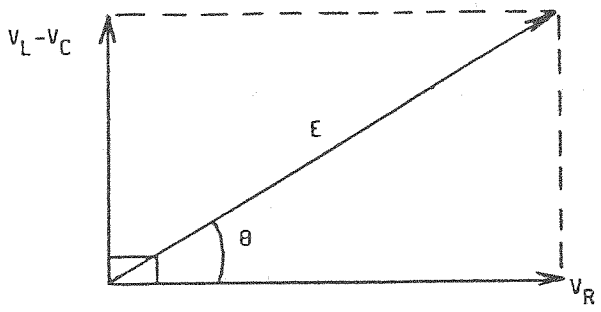
This means the power consumed is negative, which we take to mean that power is returned from the circuit. At other times of course the power consumed is positive. In a pure capacitor or inductor the power consumed (positive) will equal the power returned (negative), i.e. the net power consumption is zero.

However, in practice, actual capacitors and inductors are not pure capacitances and inductances. They do have some resistance. As a result the current and voltage are no longer out of phase by 90° but by some smaller angle. This has the effect that the positive power parts of the cycle do not equal the negative power parts. In fact the positive power is greater, indicating that power is consumed, i.e. the net power consumption is no longer zero.

The consumption of power will take place only in the resistive parts of the circuit.

$$\mathbf{P = I_R \cdot V_R} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad \textbf{XVI}$$

Now we have seen that for an L-C-R series circuit we have the following voltage vector diagram.



From this we can see

$$\cos\theta = \frac{V_R}{E}$$

$$\therefore \quad V_R = E\cos\theta$$

Substituting this in (XVI) we have

$$P = I_R \cdot E\cos\theta$$

This is the actual or REAL power consumed in an A.C. circuit.

However, the APPARENT power drawn from the A.C. supply is given by

$$\text{Power} = \text{Current} \times \text{Applied Voltage}$$

$$\text{Pa} = I_R \times E$$

$$\therefore \quad \frac{\text{Real Power}}{\text{Apparent Power}} = \frac{I_R \,.\, E\cos\theta}{I_R E} = \cos\theta$$

This ratio is called the POWER FACTOR of the circuit.

Looking back to the diagram in section 25.20 we can see that

$$\cos\theta = \frac{R}{Z}$$

$\therefore$ POWER FACTOR is the ratio of resistance to impedance

NOTE When there is no reactance $Z = R$ and $\cos\theta = 1$

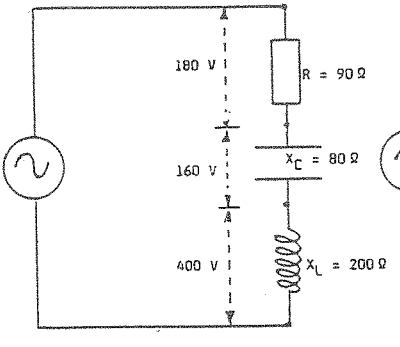
i.e. the power factor is 1 (corresponding to $\theta = 0°$).

When transmitting electrical energy in the form of A.C. along cables, it is desirable to keep the power factor as close to 1 as possible. This means zero phase angle, zero reactance and for the circuit to look to the supply exactly like a pure resistance. We have seen that this condition is called RESONANCE and is possible to achieve.

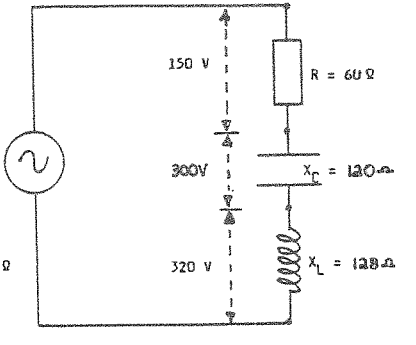
EXERCISE E

Find the power factor in the following circuits:

1



2.



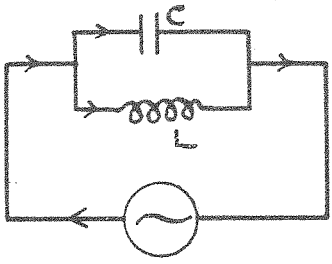
25.23 SUMMARY

	CIRCUIT	FORMULA	NOTE
(1)	Capacitive	$X_C = \frac{1}{2\pi fC}$	Current leads voltage by 90°
(2)	Inductive	$X_L = 2\pi fL$	Voltage leads current by 90°
(3)	C-R	$E = \sqrt{V_R^2 + V_C^2}$	
(4)	L-R	$E = \sqrt{V_R^2 + V_L^2}$	
(5)	L-C-R	$E = \sqrt{V_R^2 + (V_L - V_C)^2}$ $Z = \sqrt{R^2 + (X_L - X_C)^2}$	
(6)	RESONANCE in L-C-R	$X_L = X_C$ $f_r = \frac{1}{2\pi\sqrt{LC}}$	Circuit is purely resistive
(7)	POWER FACTOR in L-C-R	Power factor $= \cos\theta = \frac{R}{Z}$	

25.24 THE A.C. CIRCUIT IN PARALLEL

Take first the simplest case.

Consider the circuit shown, which contains a capacitor and an inductance in parallel and is assumed to have negligible resistance.



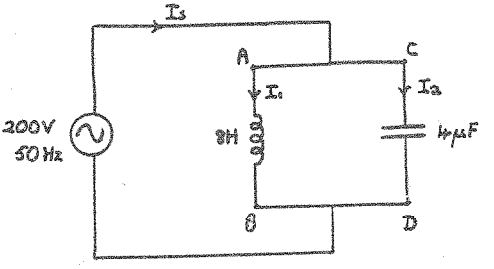
For such a circuit, it can be shown that the impedance Z is given by

$$Z = \frac{1}{\frac{1}{\omega L} \sim \omega C} \quad \text{..... XVII}$$

Since there is no resistance, the currents in the two branches will be in antiphase. Hence, the *magnitude* of the main current will be the arithmetical difference of the branch currents, and its *phase angle* will lag by 90° if the current through the inductance predominates, and conversely.

WORKED EXAMPLE 7

In the circuit illustrated find the current in each branch and the supply current.



Both branches, AB and CD, have the same p.d., 200 V, across them because they are in parallel.

The supply current I_S will be the vector addition of I_1 and I_2.

Using Ohm's Law for the inductor we have

$$200 = I_1 \times X_L$$
$$200 = I_1 \times 2\pi fL$$
$$200 = I_1 \times 2\pi \times 50 \times 8$$
$$I_1 = 79{\cdot}58\ mA$$

Using Ohm's Law for the capacitor we have

$$200 = I_2 \times X_C$$
$$200 = I_2 \times \frac{1}{2\pi fC}$$
$$200 = I_2 \times \frac{1}{2\pi \times 50 \times 4 \times 10^{-6}}$$
$$I_2 = 251{\cdot}3\ mA$$

I_1 and I_2 are 180° out of phase with each other (i.e. antiphase).

Thus $I_S = I_2 - I_1 = 171{\cdot}7\ mA$.

Note that since I_2 is bigger than I_1 the supply current I_S will be capacitative in character. This means it will LEAD the supply voltage.

25.25 RESONANCE IN A PARALLEL CIRCUIT

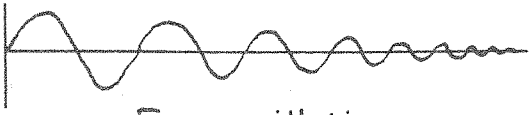
From the form of equation XVII, we can see that the impedance becomes infinite if $1/\omega L = \omega C$.

$$\text{i.e., if } \omega = \frac{1}{\sqrt{LC}} \text{ and } f = \frac{1}{2\pi\sqrt{LC}} \ldots\ldots\ldots \text{ N.B.}$$

This is resonance in a parallel circuit of zero resistance.

This expression for resonant frequency is the same as that already derived for the series circuit, but note that this time, it makes the impedance very large and therefore, the external current, that is, the current passing through the circuit as a whole, negligible. Such a circuit is called a **rejector circuit.**

Note that although at resonance the impedance is infinite, there is still current flowing in the two branches of the circuit, as a result of the condenser charging and discharging through the inductance. Such "circulations" are called "oscillatory currents".

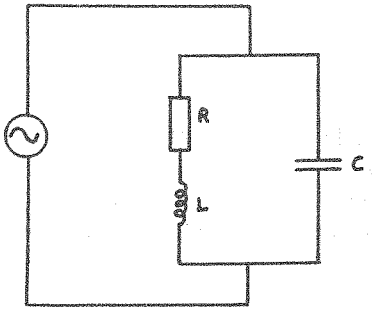


Free oscillation.

Ignoring resistance, as we have done here, the type of oscillation is called "**free oscillation**". The oscillations would die away because, in fact, there is always some resistance.

25.26 PARALLEL CIRCUIT WITH RESISTANCE

Capacitors can be made almost perfect, but all inductances have resistance. In such cases, the whole problem becomes much more involved and we can only state the result without offering any derivation. We shall deal only with the case of such a circuit when in resonance.



For resonant frequency in such a circuit, we have,

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} \quad \text{XVIII}$$

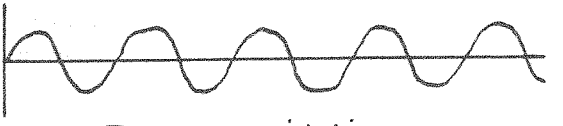
This is the general formula for resonant frequency in a parallel circuit.

And finally,

$$Z = \frac{L}{CR} \quad \text{XIX}$$

which is the impedance of a parallel circuit at resonance.

It will be noted that in **XVIII**, if we put $R = O$, we get $f = 1/2\pi\sqrt{LC}$ as before, and in **XIX**, if $R = O$, $Z =$ infinity.



Forced oscillation.

As given by **XVIII**, this is the frequency at which there will be oscillatory currents in the two branches. This time, however, since R is taken into account, there will be a make-up current in the external circuit which will keep the oscillatory circuit in operation. Such an arrangement is called **"forced oscillation"**.

It is at this point that we must leave it to more advanced works to take up the tale!

TEST PAPER 25
A.C. Circuits

All currents are A.C. and R.M.S. values unless otherwise stated.

1. What is the reactance of a coil of $1000\mu H$ and negligible R when the applied voltage is *(a)* 50 Hz *(b)* 5000 Hz? What will the current be in each case if the applied voltage is 10 V?
2. A capacitor of $2\mu F$ is in a 200 V 50 Hz circuit. Calculate the reactance and the current in the circuit.
3. The R.M.S. values of e.m.f. and current across a capacitor are 300 V and 15 A respectively. What are their peak values?

4. A resistor of 10050 ohm is in series with an inductance of 20 H and negligible R in a 230 V 50 Hz circuit. Calculate the current in the circuit and the power taken.
5. An inductance of 8 H is in a 240 V 50 Hz circuit. Calculate the reactance and the current in the circuit.
6. A H.F. circuit in which the applied voltage is 265 V at 10^6 Hz, has a 500 ohm resistor in series with a capacitor of 300×10^{-12} F. Find the impedance and current in the circuit.
7. A 200 V 50 Hz supply is applied to an inductance of 8 H in series with a resistor of 500 ohm. What R.M.S. current flows? What power is supplied?
8. A 200 V 50 Hz supply is in series with a capacitor of $2\mu F$ and a resistor of 500 ohm. Find the current in the circuit.
9. An inductance of 8 H with resistance 100 ohm is in series with a $30\mu F$ capacitor and a 2000 ohm resistor in a 200 V 50 Hz circuit. Calculate the impedance, the current in the circuit and the power supplied.

   Find also the resonant frequency of the circuit and the impedance of the circuit at resonant frequency.
10. A 200 V supply is in series with a 100 ohm resistor and an inductance of 8 H and negligible resistance. Calculate the volts drop *(a)* across the resistor and *(b)* across the inductance, if $\omega = 100$.
11. An inductance of 4 H and resistance 200 ohm is in series with a 1000 ohm resistor and a $64\mu F$ capacitor in a 200 V 50 Hz supply. Calculate the current through the generator and the power taken.
12. Calculate the volts drop across each of the three circuit components in Question 11.

    Find also the resonant frequency of the circuit and state the impedance of the circuit at resonant frequency.
13. A $4\mu F$ capacitor is in parallel with a 8 H inductance in a 200 V 50 Hz circuit. By direct method, calculate the impedance and the current through the generator.
14. A $20\mu F$ capacitor is in parallel with an inductance of 8 H and resistance 100 ohm, in a 200 V circuit. Calculate the resonant frequency of the circuit and the impedance at this frequency.

15. A 200 V 50 Hz generator is connected to a $30\mu F$ capacitor in parallel with a 12 H inductance of negligible resistance. Calculate the current in each branch.

    Calculate the impedance for the whole circuit.

16. In the same circuit as Question 15 it is desired to adjust the frequency so as to achieve resonance with impedance = infinity. Calculate the resonant frequency. What is the current in the external circuit now?

17. A 200 V 50 Hz generator is supplying current to a $9H$ inductance of resistance 100 ohm in series with a 1000 ohm resistor. Calculate the volts drop *(a)* across the inductance *(b)* across the resistor.

    What is the current in the circuit?

18. A $12\mu F$ capacitor is in parallel with a $10H$ inductance of zero resistance and is supplied from a 200 V generator. What is the current in each branch when at resonant frequency? What is the external current?

19. Give an expression for the e.m.f. induced in a conductor in the form of a loop when rotated in a magnetic field, the axis of rotation being at right angles to the field.

    A coil consists of 500 turns each of area 80 cm^2 and rotates at 1000 revs/min in a magnetic field of 0·3 Wb/m^2, the axis of rotation being perpendicular to the field. Find the e.m.f. in the coil when its plane makes 0°, 60° and 90° respectively with the field. In the first named case, give also the R.M.S. value.

20. A $6\mu F$ capacitor is in parallel with an inductance of 6 H and resistance 200 ohm, the whole being in circuit with a 200 V generator. Calculate the resonant frequency and the impedance at this frequency. What is the magnitude of the current in the external circuit?

CHAPTER 26

ELECTRONICS

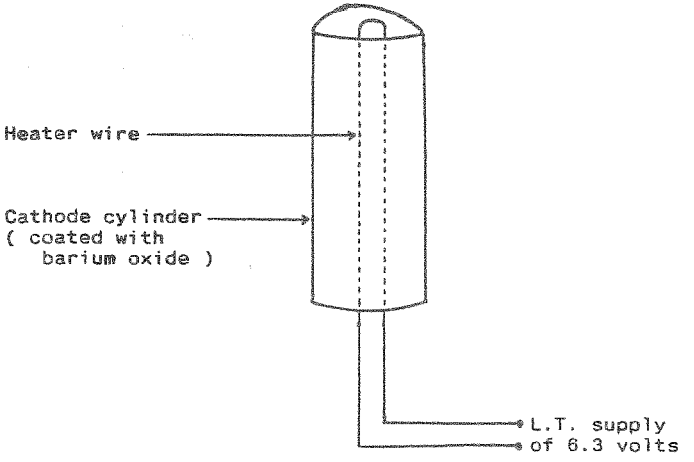
26.1 THERMIONIC EMISSION

So far, the general principles of electricity have been discussed, and, in Chapter 21 we have talked about the construction of the atoms in a conductor, together with a description of the orbital path taken by the electrons about the nucleus. There is one particular sentence one should note, and that is the statement that the motion of free electrons "is quite haphazard and on the whole they do not progress far in any given direction". This of course applies to the conductor before the application of an e.m.f. and is true under normal conditions. A rather interesting phenomenon occurs however, if we apply heat to the conductor. First of all, as the temperature of the conductor rises, the electrons become more and more agitated until what could be described as a violent movement in all directions eventually takes place. After a while, at a certain temperature (depending upon the type of material), the electrons become sufficiently agitated to actually leave the conductor. This process, of emission of free electrons by the application of heat, is called "**thermionic emission**".

Also within the whole field of electricity, that branch of study (itself a wide one) which deals with circuits in which the movement of *free* electrons is involved, is termed **electronics**, thus making some distinction between it and the study of currents which are constrained within the path laid down by a conductor. The borderline is, of course, not a rigid one.

26.2 CATHODE AND SPACE CHARGE

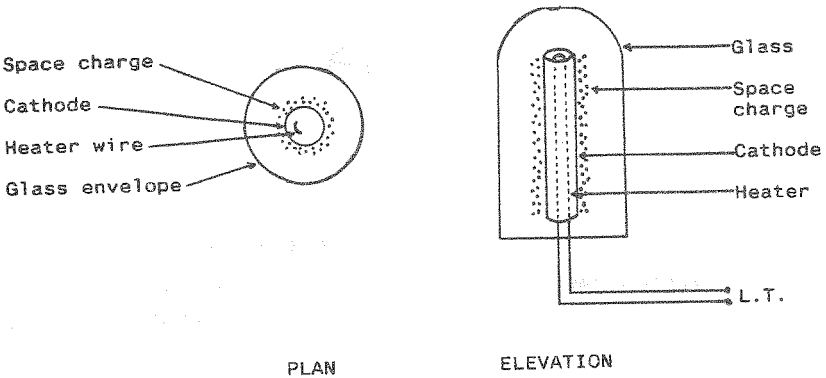
The way in which the heat is applied to the conductor is not of course in such a crude fashion as the application of a flame (although this would work), but by a more sophisticated approach using a heater wire surrounded by a cylinder.



Cathode and heater.

The heater wire is connected to a low voltage supply of usually 6·3 volts (known as the **low tension voltage**). As the current flows through the wire it becomes hotter, and the heat so produced is radiated to the cylinder which surrounds it. The cylinder is coated with a suitable substance (barium oxide) which will emit electrons easily at a comparatively low temperature, so that as the temperature rises the free electrons are emitted and form a "cloud" around the outside of the cylinder. The cloud is called a **Space Charge**.

The space charge does not, as might at first be supposed, drift away. When the electrons leave the conductor, the conductor becomes deficient of free electrons. It is then predominately positive and as such is said to be positively charged. This charge will then act as an attractive force to the negative electrons, in opposition to the heat, which is trying to repel them. A balanced state is reached where the two forces are equal and opposite. The cylinder is mounted (with the heater wire) inside an evacuated glass envelope so that no uncontrolled forces can act on the space charge. The charge will remain static around the cylinder until such time as the heat is removed, when the electrons will subside back to the cylinder. The cylinder is called the **Cathode**.



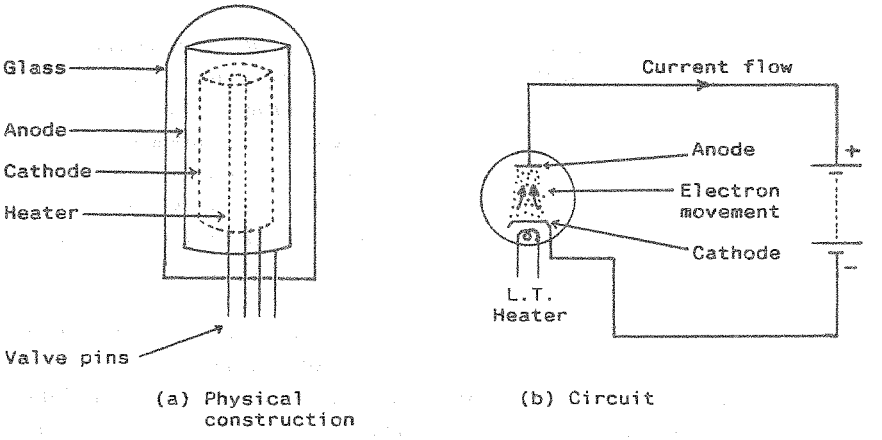
Thermionic emission.

26.3 THE DIODE VALVE

Having seen what we mean by thermionic emission, let us now consider an important application of it, as in the thermionic valve, taking the simplest case first.

A Diode valve consists of an evacuated glass envelope containing a heater together with a cathode. It also contains a second electrode which has been introduced into the envelope, known as an **Anode**. The anode (like the cathode) is also cylindrical in shape and designed to surround the cathode at some distance from it. The physical construction of the valve is shown in the left-hand diagram, but its action is most readily understood by reference to the symbol used and shown in the right-hand diagram.

When the valve is switched on, and after it has warmed up, the cathode emits electrons which give rise to the space charge surrounding the cathode. If, then, a positive potential is applied to the anode, the negative space charge is attracted away from the cathode and the electrons move towards the anode. Electrons are thus flowing from a negative potential towards a positive potential (which is the *true* nature of the flow of an electric current). The battery which provides the potentials is so connected that the positive pole of the battery is connected to the anode, and the negative pole is connected to the cathode. The connection made to the cathode will ensure a continuous supply of electrons as these are used up by the flow taking place.



The diode valve.

It is interesting to see where the name valve comes from. Clearly, if we apply a negative voltage to the anode, no current will flow since the space charge will be repelled. Hence, the valve, like any mechanical valve, will allow a movement to take place in one direction only, and not in the other direction. The word Diode comes from the fact that we are using two electrodes (cathode and anode) only. The heater, essential as it is to the efficient operation of the valve, is not considered as an electrode when naming the valve.

26.4 HALF VALVE AND FULL WAVE RECTIFICATION

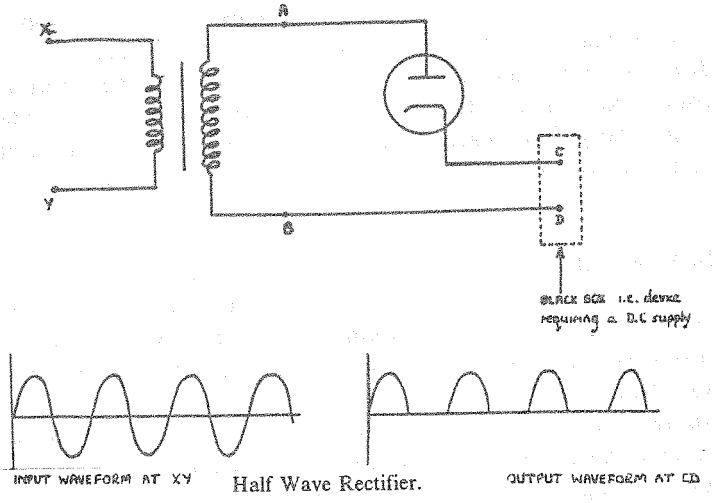
Since the diode valve allows current to flow in one direction only, it has a direct application in converting alternating current into direct current. The ability to do this is advantageous in more ways than one. For example, if we have a low value of alternating voltage and we require a high value of direct voltage, then the low value of A.C. can be increased in value using a transformer and the resulting high value of A.C. can then be converted (or, to use the correct term, "rectified") using a diode valve, into D.C. In effect, this means that A.C. is far more pliable than D.C. — it can easily be changed in value (by a transformer) and it can easily be converted into D.C. should we require it. For this reason, A.C. is used in preference to D.C. in modern ships.

Now, all electronic equipment requires D.C. to power it. The power in a "black box" — whether the box be an ordinary radio set or a highly sophisticated computer — is always provided by D.C. The amount of D.C. required varies according to the equipment. Many transistorised radio receivers these days use only a 9 or 6 volt D.C. supply to operate them, whilst mains receivers may need 220 volts of D.C. to operate them. The unit used to take the A.C. mains, step it up or down, and rectify it to D.C. is called the **Power Pack**. It is as essential to any "black box" as the engine is to a car in providing the power to run.

There are two kinds of power pack. The first is economical to produce and is used whenever possible, but is less efficient than the second. This is the **Half Wave Rectifier** type. The second is the **Full Wave Rectifier** type which is used whenever it is needed. The diagrams below show the two circuits.

26.5 HALF WAVE RECTIFICATION

Consider the Half Wave Rectifier in the diagram below. The



Half Wave Rectifier.

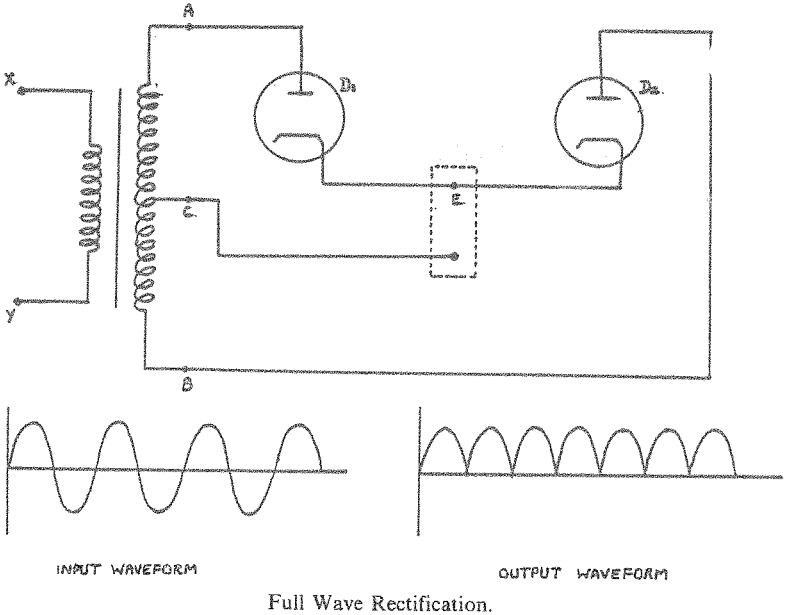
A.C. mains are connected to the primary winding of the transformer at the terminals X and Y. The turns ratio of the transformer governs the A.C. output voltage which appears at the secondary winding terminals A and B. (A 2:1 step up ratio of the transformer will convert the input voltage of 220 volts into 440 volts at the

secondary). The action of the rectifier is considered by working through the two half cycles of the A.C. input. During the first half cycle, let X be + ve and Y be − ve. Then, A is + ve and B is − ve. The anode of the diode is therefore at a positive potential so that it will conduct. Electrons will then flow from B through the black box to A.

During the second half cycle. Y is + ve and X is − ve. Thus, B is + ve and A is − ve. This means that the anode of the diode is at a negative potential and it will not conduct. Thus, current flows through the black box only during alternate half cycles. It flows in one direction only however, so we may say that the unit is a half wave rectifier. As may be imagined, the fact that it only works on alternate half cycles means that it is not very efficient. A graph of the output current is shown underneath the circuit diagram. Note that the heater circuit is not shown on the diagram.

26.6. FULL WAVE RECTIFICATION

By using two diodes, it is possible to convert each half cycle into D.C. and so provide a more efficient rectifier. In the diagram below the diodes are shown as separate valves. It is possible however, to



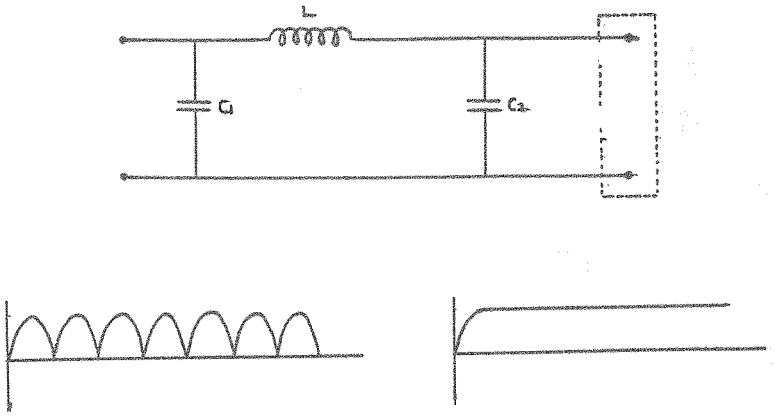
Full Wave Rectification.

use a "double diode" where the two diodes are contained in the one glass envelope, but for simplicity we will consider the action assuming two valves are used. Also, the heater voltage has been omitted for clarity.

The transformer secondary winding has a connection at a mid-point on the winding known as the centre tap. This is connection C in the figure. It is used as the common terminal for both half cycles and is connected as the − ve terminal of the black box. Once again, as in the case of the half wave rectifier, let us consider the action over the full cycle by taking each half cycle in turn. During the first half cycle let X be + ve and Y − ve. This means that A is + ve and B − ve. The important point is that C is also − ve with respect to A, and hence diode D_1 will conduct allowing current to flow through the black box from C to E. During the second half cycle, B is + ve and A is − ve. Thus D_1 is unable to conduct, but D_2 will take its place, C is *still* − ve, but with respect to terminal B this time, so current once again flows through the black box from C to E. A graph of the output current is shown in the figure.

26.7 SMOOTHING CIRCUITS

The output current in both half and full wave rectification is in the form of pulsating D.C. That is to say, the output is not a



Smoothing circuit.

smooth steady flow of current but follows the pattern indicated in the diagram above. This can be improved upon by the use of a smoothing circuit which has the job of taking the pulsations of current and levelling them off into a steady flow through the black box.

There are one or two different types of smoothing circuits which are used by manufacturers but they all fall into the category of the circuit illustrated in the diagram.

In this circuit; there are two capacitors, C_1 and C_2, and an inductance coil. The capacitors are called "reservoir capacitors" since they each act in a similar fashion to a water reservoir. Whenever a water reservoir is used, it provides a continuous flow of water in the absence of rainfall. Of course, it must be topped up from time to time and provided this occurs at fairly regular intervals the reservoir is able to give out a steady supply of water to the consumer. Bearing this analogy in mind, the action of the circuit is as follows:

The input current enters the circuit and fills the capacitor C_1. It will also pass through L and partially fill C_2.

The two capacitors very soon completely "fill" with the pumping up effect of the incoming pulsations of current, and are then in a position to give off the current to the external circuit during the interval between pulses. Thus the flow of current to the black box soon becomes much more of a steady flow than before.

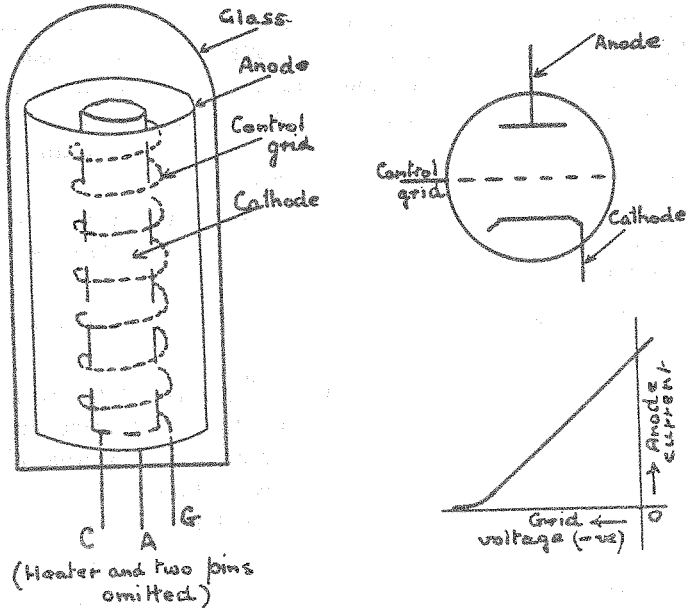
The action of the coil L is also important, since this is able to provide that extra flow of current to ensure a steady movement of electrons. (Reference has already been made to the "inertia" of a coil — in this case we may liken it to a flywheel which when set in motion is difficult to stop). When the current rises in the coil with the incoming pulses, a magnetic field is built up around it. If the current then falls, as the capacitor C_1 is drained, the magnetic field falls with it and in doing so cuts the turns of the coil and induces an additional e.m.f. which tries to keep the current flowing.

Thus the collapsing current flow which may occur is re-energised by the presence of the coil, and this assists in maintaining a steady flow of current. The net result of all this will be a continuous flow of current in the black box, and we can say that we

now have pure D.C. instead of the pulsating D.C. derived from the rectifier.

26.8 THE TRIODE VALVE

We have seen that a diode valve contains two electrodes. The triode valve has the same electrodes as a diode but an additional electrode in the form of a coil of wire is placed between the cathode and the anode. This is called the **grid**. (See the diagrams below).



(*a*) Physical construction of triode. (Heater omitted for clarity.)

(*b*) Symbol.

The Triode Valve.

The grid is placed between the cathode and anode so that it can be used to *control* the total flow of current which takes place through the valve. The full name for this electrode is in fact the **control grid** since in other, more advanced valves, other grids are inserted which have a different use. (We are not concerned here with anything more advanced than the triode valve however so any reference to the grid will be assumed to be the control grid).

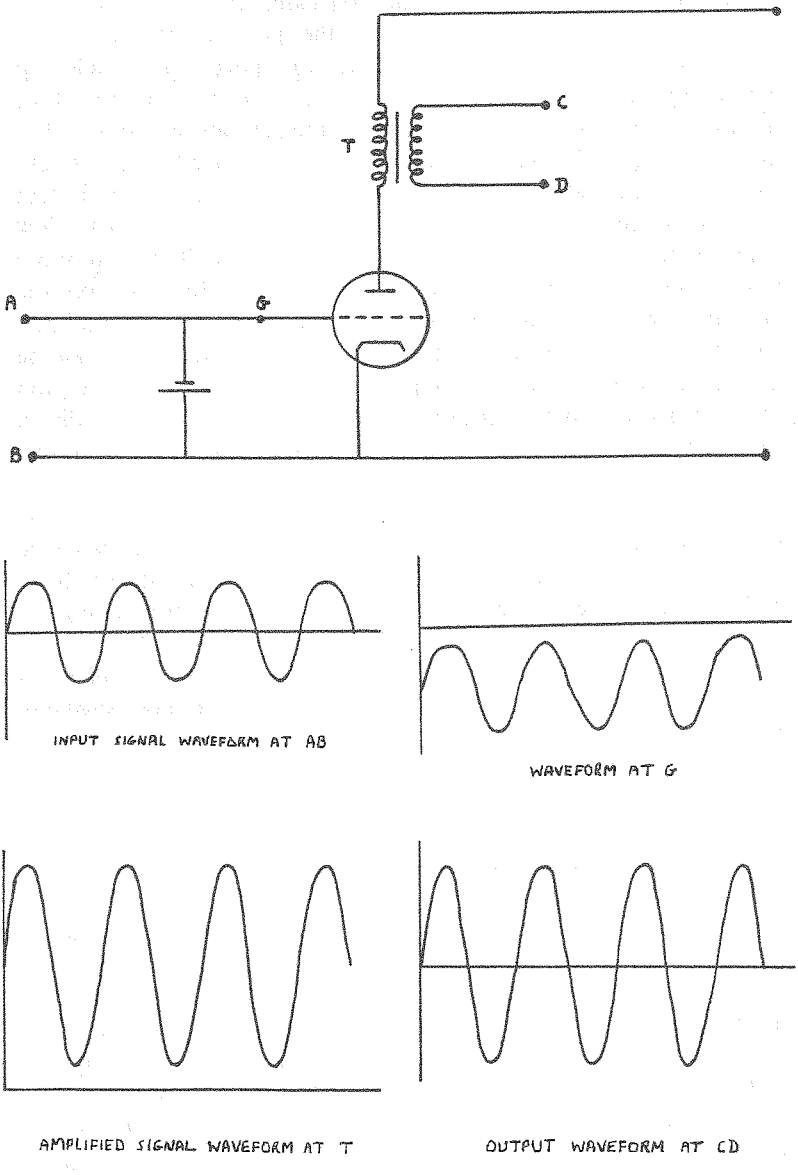
To understand the action of the grid, and the manner in which it

effects the flow of electrons let us first consider two extremes of voltage applied to the grid. In the first case, let us suppose that a high *negative* voltage is applied to the grid. In this case, the electrons surrounding the cathode when the space charge builds up will be repelled by the negative voltage on the grid and there will be no flow of current taking place between the cathode and anode. The anode voltage will have little effect in this case since the grid is very close to the cathode. We say that the valve in this condition is "cut off". (In effect it is analogous to a gate which is closed). Now consider the other extreme of voltage on the grid. Let us suppose that a *positive* voltage appeared on the grid. In this case the electrons from the cathode would be attracted to the grid and would *not* reach the anode at all. The grid would in this case be acting like an anode, and although there would be a strong emission of electrons from the cathode they would never reach the anode. In this case we say that grid current is flowing. Thus, going from one extreme to the other we find that in both cases there is no current reaching the anode. The grid voltage lies somewhere between these two values however, and it is made to be variable. It will always be of some *negative* value, since as we have seen, any positive voltage creates a flow of grid current and prevents the current reaching the anode.

Starting then from the high negative voltage on the grid when we have complete cut off, imagine the effect of gradually increasing the voltage on the grid so that it becomes less and less negative. The electrons will gradually be permitted to flow towards the anode with the flow increasing as the grid voltage approaches a zero value. Maximum current will flow when the grid voltage arrives at zero. (The valve in the circumstance is then identical to the diode valve with only two electrodes).

Thus, variation of the grid voltage over a negative range of voltages, will be analogous to the gradual opening of the gate between cathode and anode. The range of voltage concerned here are very important since it is this range which gives the clue to the action of the valve as an amplifier. Due to the proximity of the grid to the cathode it is only necessary to change the voltage from, say, minus four volts to zero to go from complete cut off to full flow of current. The variation of the valve current with grid voltage is shown in the diagram.

26.9 THE AMPLIFIER



One valve amplifier

The triode valve forms the basis of all valve amplifiers. By amplification we mean the changing of a weak voltage into a strong voltage of greater value capable of doing a positive job of work. The weak voltage is frequently referred to as the signal voltage since in most cases it is obtained from a radio signal over a long distance and is therefore not particularly powerful. The diagram shows a circuit which can be used to amplify such a signal.

The battery connected between the grid and the cathode is placed so that it has its − ve terminal on the grid. This gives the grid a negative **Bias**, and we know that to operate satisfactorily the grid must be negative. The incoming signal voltage is connected between the grid and the cathode so that the net result of both of these voltages will be a fluctuation of voltage which is always negative. The waveforms corresponding to the input signal and to the input signal after the application of negative bias are shown above.

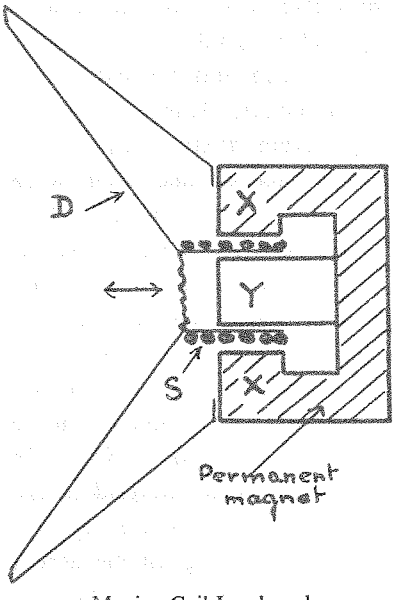
The effect of this fluctuating grid voltage is that the valve current will fluctuate, (with the swing of the "gate") and we will therefore also have a rise and fall of current in the primary winding of the transformer shown in series with the anode. Now, the important point is this. Due to the proximity of the grid to the cathode, the small fluctuation of grid voltage produces a *large* change in the anode current and hence a *large* voltage change across the terminals of the primary winding of the transformer. Thus, we have amplified the signal voltage to some value capable of doing a job of work.

It will be noticed that the output voltage from the amplifier is taken from the secondary terminals of the transformer. The reason for this is that the primary current will contain both the amplified current and the direct current provided from the positive terminal connected to the valve anode. If it were not for this D.C. of course the valve would not work, but we do not need it thereafter. Hence, we pass the combined currents through the transformer primary in order to separate the A.C. from the D.C. in the secondary.

The reader may have realised by this time, that the positive and negative D.C. supply to the valve is provided by a power pack (not shown in the figure) details of which we have already discussed.

26.10 THE LOUDSPEAKER

The diagram below shows a cross section of a typical loud speaker. A permanent magnet is shown which is ring shaped and which will allow lines of force to traverse the air gap between X and Y. Y forms the return path for the lines of force.



Moving Coil Loudspeaker.

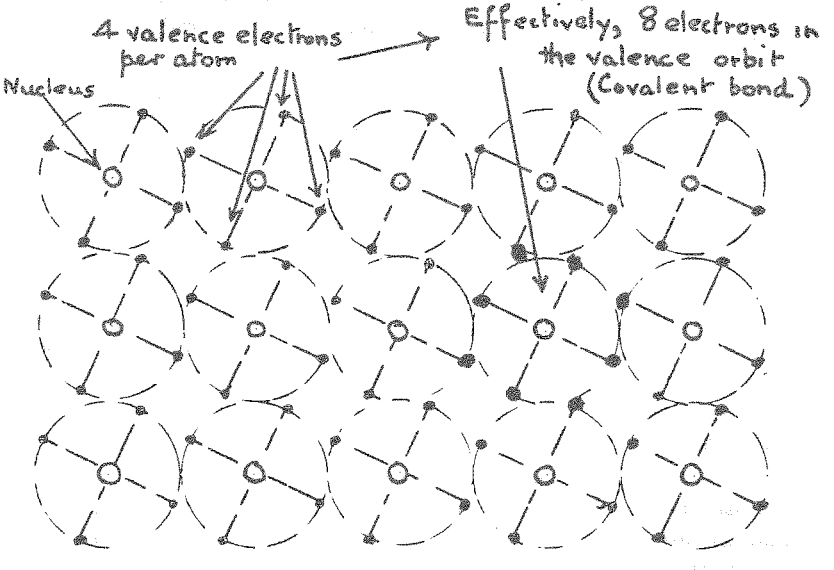
A coil, S, is placed in the circular air gap and is able to move in and out of the gap at will. It is connected to a conical diaphragm D. Now, if alternating current is passed through the coil, then the magnetic field which it sets up will react with the main field passing through the air gap, and the effect will be that the coil will move in and out according to the change in direction of the alternating current. (The motor principle is operating here). This creates a corresponding movement of the diaphragm and hence projects vibrations into the air.

In practice, the speaker coil is connected to the secondary winding of the transformer shown in the circuit diagram of the previous section (terminals CD). Thus, if we have a signal voltage of an audio frequency at the terminals A and B, it will be amplified and can be fed to the loudspeaker which may be at any remote position.

The whole subject of amplification is of course a very wide one and we have only dealt here with the straightforward case. This might be a good point at which to leave it and re-orientate our thoughts along slightly different lines.

26.11 SEMICONDUCTORS

In recent years, the use of valves in power packs and in amplifiers has been superseded by the use of "**solid state**" devices. Now, a "solid state device" is the expression used to describe a particular piece of electronic equipment which achieves the same result as a valve but which requires no heater (and is therefore not concerned with thermionic emission) and which consists of a solid piece of matter through which electrons move. To understand the way in which these devices work let us first of all look to the crystalline structure of a certain material called **germanium**.



Covalent bonds in germanium.

The structure of an atom is such that the nucleus is surrounded

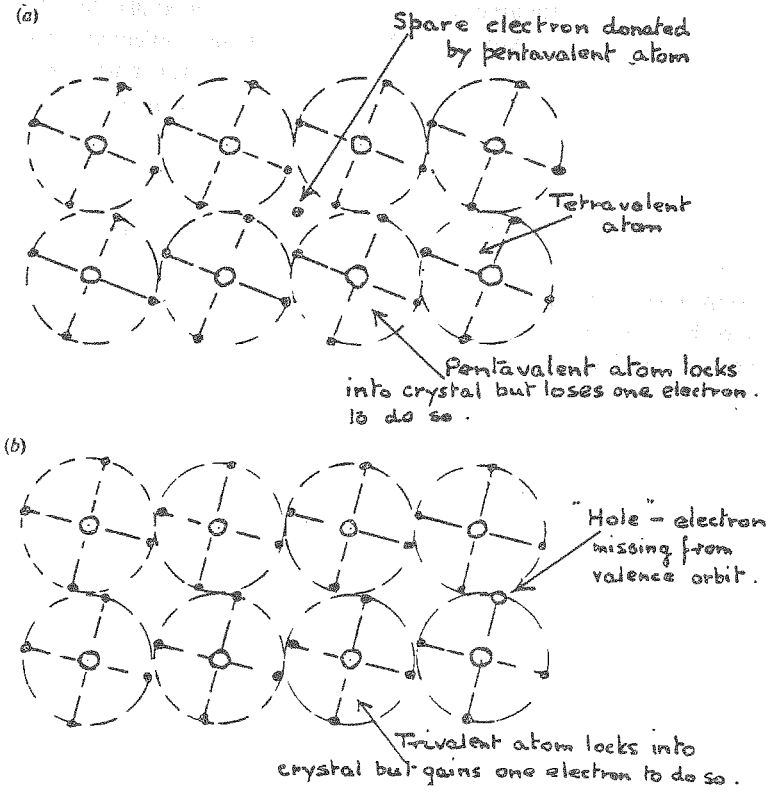
by a number of electrons which orbit the nucleus in a similar fashion to the orbit of the planets about the sun. (This is perhaps over-simplified but is suitable as a first approach). The number of electrons per atom, known as the **atomic number**, can be anything up to about 100. Substances which have atoms all of the same atomic number are called **elements**, and there are two elements most important in this particular field which are called **silicon** and **germanium**. Their atomic numbers are 14 and 32 respectively. We are not concerned however with the total number of electrons in orbit in these elements but only with those in the outermost orbital path. These electrons are called **valence electrons**, (this term first arose in chemistry since the chemical behaviour of substances depends mainly on the number of these outer electrons), and in both silicon and germanium there are four valence electrons. The diagram shows the structure of germanium — a similar diagram could be used for silicon of course since the structure of the four valence electrons is the same. This means that both germanium and silicon can be described as having **tetravalent atoms**.

Now, one of the interesting features about atoms, is that the full capacity of the outermost, valence, orbit is only eight electrons and if eight are not present they show a tendency to fill the vacancy from another atom if possible. In the case of germanium, or silicon, this results in each atom sharing its valence electrons with its neighbouring atom so that every atom has in effect **eight** electrons in its valence orbit. This causes the atoms to bond together, and the tie so created is called the **covalent bond**. The net result of all this is that since every atom is identical to its neighbour the structure is perfectly regular in three dimensions. Dimension in one plane only is shown in the diagram above but one can imagine that in effect the structure is like a box containing tennis balls fully packed out. The analogy falls down however if we imagine a part of it to be broken off. In this case the broken pieces have flat surfaces at definite angles to one another, and this of course is applicable to the description of a crystal.

The electrical characteristic exhibited by germanium and silicon is such that the material forms an almost perfect insulator. The covalent bonds prohibit the free movement of electrons in all cases except in the extreme case of applying a very high potential which breaks down the bonds completely.

26.12 TRIVALENT AND PENTAVALENT ATOMS

The elements of phosphorous, arsenic, and antimony all have five electrons in their valence orbit and these are called **pentavalent** atoms. Similarly, aluminium, indium and gallium have only three valence electrons per atom so that they are called **trivalent** atoms.



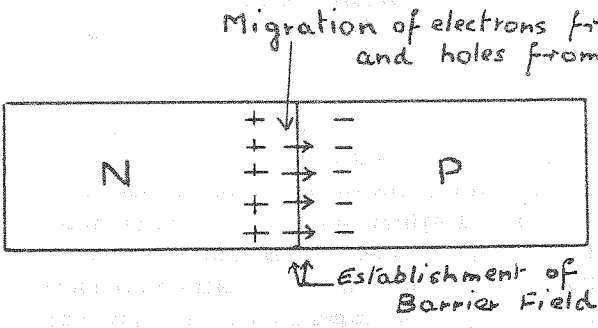
(*a*) N Type material. (*b*) P Type material.

A rather interesting material is produced when a small amount of pentavalent material is introduced into the tetravalent germanium we have previously discussed. The resulting material of course, can be described as a hybrid, since it has been produced artificially by the injection of one crystal into another. A representation of this arrangement is shown in diagram (*a*) above and we see here that the presence of the pentavalent atom produces

a "spare" electron which is unable to locate itself naturally into the crystal. In fact, the spare electron moves from one atom to another in its attempt to locate a free space. This electron is similar to the "free" electrons we have discussed in previous chapters. The injection of the pentavalent atom is called **injecting a donor atom** since an electron is donated, and the material so formed is called "**N type**" material. The reason for this is that we have injected what appear to be spare electrons into the germanium; *but*, and this is most important, the material as a whole is electrically *neutral*, since the pentavalent atom itself is neutral.

A similar arrangement, but this time by using trivalent atoms, can be achieved where the resultant crystal contains a number of atoms with only three electrons in the valence orbit. The absence of one electron in this atom constitutes the presence of a "hole" which is soon filled by an adjacent valence electron. As a result of this, the "hole" has now moved, and in this fashion we have a movement of holes taking place in the same way as the movement of electrons in the N type material. The best analogy of this hole movement, is that a row of seats in a cinema where there is one vacant place in the centre. As each person moves up, the vacancy, in effect, can be said to move down. This type of material is called "**P type**" material, and the impurity atom injected into it is called an **acceptor impurity**. See diagram (*b*). Once again, P type material as it stands, is electrically neutral in spite of the fact that a vacancy, or hole, is present. The hole is created by an electrically neutral trivalent atom.

26.13 THE P-N JUNCTION

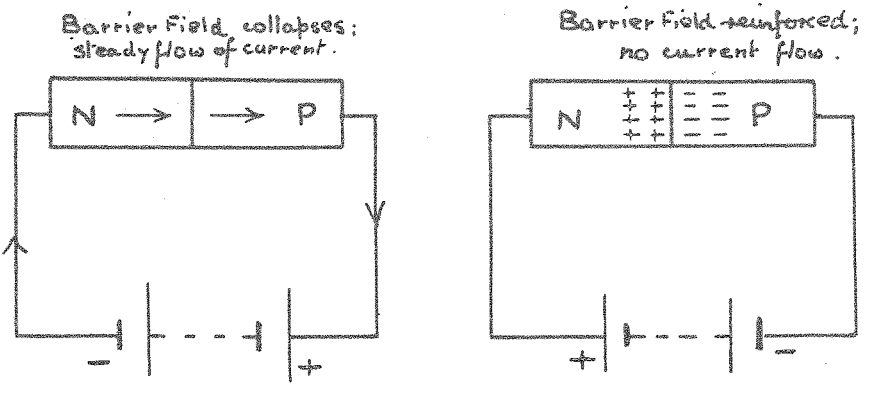


The P-N Junction.

Now, if we place N type material (which, remember, contains a number of free electrons) adjacent to P type material containing a number of holes there will be a migration of electrons in the immediate area of the junction from the N type into the available holes of the P type. This means that the atoms near the junction in the P type material now become − ve'ly charged, and the atoms near the junction in the N type material become + ve'ly charged. This prevents further general movement of electrons since the − ve build up on the P type material repels any further advance of electrons. In fact, an electrostatic field will now exist across the junction which is called the **barrier field**. (See the diagram).

26.14 THE SEMICONDUCTOR DIODE

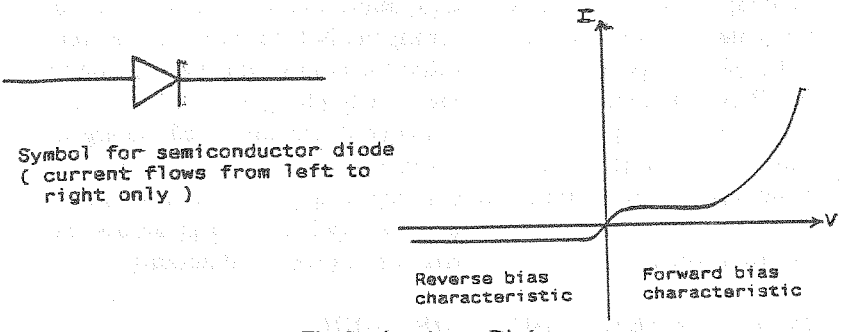
The P-N junction so described shows some very interesting characteristics. For example, if we connect the N type material to the − ve pole of a battery, and the P type material to the + ve pole then current flows easily. This is because the + ve pole of the battery attracts the − ve electrons away from the junction and the barrier field is broken down. Current then flows freely as it would in a valve diode with the + ve supply connected to the anode. This is called **forward biasing** the diode. (Diagram (*a*) below).



(*a*) Forward Biased Junction. (*b*) Reverse Biased Junction.

If however, we place the + ve pole of the battery on the N type material and the − ve pole on the P type material the barrier field is *strengthened*, and no current flows. This is called **reverse biasing** the diode. (See diagram (*b*) above).

The symbol for the diode is shown in the diagram below, together with a graph of the electrical characteristics.



The Semiconductor Diode.

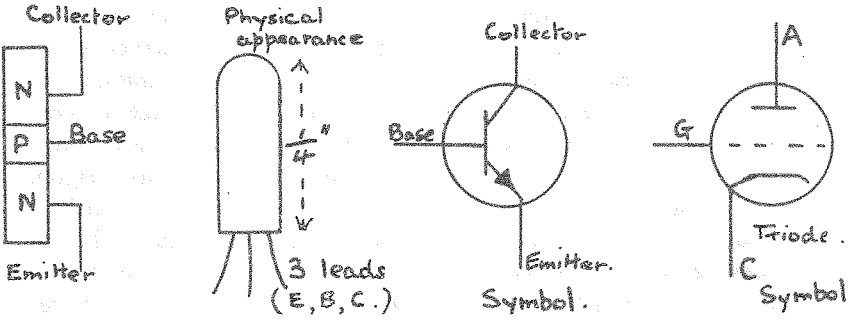
Clearly, the application of this material is such that it can, and does, replace the old fashioned valve diode. The semi-conductor diode operates most readily on low voltage and light current which makes it ideal for this area of radio and electronic engineering.

26.15 THE TRANSISTOR

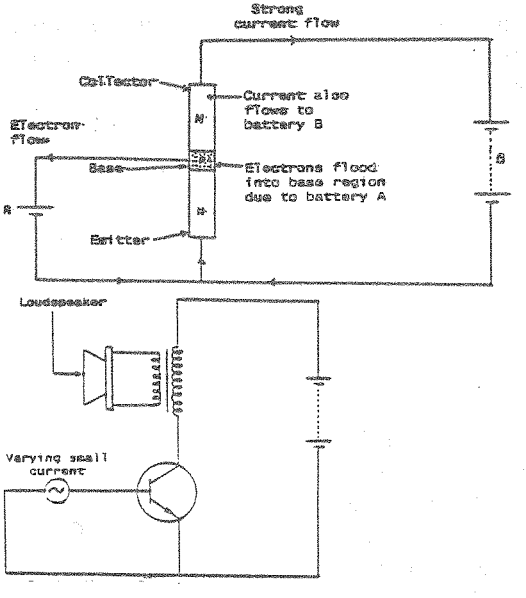
The purpose of the transistor is to replace the triode valve and it is one step on from the semi-conductor diode discussed in the previous paragraph. In the semi-conductor diode we have the P-N junction and the barrier field which exists across its interface. In the transistor an additional junction is introduced in order to create either a P-N-P sequence or an N-P-N sequence. The action of both types is very similar and we will discuss the N-P-N type only here and then briefly indicate the difference for the P-N-P type.

Physically, the N-P-N transistor consists of a thin layer of P type material sandwiched between two wider slabs of N type material. (See the diagram). An electrical connection is made to each section and the whole unit, which is cylindrical in shape, is of the order of a quarter inch in length and a tenth of an inch wide. There are variations in size, of course, depending on the current carrying capacity of the transistor and the use to which it is to be put. The three parts of the transistor are known as the **emitter**, **base**, and **collector**. The relationship between each part and the electrodes in a triode valve are such that the emitter corresponds to the cathode, the base corresponds to the grid, and the collector

corresponds to the anode. All these points are illustrated in the diagram below.



The Transistor (N-P-N).



The transistor as an amplifier.

The action of the N-P-N transistor as an amplifier is best understood by reference to the diagram above. Let us assume that a low voltage battery is connected between the emitter and base so

that the negative terminal is on the emitter and the positive terminal is on the base. In this case the P-N junction of this section is forward biased. If we also connect a battery (of higher voltage) between the emitter and collector such that the positive pole is on the collector and the negative pole is on the emitter then electrons will flow between emitter and collector. The presence of this current is however governed by the presence of the biasing voltage between the emitter and base. The reason for this is that the forward biased emitter/base generates a flow of electrons on to the base region. This floods the base with electrons, and the presence of a positive voltage on the collector attracts the electrons on to the collector, breaking down this barrier field and hence closing the circuit between emitter and collector. If the forward bias between emitter and base were removed (or reversed), then no electrons would move into the base region so that no current could then flow between emitter and collector. Indeed, the amount of current which actually flows is governed by the degree of biasing which exists between emitter and base. If the voltage between these two is allowed (or designed) to vary, then the number of electrons reaching the base region also varies, with the result that, in turn, the current to the collector must vary.

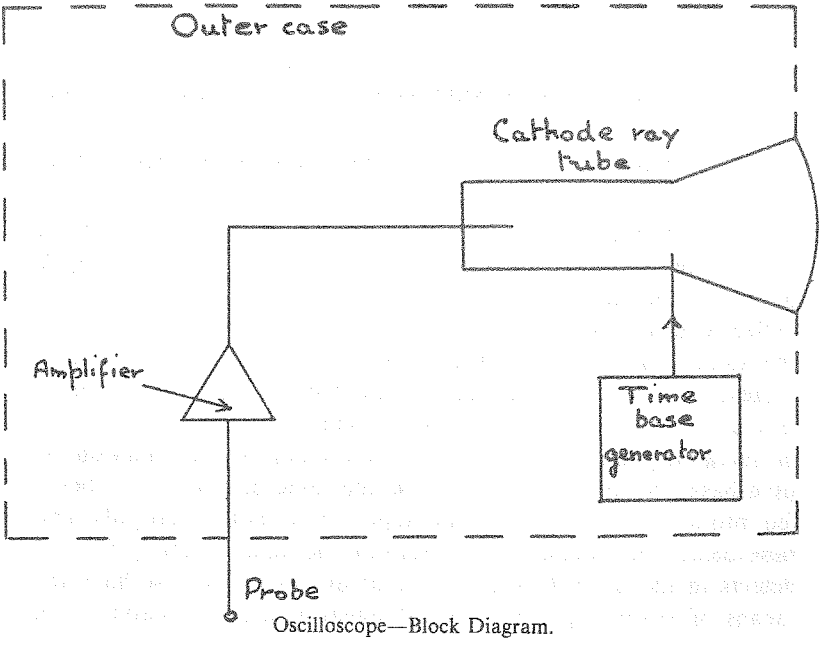
A very small current on the base can in this fashion generate a large flow of current to the collector. In other words, the action of the transistor is identical to the action of a triode valve, except that in the case of the triode the action is governed by the size of voltages on the grid whilst in the case of the transistor the action is governed by the amount of current which flows on to the base area. The electrical circuit around the transistor is the same as that around the triode valve.

The difference between the N-P-N and P-N-P type of transistor is in the polarity of the external circuit, and in the analysis of the action. In the P-N-P type the negative terminal of the battery between emitter and collector is placed on the collector. Also, the battery between emitter and the base is connected in the reverse direction to that of the N-P-N type, i.e., with the negative terminal on the base and positive on the emitter. Thereafter the analysis of action is really the same except that we should deal with movement of holes rather than movement of electrons. Finally, a word about the power supplies. The normal D.C. voltage required for transistor

operations is 6 or 9 volts which replaces the 220 volts needed for valve operation. This reduces considerably the dangers of electric shock and simplifies the design of equipment.

26.16 THE OSCILLOSCOPE

The purpose of an oscilloscope is to present visually, with the aid of a cathode ray tube, information of an electrical nature. For example if we wish to see the shape of the voltage waveform which is present on the grid of a triode valve we can connect an oscilloscope to that point and view it on the associated cathode ray tube. It is also possible to view the shape of the output voltage of the triode by connecting the oscilloscope to the anode. In this way it is possible to fault find since if the right shape appears on the grid, but the wrong shape appears on the anode there must be a fault in the valve itself. Another application, frequently used in hospitals, is to use a patient's heartbeat to produce an electrical impulse and then connect an oscilloscope to the impulse. In this way it is possible to view any irregularity in beat and also in the strength of beat — a passing nurse can quickly check that all is well.



Oscilloscope—Block Diagram.

The complete oscilloscope is made up of three essential parts. The first is the **cathode ray tube** which is an electrostatic type. (The electromagnetic type is used in a ship's radar). The second is the **time base generator** which can be regarded as an electronic stop watch. The third is the **amplifier attached to the probe** which is used to pick up the signal we wish to view. A block diagram of the three units is shown in the diagram. The units are housed in a container which can be carried about (or mounted on a trolley).

It is beyond our present scope to deal with the details of a cathode ray tube (C.R.T.), a subject which is treated fully in most textbooks on radar. It should be noted however that the electrostatic type is used in preference to the electromagnetic type since it is lighter and hence is more portable. Even so, the complete oscilloscope is quite a big piece of equipment and reference has already been made to the usefulness of a trolley.

For the reader who has some familiarity with the C.R.T., the X plates, in conjuction with the time base generator, cause the electron stream within the C.R.T. to form a *horizontal* fluorescent line on the face of the C.R.T.

The length of this line corresponds to a fixed interval of time — hence it is termed the **time base**. This time may be very short — of the order of microseconds. With its aid, we can measure very short periods indeed.

The purpose of the Y plates in the C.R.T. is to cause *vertical* movement of the electron stream.

Now, the top Y plate is used to portray the voltage which we wish to see. Suppose this is a radar pulse. If we connect the top Y plate, via the amplifier, to the radar pulse, then the spot will be deflected upwards on the cathode ray tube of the oscilloscope. At the same time we adjust the speed of the time base so that it corresponds to the repetition rate of the radar pulse. This is done by trial and error. Once the time base coincides with the repetition rate of the radar pulse, then the radar pulse will appear stationary on the time base and never fade as long as the same information is being fed into it — i.e., with the normal repetition of both radar pulse and time base. This enables us to examine the pulse carefully for any defects in its shape. One can see that in this fashion we have the means of examining any electrical signal which is periodic — i.e.

any which is constantly repeating itself — like the patient's heart beat mentioned earlier.

TEST PAPER 26
Electronics

1. Explain the terms thermionic emission, space charge, cathode, anode, indicating the relationship between them in a simple diode.
2. With the aid of diagrams describe the action of an electrostatic cathode ray tube.
3. What is a triode and why is it so called? With the aid of a sketch describe the action of a triode.
4. Why are tetravalent, pentavalent, trivalent, atoms so called? Discuss valence electrons from the point of view of what happens when pentavalent material is introduced into tetravalent material.
5. Explain with the aid of a sketch, the terms barrier field, forward biasing, reverse biasing.
6. What is meant by the P–N junction? Show how it is employed in the N–P–N– sequence to form a transistor.
7. Describe carefully the effects of varying the grid voltage in a triode valve from small negative to small positive voltage.
8. With the aid of a sketch, explain how a diode may be used as a half wave rectifier.
9. What is the purpose of a "smoothing circuit"? Draw a wiring diagram for such a circuit and explain the functions of the various components in it.
10. How may the amplification of a weak alternating voltage be achieved? Discuss carefully with the aid of a circuit diagram.
11. Explain carefully the essential difference(s) between an N–P–N sequence and a P–N–P sequence.
12. Discuss briefly, with examples, the essential purpose of an oscilloscope. Illustrate its principal components in a block diagram.
13. State briefly the purpose of *(a)* a transformer *(b)* a power pack. Discuss the purpose(s) for which these two may be used in conjunction with one another.
14. What are "semi-conductors"? Discuss briefly the properties of some of the elements which are important in this connection.

CHAPTER 27

LIGHT

Rectilinear propagation. Optical instruments.

27.1 NATURE OF LIGHT

Light is a form of energy. We have already seen that other forms of energy are heat energy, chemical energy, electrical energy, mechanical energy, and, in the case of electricity, we said there was "something intangible about it — it was only recognised by its results".

We might use the same words about **light energy**. It travels through an intervening medium without having any effect on that medium, and without itself being visible. It only becomes recognisable when it falls on some body which is then said to be **illuminated**.

Light energy reaches us through space from the sun.

To make light on the Earth, we usually have to make a body so hot as to become incandescent, when it radiates light energy as well as heat energy.

27.2 PATH OF LIGHT

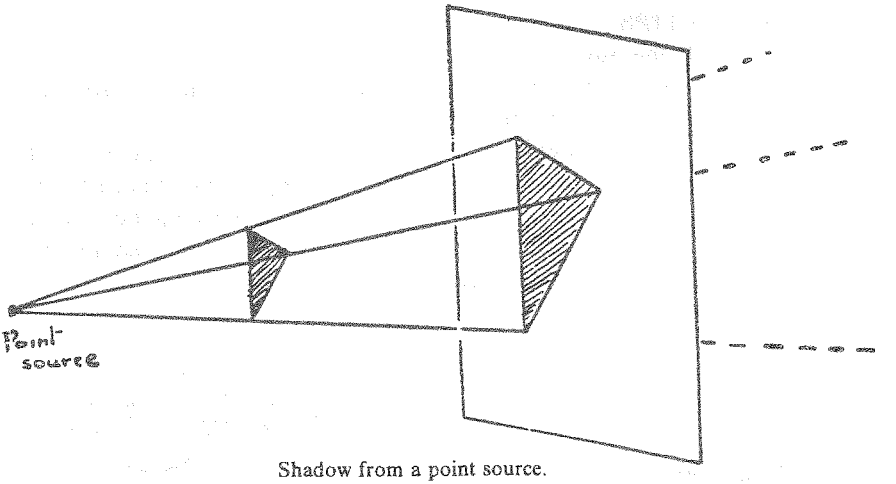
When light is travelling through a medium of uniform density, it does so in a straight line. This is called the **rectilinear propagation of light**.

Light may thus be represented by straight lines called **rays**. A collection of rays is called a **beam** or **pencil**.

27.3 SHADOWS

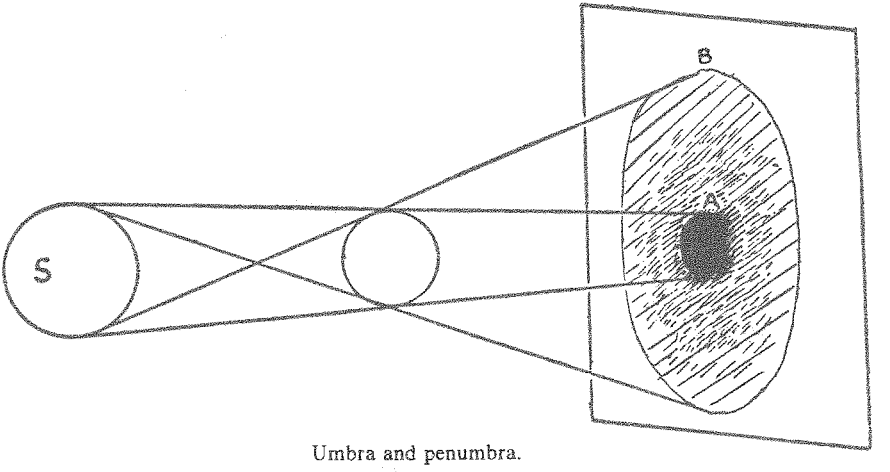
When a opaque object is placed in the path of a beam of light it will cast a shadow on a screen suitably placed.

If the source of light is a point, then the shadow is distinct and of the same shape as the object, though enlarged, as in the diagram.



Shadow from a point source.

When the source is larger, the shadow is blurred, not sharp, and the effect shown in the diagram below is obtained.



Umbra and penumbra.

The black central part, which receives no light from any part of the source *S*, is called the **umbra**, whilst the surrounding part is called the **penumbra**.

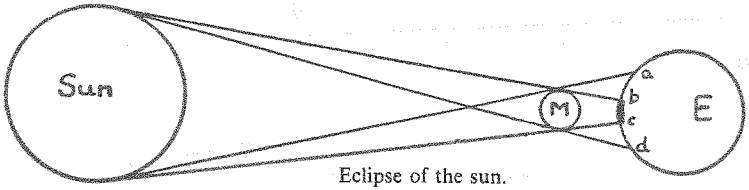
There is no distinct line of demarcation between them, however. From full shadow at *A*, there is a gradual change to full illumination at *B*.

27.4 ECLIPSES

(*a*) Of the Sun

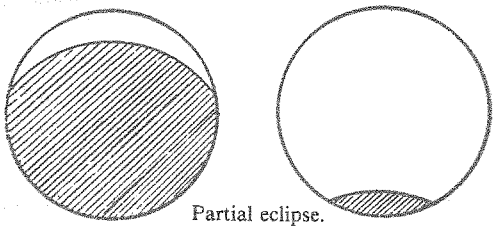
Eclipses of the sun can be either **total**, **partial**, or **annular** (ring-shaped — not to be confused with "annual").

The moon revolves in an orbit round the Earth, and when it passes directly between the Earth and the sun, so that the centres of the three bodies are in line, or nearly in a line, then some portion of the Earth will be in the moon's shadow. The sun is then said to be **eclipsed** (see the diagram below).



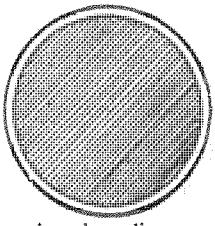
Eclipse of the sun.

To an observer between *b* and *c* in the **umbra**, the eclipse would be **total**, that is, he would be unable to see any portion of the sun.



Partial eclipse.

An observer in the **penumbra**, between *a* and *b*, or *c* and *d*, would see a portion only of the sun, i.e., the eclipse would be **partial**. How much he saw would depend on his position. Near *b* or *c*, the eclipse would be nearly total; as he approached *a* or *d* he would see more and more of the sun, until, outside of *a* or *d*, there would be no eclipse at all.



Annular eclipse.

The angular diameters of both sun and moon vary, and there are periods, as one may see in the *Nautical Almanac*, when the moon's diameter is less than the sun's. If an eclipse should occur at such a time, even though the centres may be exactly in line, the moon is not big enough (as viewed from the Earth) completely to cover the sun. Thus, the effect in the diagram above is obtained, a ring of the sun still being visible around the outer edge of the moon. Such an eclipse is called **annular**.

(*b*) **Of the Moon**

An eclipse of the moon is caused when the moon moves into the shadow cast by the Earth, and may be either **total** or **partial** (see the diagram below).

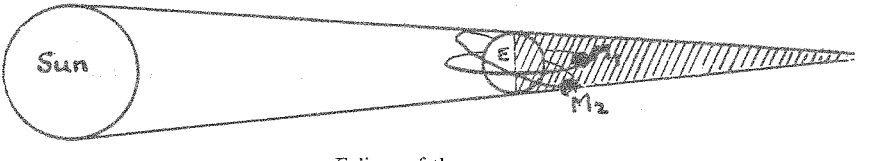
At M_1, the moon is completely in the umbra, and a total eclipse occurs. Even during the period of totality the moon is still faintly visible.

The plane of the moon's orbit round the Earth is constantly tilting however — tilting that is, in relation to the plane of the Earth's orbit round the sun. It is this that causes the moon's declination to change so rapidly. Thus, on another occasion, the plane of the moon's orbit might be as shown for M_2, so that the centres of the three bodies are at no time in line. In this case, a **partial eclipse** occurs.

(*c*) **Further Note**

It will be noted that a solar eclipse is only visible over a very narrow belt of the Earth's surface — actually much narrower than that shown in the first eclipse diagram.

The reader should make a point of looking in the current *Nautical Almanac*, where details of solar eclipses, with paths of visibility are given.



Eclipse of the moon.

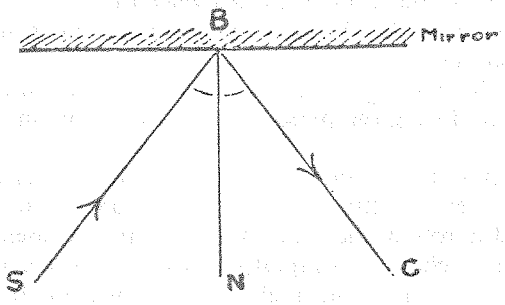
A lunar eclipse, however, as may be seen from the diagram above when it occurs, is visible over the whole of the dark hemisphere of the Earth.

This explains how it is that most of us in a lifetime see several lunar eclipses, but very few are destined ever to see an eclipse of the sun.

27.5 REFLECTION OF RAY OF LIGHT BY PLANE MIRROR

The terms used in connection with this can be understood from this figure.

A ray from a source of light S falls on a mirror at B, SB is the **incident** ray, BC is the reflected ray, BN is the **normal** or perpendicular to the reflecting surface.



Reflection by plane mirror.

Angle NBS is known as the **angle of incidence.**

Angle NBC as the **angle of reflection.**

When a beam of light strikes the surface of a mirror most of the rays are "sent back" or reflected.

Learn the following **Laws of Reflection.**

(*a*) The angle of incidence is equal to the angle of reflection.

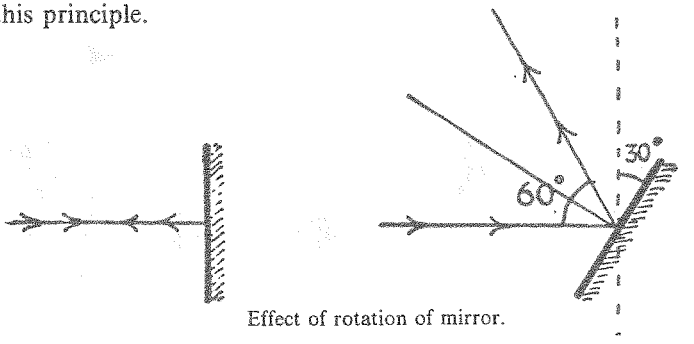
(*b*) The incident and reflected rays are in the same plane as the normal to the reflecting surface.

The former is a convenient way of saying that the angle at which a ray strikes a reflecting surface is the same as the angle at which it leaves — like a snooker ball striking the cushion.

27.6 EFFECT OF ROTATION OF MIRROR

By means of a lantern a beam of light is directed normally (perpendicularly) on to a mirror. The mirror is then turned through an angle of, say, 30°. By the application of the first law of reflection it should be clear that the reflected ray rotates through 60°, i.e., **the reflected ray rotates through twice the angle of rotation of mirror.**

The **sextant** offers an interesting example of the application of this principle.



Effect of rotation of mirror.

27.7 PRINCIPLE OF THE SEXTANT

The operation of the sextant depends on two optical laws, both of which have been referred to:

(1) that the angle of incidence is equal to the angle of reflection.

(2) that the angle through which a reflected ray is rotated is **twice** the angle through which the mirror has been rotated.

These two together constitute which is usually described as the **principle of the sextant.**

In both, all the rays, incident and reflected, lie in the same plane.

The first is easily understood. For instance, in the diagram below, it tells us what happens each time the ray meets a reflecting surface, as at I and H, where the equal angles (incidence and reflection) are clearly marked.

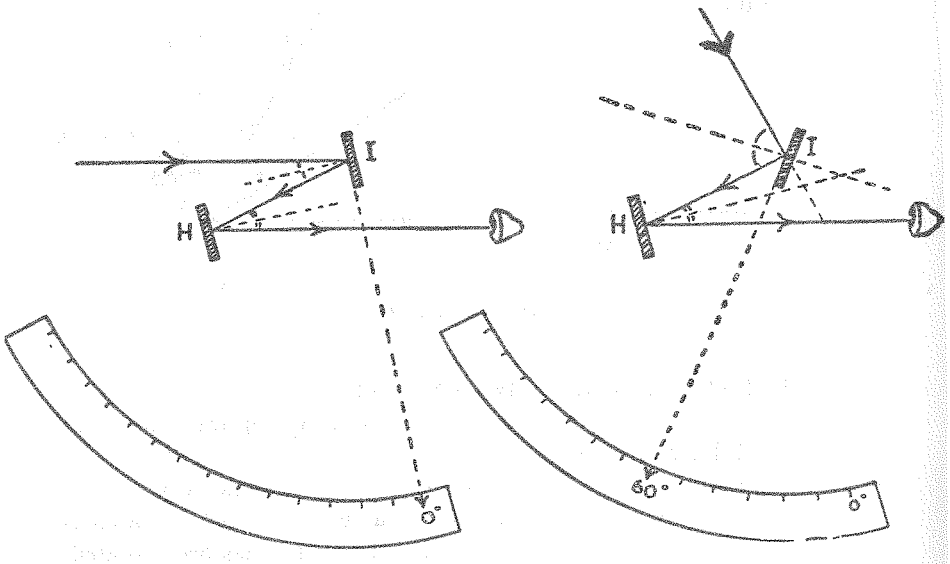
The second might be considered a bit further.

It is assumed that the reader is familiar with the constructional details of the sextant, and he should take every opportunity of handling one. But for guidance:

I is the index glass, into which the ray of light first enters.

H is the horizon glass, in which (as a result of the reflected ray entering the eye), the object appears to be seen, and also part of the horizon is visible. Thus the object can be brought in contact with the horizon and its altitude measured.

Well now, for a vertical sextant reading of 0°, the incident ray is reaching the eye from an object on the horizon. Its altitude is zero (see diagram (*a*) below). In this position the index glass and the horizon glass are **parallel.**



(*a*) Mirrors parallel. Altitude zero. (*b*) Mirrors inclined at 30°. Altitude 60°.
Principle of the sextant.

If we now rotate the index mirror through 30°, then according to the second law the angle of reflection rotates through 60°. This means the incident ray has also rotated through 60°, which is only another way of saying that we are now catching a ray of light from an object whose altitude is 60°, even though the angle of inclination of the index mirror to its original zero position is only 30°.

It is for this reason that the sextant, although only a sixth part of a circle, can read angles of twice that amount, namely, 120° (actually, most "sextants" are rather more than a sixth part, and it follows they can read angles rather more than 120°, usually up to about 140°).

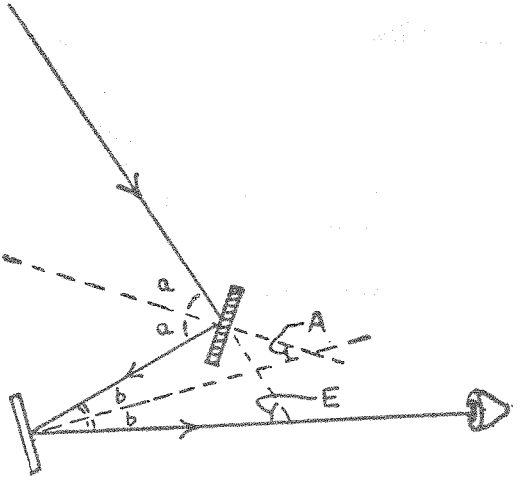
Of course, it would be inconvenient to have to multiply the angle of inclination of the mirrors by 2 in order to obtain the desired angle, so this is done for us in the graduation of the scale, e.g., in diagram (*b*) above, although the inclination of the mirrors is 30°, the angle read off on the arc of the sextant is 60°, as shown.

27.8 GEOMETRICAL PROOF

The two laws quoted above are sometimes run into one, as follows:

Principle of Sextant — Alternative definition:

When a ray of light passes through two reflections in the same plane, the angle between the first and last directions of the ray is double the angle of inclination of the mirrors to each other.



Principle of sextant, geometrical proof.

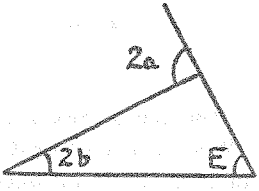
In this form, this leads to the following easy geometrical proof.

In the diagram above, E is the angle between the first and last directions of the ray, i.e.., E is the altitude.

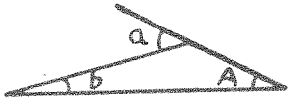
Since the angle between the two normals to two straight lines is equal to the angle between the lines themselves, A, the angle between the normals, is the angle of inclination of the mirrors.

Proof:

Since in any triangle the exterior angle equals the sum of the two interior and opposite angles, we have, from the diagrams shown.



$$2a = 2b + E$$
$$E = 2a - 2b \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$



and again,

$$a = b + A$$
$$A = a - b$$
$$2A = 2a - 2b$$

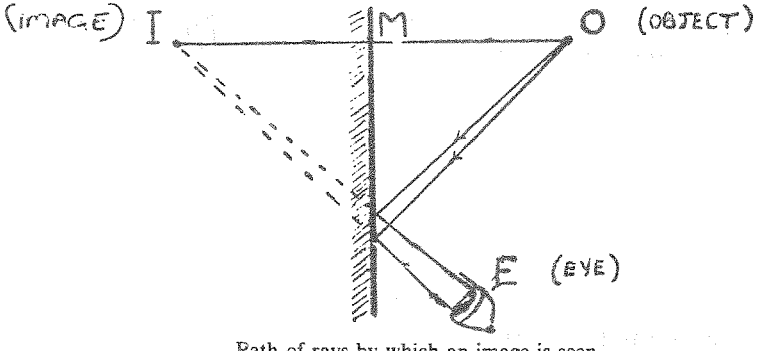
whence $E = 2A$, that is,

the altitude = twice the angle of inclination of the mirrors.

Note: Although in the interests of clarity we have referred throughout to "altitude", the principle is not affected if angles are taken in the horizontal plane.

27.9 POSITION OF IMAGE IN A PLANE MIRROR

When an object is placed before a plane mirror, an image of the object is seen in the mirror.

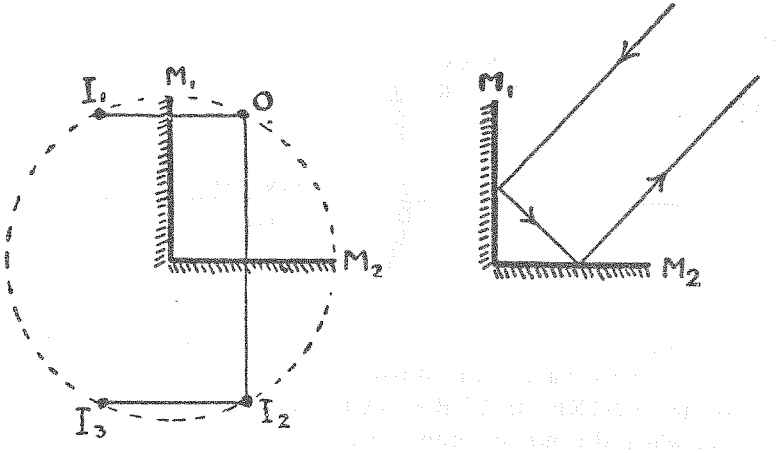


Path of rays by which an image is seen.

The diagram above shows the path of the rays by which the image is seen, and from the geometry of the figure, it is easy to see that the **image lies on the normal, *OMI* and in such a position that** $IM = OM$.

27.10 REFLECTION AT TWO PERPENDICULAR SURFACES

Two mirrors at an angle form varying numbers of images, the actual number depending on the angle between the mirrors. When mirrors M_1 and M_2 are arranged perpendicular to one another, *three* images of an object O will be visible. The images I_1 and I_2 are obtained each by *one* reflection from the mirrors M_1 and M_2 respectively. The third image I_3 is obtained by rays which are reflected twice before reaching the eye. It can be thought of as the image of I_1 in mirror M_2 or the image of I_2 in mirror M_1.



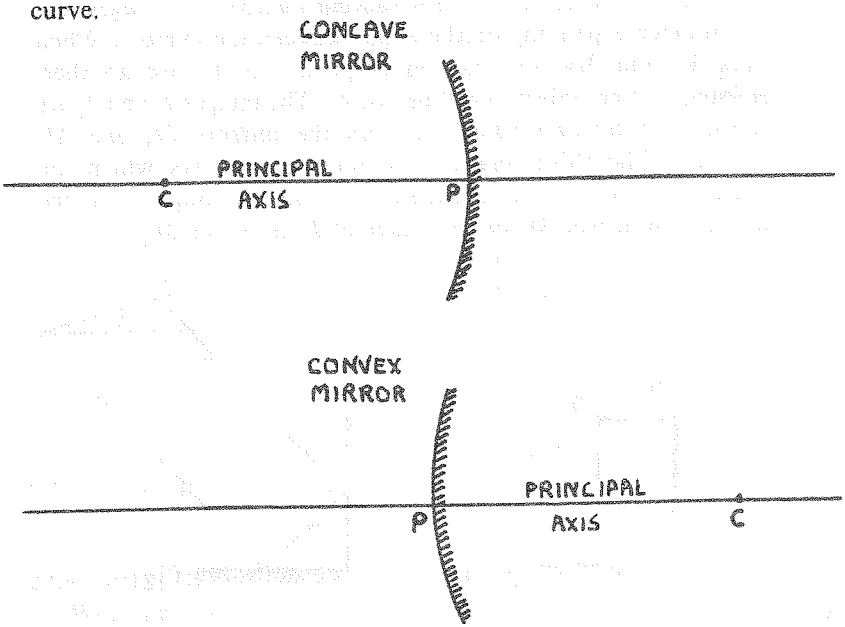
Reflection by two mirrors at right angles.

Note that a ray which is incident at an angle of 45° to either mirror will after 2 reflections be reflected along a path parallel to its original path.

27.11 REFLECTION IN A CURVED MIRROR

The laws of reflection still hold true for reflection of light at a curved surface. The most common type of curved mirror is the spherical mirror. This is constructed by silvering a piece of glass which is part of the surface of a hollow glass sphere.

Spherical mirrors can either be CONCAVE, silvered on the outside of the curve, or CONVEX, silvered on the inside of the curve.



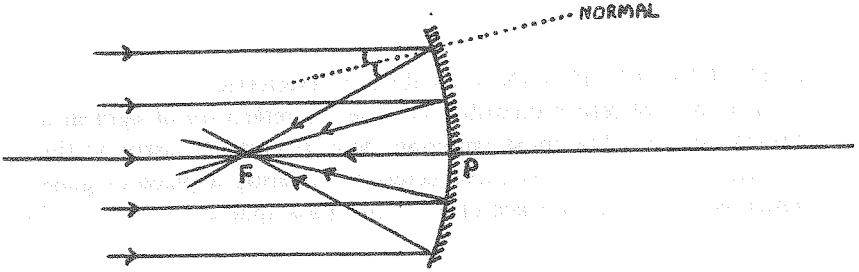
The point *P* is the POLE of a spherical mirror and is the mid-point of the surface of the mirror.

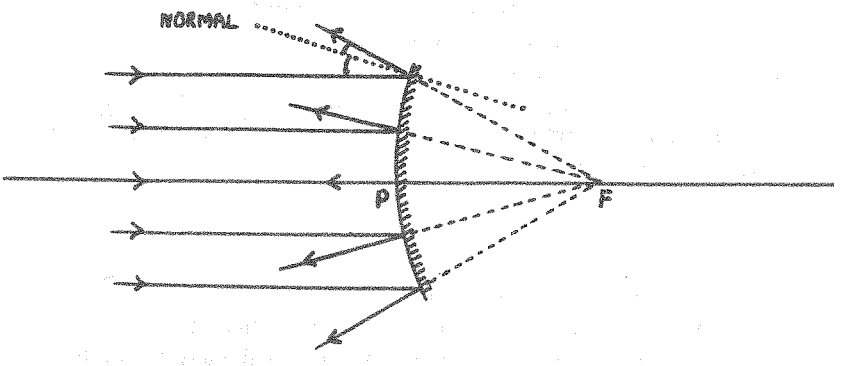
C is the CENTRE OF CURVATURE and is the centre of the sphere of which the mirror forms a part.

The distance *CP* is called the RADIUS OF CURVATURE.

The line joining *C* to *P* is called the PRINCIPAL AXIS.

Obviously these two types of mirror reflect light in different ways and they are distinguished from each other by the effect they have on an incident beam of parallel light.





The diagrams illustrate the reflection of a parallel beam of light in each type of mirror.

We can see that the CONCAVE mirror is a CONVERGING mirror i.e. it reflects the parallel beam of light inwards.

The CONVEX mirror is a DIVERGING mirror i.e. it reflects the light outwards.

For both mirrors F is called the PRINCIPAL FOCUS or focal point.

In the case of the concave mirror it is defined to be the point on the principal axis to which rays of light close to and parallel to the principal axis converge after reflection.

In the case of the convex mirror it is defined to be the point on the principal axis from which rays of light close to and parallel to the principal axis appear to diverge after reflection.

In this later case because the rays of light do not actually pass through F it is called a VIRTUAL focus.

For the concave mirror we have a REAL focus — the rays of light actually pass through it.

For both mirrors the distance FP is called the FOCAL LENGTH. It is denoted by f.

For both mirrors

the radius of curvature = twice the focal length

$$r = 2f$$

Note that a ray of light incident along the principal axis is, in both cases, reflected back along the principal axis.

27.12 IMAGES IN SPHERICAL MIRRORS

If an object is placed in front of a spherical mirror then, in much the same way as for a plane mirror, an image is formed by the mirror. However the nature, size and position of such an image is generally not the same as for a plane mirror but depends on the position of the object and the type of mirror.

Ray diagrams can be used to determine the nature, size and position of images.

Four particular rays of light are available for drawing but only two are necessary to fix the image. The rays are:

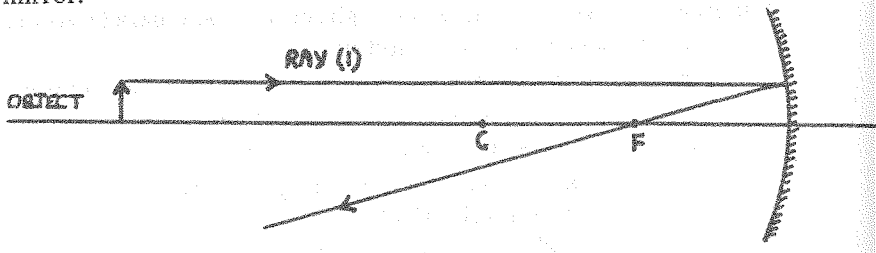
(1) A ray of light from the top of the object parallel to the principal axis. After reflection this will pass through, or appear to pass through, the focus.

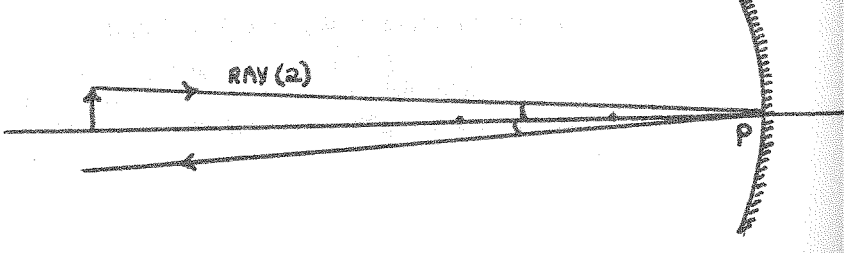
(2) A ray of light incident at the pole of the mirror. This will be reflected back at the same angle to the principal axis.

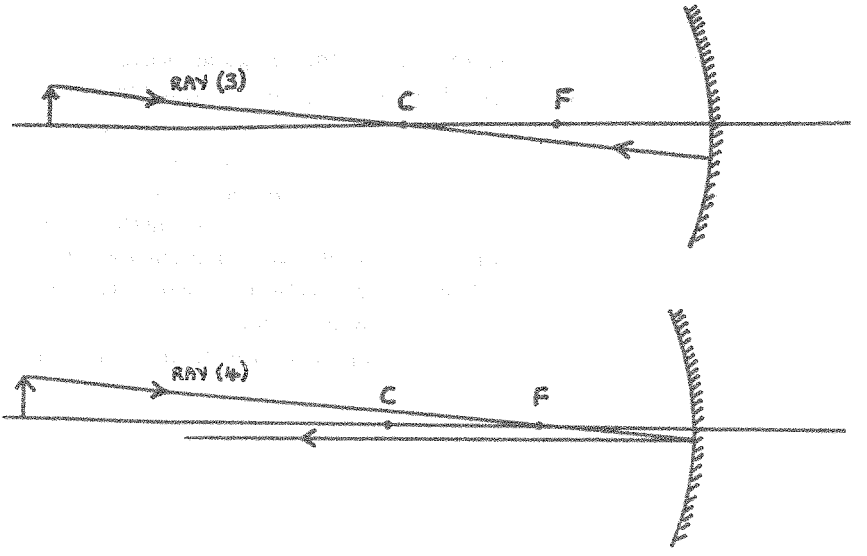
(3) A ray of light passing through the centre of curvature. This will be reflected back along its own path.

(4) A ray of light passing through the principal focus. After reflection it will be parallel to the principal axis.

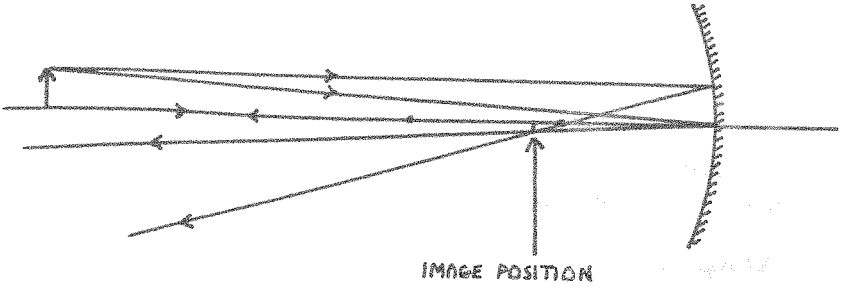
To illustrate each of these rays the diagrams below use the example of an object placed some distance in front of a concave mirror.







These rays are not drawn on separate diagrams as shown above but ANY TWO are drawn on the same diagram. Thus, for example, drawing rays (1) and (2) on the same diagram we have



The image will be found where the two reflected rays intersect. From the above diagram we can see that the image is

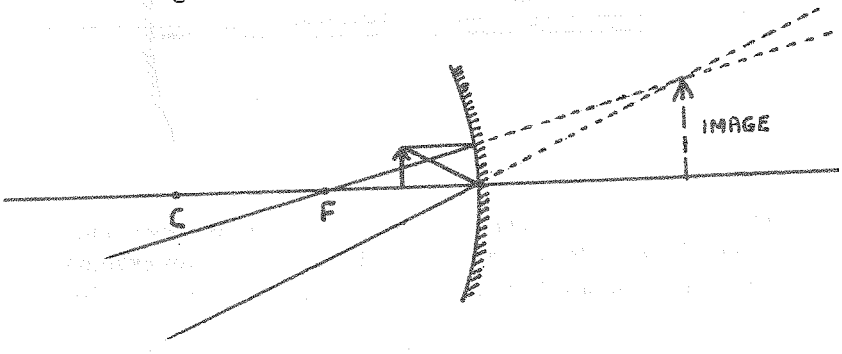
(1) between C and F
(2) real and inverted
(3) diminished in size

27.13 REAL AND VIRTUAL IMAGES

In the above section we referred to the image as being "real". This is because rays of light actually do pass through the image position.

However it is possible to obtain images for which the rays of light only appear to pass through the image position. Such an image is called a virtual image. For plane and curved mirrors such images are formed behind the reflecting surface and as such will not appear on a screen placed at the image position. A real image will appear on a screen placed at the image position.

The following diagram is an example of the formation of a virtual image.



The image obtained is (1) virtual
(2) erect
(3) magnified

In general if an image is real it will always be inverted whilst if it is virtual it will be erect.

EXERCISE A

1. A concave mirror of focal length 40 mm has an object 10 mm tall placed in front of it. By drawing ray diagrams to scale determine the nature, position and size of the image formed when this object is placed (*a*) 20 mm (*b*) 40 mm (*c*) 60 mm (*d*) 80 mm (*e*) 100 mm in front of the mirror.
2. Repeat question 1 for a convex mirror of the same focal length.

27.14 IMAGES IN SPHERICAL MIRRORS

From the above exercise it can be seen that, irrespective of the position of the object, the image formed in a convex mirror is always virtual, erect and diminished. Such mirrors are sometimes used as car driving mirrors.

The image formed in a concave mirror can be real or virtual, erect or inverted, diminished or magnified or remain the same size. It all depends on where the object is placed. When the object is placed between the focus and the mirror, as shown in the diagram in section 27.13, then this describes the situation with a shaving mirror. When the object is placed at the focus then this describes the situation when a light bulb is placed in front of a curved reflecting surface in a torch or searchlight — a parallel beam of light is produced.

27.15 SPHERICAL MIRROR CALCULATIONS

Drawing scale ray diagrams can be a long-winded and inaccurate method of determining images. Formulae exist which enable the image's position, size and nature to be calculated.

If an object is placed a distance u in front of a spherical mirror of focal length f and this results in an image being formed at a distance v from the mirror then it can be shown that the relationship between u, f and v is given by

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

It is necessary to use a sign convention with this formula. This states that "REAL IS POSITIVE, VIRTUAL IS NEGATIVE".

Thus since a concave mirror has a real focus its focal length will be positive but the virtual focus of the convex mirror means its focal length will be negative.

WORKED EXAMPLE 1

Calculate the position of the image formed in a concave mirror of focal length 40 mm when an object is placed (*a*) 20 mm (*b*) 100 mm in front of it.

(*a*) $u = +20$ mm
$v = ?$
$f = +40$ mm (Concave mirror has REAL focus)

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} + \frac{1}{20} = \frac{1}{40}$$

$$\frac{1}{v} = \frac{1}{40} - \frac{1}{20}$$

$$v = -40 \text{ mm}$$

The negative sign indicates the image is virtual. This means it is 40 mm BEHIND the mirror. Also, being virtual, the image must be erect.

(*b*) $u = +100$ mm
$v = ?$
$f = +40$ mm

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} + \frac{1}{100} = \frac{1}{40}$$

$$\frac{1}{v} = \frac{1}{40} - \frac{1}{100}$$

$$v = +66^2/_3 \text{ mm}$$

This time the image distance is positive indicating that the image is real. This means it is $66^2/_3$ mm IN FRONT OF the mirror. Also, being real, it is inverted.

WORKED EXAMPLE 2

Repeat the calculations of Worked Example 1 for a convex mirror of focal length 40 mm.

(*a*) $u = +20$ mm

$v = ?$

$f = -40$ mm (Convex mirror has a VIRTUAL focus)

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} + \frac{1}{20} = \frac{1}{-40}$$

$$\frac{1}{v} = -\frac{1}{40} - \frac{1}{20}$$

$$v = -13^1/_3 \text{ mm}$$

The negative sign indicates the image is virtual. Therefore it is behind the mirror and erect.

(*b*) $u = +100$ mm

$v = ?$

$f = -40$ mm

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} + \frac{1}{100} = \frac{1}{-40}$$

$$\frac{1}{v} = \frac{1}{-40} - \frac{1}{100}$$

$$v = -200/_7 \text{ mm } (28{\cdot}57 \text{ mm})$$

Again the image is virtual, erect and behind the mirror.

27.16 LINEAR MAGNIFICATION

This is defined to be $m = \dfrac{\text{height of image}}{\text{height of object}}$

The linear magnification m can be found from the ratio of the image distance to the object distance.

$$\text{i.e. } m = \frac{v}{u}$$

m is a pure ratio — it has no units.

WORKED EXAMPLE 3

Assuming the object used in Worked Example 1 was 6 mm tall calculate the height of the image in each case.

(*a*) $u = 20$ mm and $v = -40$ mm

$$m = \frac{v}{-u}$$

$$= \frac{-40}{20}$$

$m = -2$ the negative sign merely indicates the image is virtual.

The linear magnification is 2. The image is 2× the height of the object i.e. is 2×6 mm $= 12$ mm tall.

(*b*) $u = 100$ mm and $v = 66^2/_3$ mm

$$m = \frac{v}{u}$$

$$= \frac{66^2/_3}{100}$$

$$= \frac{2}{3}$$

The image is $\frac{2}{3} \times 6$ mm $= 4$ mm tall

EXERCISE B

1. A concave mirror of focal length 40 mm has an object 10 mm tall placed in front of it. Calculate the position, size and nature of the image formed when this object is placed (*a*) 60 mm (*b*) 80 mm (*c*) 40 mm in front of the mirror.
2. Repeat question 1 for a convex mirror of the same focal length.

27.17 REFRACTION

Light travels in a straight line only as long as it is passing through a medium of uniform density.

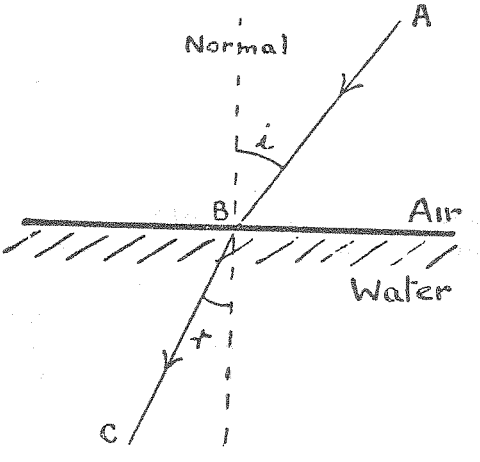
A ray of light may undergo a change of direction as it passes from one medium to another of different density — a phenomenon which is called **refraction**.

The terms used in connection with this can be learnt from the diagram below showing a ray of light passing from air into water.

AB is the incident ray, angle i is the **angle of incidence**.

BC is the refracted ray, angle r is the **angle of refraction**.

Notice that the refracted ray is on the opposite side of the normal to the incident ray and (this is important) the ray is bent **towards** the normal when passing from an optically rarer to a denser medium, (e.g., from air to glass or water), and *vice versa*.



Refraction of a ray of light.

As with reflection there are two LAWS OF REFRACTION. These state

(1) The incident, normal and refracted rays all lie in the same plane

(2) For any two given media the ratio of the sine of the angle of incidence to the sine of the angle of refraction is a constant.

This constant is called REFRACTIVE INDEX and is usually denoted by the Greek letter μ(mu) or n.

$$\text{i.e.} \quad \mu = \frac{\sin i}{\sin r}$$

Thus from air to water as above μ is $^4/_3$.

$$\text{i.e.,} \quad \frac{\sin i}{\sin r} = \frac{4}{3}$$

For example, if i (air to water) is 25°, then

$$\frac{\sin 25°}{\sin r} = \frac{4}{3}$$

$$\sin r = \frac{3 \sin 25°}{4} = 0{\cdot}3170$$

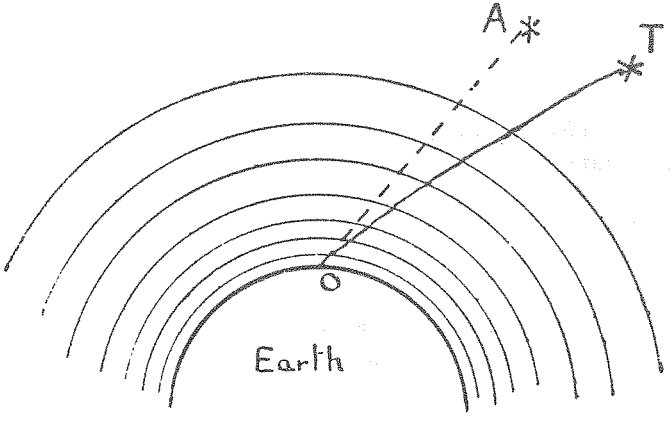
i.e., Angle of refraction = 18½°.

Table:

Refractive indices	
Air into water	$^4/_3$
Water into air	$^3/_4$
Air into glass	$^3/_2$
Glass into air	$^2/_3$

27.18 ATMOSPHERIC REFRACTION

The light from the sun, or from a star, reaches us from outer space. In order to do so, however, it has to pass through the atmosphere, very rarified when first encountered, but increasing in density as the ray approaches the observer's eye at O. Thus, the ray is progressively bent as shown in the diagram below.



Atmospheric refraction.

Remember that we see a thing in the direction in which the light from it enters the eye.

Hence, to an observer at O, the star whose true position is at T would *appear* to be at A. In short, atmospheric refraction makes the altitude of a body seem greater than it really is, and for this reason, the correction for refraction is always *subtracted* from the observed altitude.

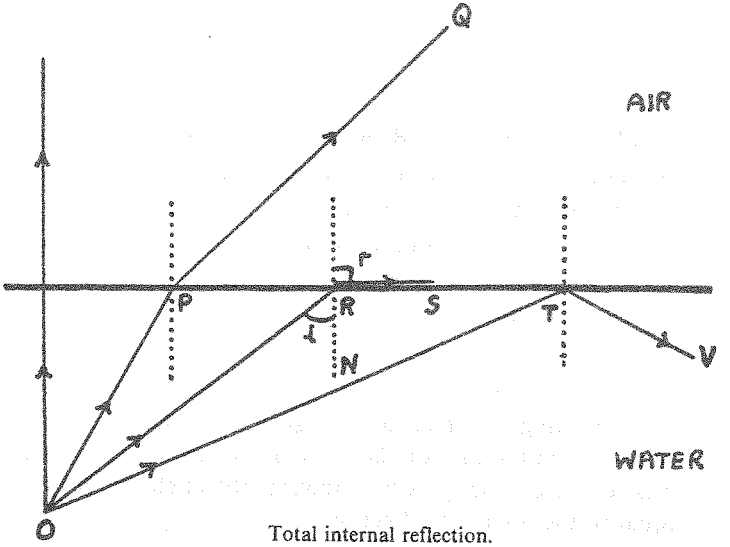
An interesting effect of atmospheric refraction is provided at the time of a total eclipse of the moon. As explained earlier, even during the period of totality, the moon still remains faintly visible. Where is the light coming from?

What we are seeing is light from the sun which has passed through several hundred miles of the Earth's atmosphere, being refracted as it does so. This refracted light is responsible for the faint illumination of the moon's disc.

27.19 TOTAL INTERNAL REFLECTION

On passing from a medium to one less dense, say from water to air, a ray inclined to the normal is bent *away* from the normal.

In the diagram below, the ray OP is refracted into the direction PQ. As the inclination of the ray to the normal is increased, there is a position in which the refracted ray lies along the surface of the water: thus the ray OR is refracted along RS.



Total internal reflection.

The angle ORN is called the **critical angle.**

Any ray, such as OT, making a greater angle with the normal than $\angle ORN$ cannot "get through" the surface and is **totally internally reflected** along TV.

Since light rays are reversible, the index of refraction for water to air must be the reciprocal of that for air to water and is, therefore ¾.

$$\text{Thus } \frac{\sin i}{\sin r} = \tfrac{3}{4}$$

At the critical angle, the angle of refraction r is 90°. Remember that the sine of 90° is unity, then

$$\sin i = \tfrac{3}{4} \text{ or more generally}$$

The sine of the critical angle is equal to the refractive index from one medium to a less dense medium.

For water to air the critical eye is $\sin^{-1}(\frac{3}{4})$ which is approximately 48½°.

From glass to air, the refractive index is approximately $^2/_3$, thus the critical angle is just less than 42°.

This fact enables us to use 45° prisms as reflectors with the avoidance of multiple reflections and tarnishing defects common to silvered plane mirrors.

27.20 PATH OF A RAY THROUGH A PRISM

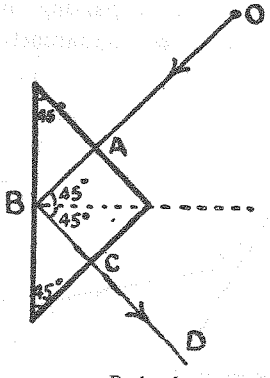
In considering the path of a ray of light through a prism, the following summary should be kept in mind.

(1) On going from a denser into a rarer medium, the ray is refracted away from the normal. From a rarer into a denser medium, towards the normal.

(2) A ray meeting a surface at right angles passes straight through without being refracted or deviating at all.

(3) A ray meeting a surface at an angle greater than the critical angle (42° in the case of glass into air) cannot pass through. It is reflected exactly as by a mirror, the angle of incidence equal to the angle of reflection.

A few examples are given below.

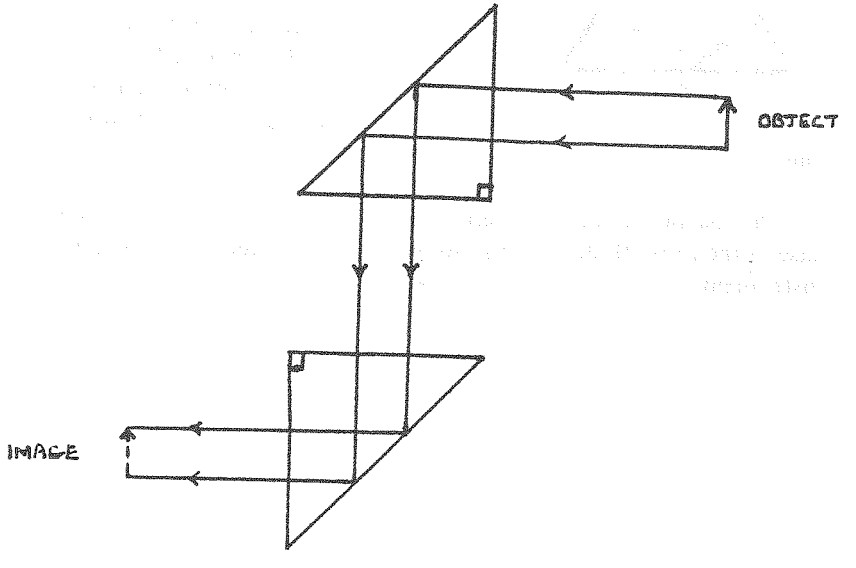


Path of a ray through a 45° prism.

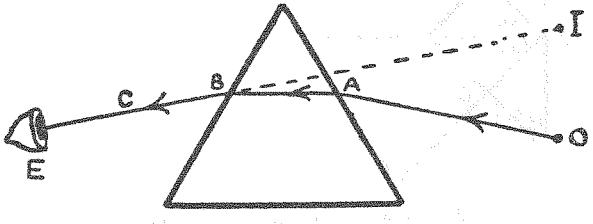
From an object at *O* in the diagram above a ray is incident normally at *A* and suffers no refraction. Passing on through the glass to *B* it is incident at 45°. Since this exceeds the critical angle, it is reflected at *B* and the ray continues to *C* where no refraction occurs, and passes out of the prism again, towards *D*.

This type of prism is much used in prism binoculars, as we shall see later.

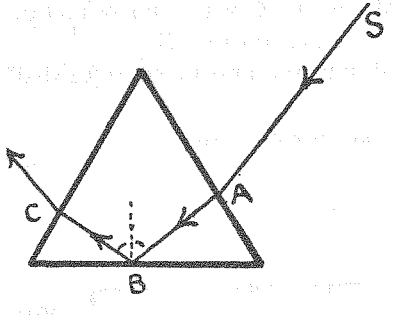
It is also used in the periscope as shown below



The diagram below, shows a ray passing through a prism, from *O*, being refracted at *A* and *B*, and finally passing out in the direction *C*. An eye at *E* would see the object, apparently at *I*.



Path of a ray through a 60° prism.



Important case of a 60° prism.

The diagram opposite shows a particularly important case. This time we have rotated the prism slightly, and are considering a ray from *S* (say, from the sun). It is refracted at *A*, but at *B* is totally reflected to *C*. Here it is refracted again, passing out in the direction *D*.

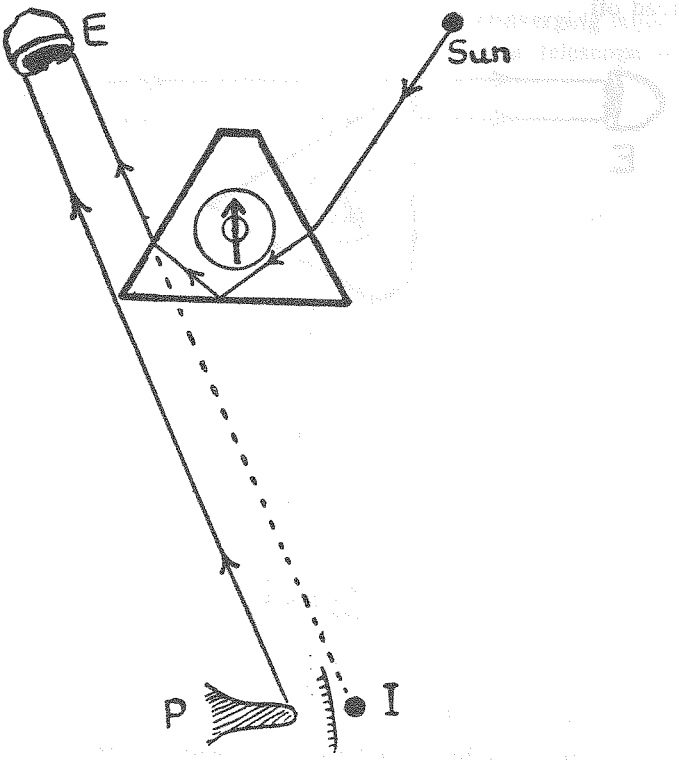
This particular arrangement has a practical application in the case of the **azimuth mirror**, and we will now consider this important instrument.

27.21 PRINCIPLE OF THE AZIMUTH MIRROR

There is more than one design of this instrument. The one we are referring to is the **KELVIN AZIMUTH MIRROR**, commonly to be found on merchant ships.

It is assumed that the reader is familiar with the purpose and general appearance of this instrument. Every opportunity should be taken of handling it and using it.

We shall only deal here with the optical principles.



Azimuth mirror, "arrow up".

(a) **Arrow up**

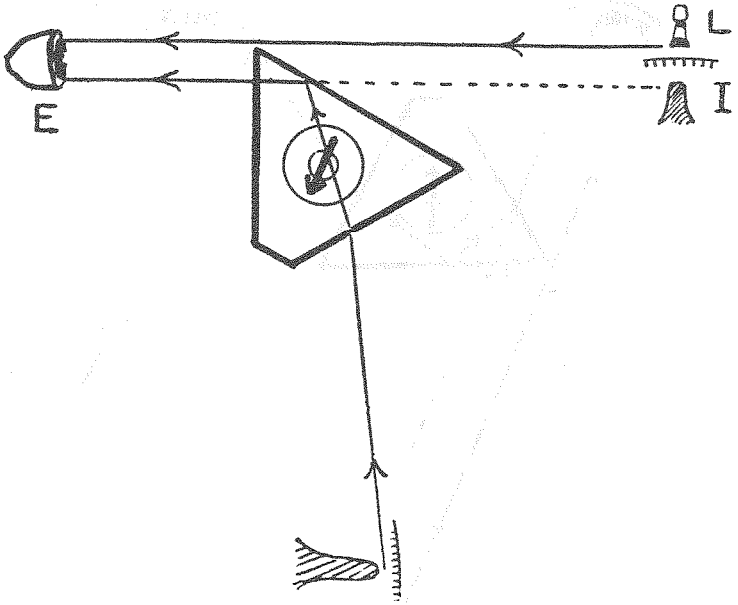
In this position, the instrument is used to obtain the bearing of a heavenly body.

The prism is a 60° one, and the diagram above illustrates what happens.

Remember that we see an object in the direction in which light from that object enters the eye.

Thus, an eye at *E* would see an image of the sun (or star) at *I*, whilst by glancing just outside the line of the prism itself, a pointer *P* is seen (by direct vision) close against the graduated rim of the compass card.

By bringing the image against the pointer, the bearing may be read off.



Azimuth mirror, "arrow down".

(*b*) **Arrow down**

In this position, the instrument is used to take a bearing of a terrestrial object.

The diagram above shows the path of the rays. In this case, an eye at *E*, will see an image of the pointer and compass card at *I*, and by raising the line of vision slightly, will see (say) a lighthouse *L*, whose bearing against the compass card can be read off.

27.22 PATH OF A RAY THROUGH A THIN LENS

The laws of refraction can be applied to lenses as to prisms.

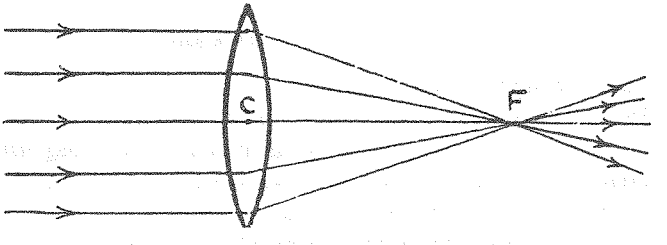
(*a*) **Convex lens**

In the diagram below we can see what happens when a pencil of parallel rays reaches the lens from a direction along the axis of the lens. Such a pencil would be from an object say, a few miles away.

Note: All rays that pass through the optical centre *C* pass straight on without deviation.

All other incident rays are refracted through a point *F*, called the **focus** of the lens.

For obvious reasons, such lenses are called **converging** lenses. They draw rays together. The object glass in a telescope or binoculars is always a converging lens.

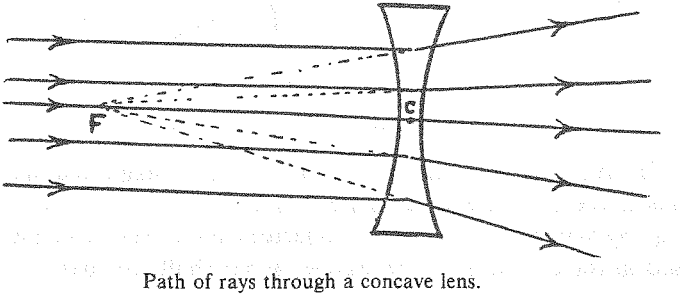


Path of rays through a convex lens.

(*b*) **Concave lens**

The diagram below shows the passage of rays through a concave lens.

In this case, all rays diverge, after refraction, from the focus *F*, which is on the same side of the lens as the incident pencil. Such lenses are sometimes called **diverging lenses.**



Path of rays through a concave lens.

27.23 IMAGE FORMED BY A LENS

To find the image of a point P, it is only necessary to draw two rays from the point; where these intersect determines the image of P.

(*a*) **Convex lens**

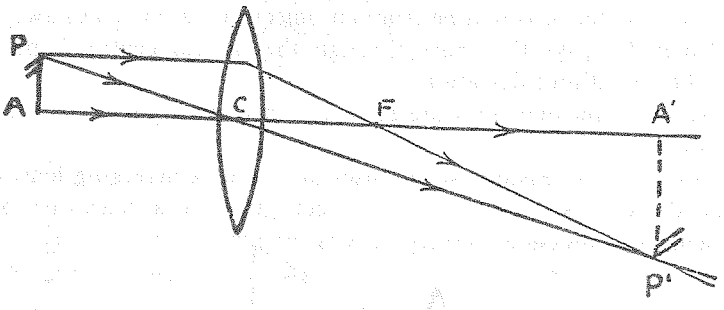


Image formed by a convex lens.

Rays through C pass straight on.

Parallel rays are refracted through the focus F.

In the above diagram the image is at A^1P^1 and is **magnified** but **inverted**. Note that it is not only reversed top to bottom, but also left to right. We shall refer to this again shortly.

If however, the object being viewed is nearer the lens than its focal length, we obtain the result shown below.

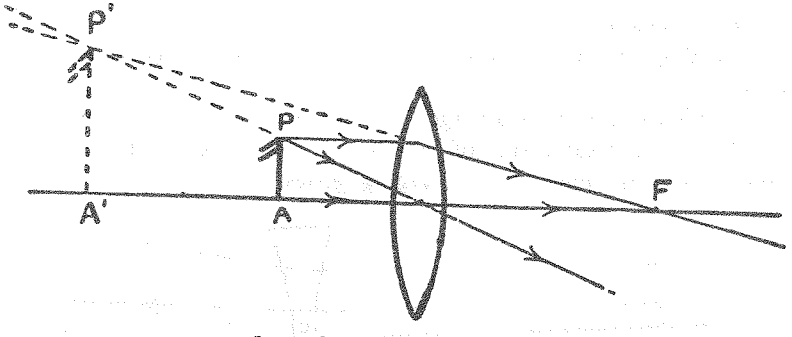


Image formed by a convex lens.

In this case, the image is at A^1P^1 and is **magnified** and **erect**. This is the principle of a single lens used as a "magnifying glass" (e.g., by watch-makers, stamp-collectors, etc.). The principle is also used in the astronomical telescope, as we shall see later.

(*b*) Concave lens

In the diagram below, the image is at A^1P^1 and is erect but diminished.

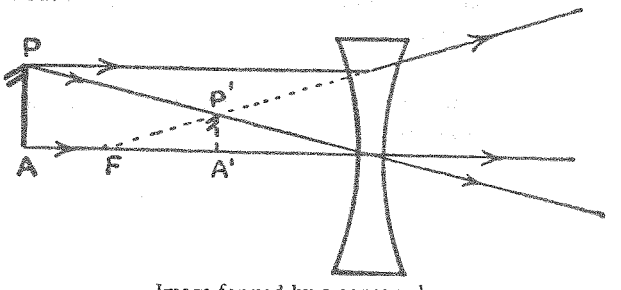


Image formed by a concave lens.

27.24 REAL AND VIRTUAL IMAGES

With regard to images being real or virtual much the same is true for lenses as for spherical mirrors.

A **real image** is one through which the rays of light actually pass after refraction in their passage through the lens..

A **virtual image**, conversely, is one through which the rays of light do not pass. It is formed by producing the refracted rays backward.

For the three diagrams drawn in the previous section the first diagram shows a REAL image whereas the other two show VIRTUAL images.

As can be seen from the ray diagrams above a convex lens has a real focus; a concave lens has a virtual focus.

A real image is always on the side of the lens remote from the object and may be received on a screen or seen by an eye so placed as to receive the rays involved in its formation (near the normal).

A virtual image cannot be received on a screen. It may however be observed by an eye suitably placed.

Again as with spherical mirrors all real images are inverted; all virtual images are erect.

EXERCISE C

1. A convex lens of focal length 40 mm has an object 10 mm tall placed in front of it. By drawing ray diagrams to scale determine the nature, position and size of the image formed when this object is placed (*a*) 20 mm (*b*) 40 mm (*c*) 60 mm (*d*) 80 mm (*e*) 100 mm in front of the lens.
2. Repeat question 1 for a concave lens of the same focal length.

27.25 IMAGES IN LENSES

From the above exercise it can be seen that, irrespective of the position of the object, the image formed in a concave lens is always virtual, erect and diminished.

The image formed in a convex lens can be real or virtual, erect or inverted, diminished or magnified or remain the same size. It all depends on where the object is placed.

Note that the above is the opposite of the situation for spherical mirrors.

27.26 THE LENS FORMULA

The formula and sign convention we used to determine the position of images in spherical mirrors can also be used for lenses as can the formula for linear magnification i.e. we can use

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f} \qquad\qquad M=\frac{v}{u}$$

WORKED EXAMPLE 4

Calculate the position, nature and size of the image formed in a convex lens of focal length 40 mm when an object 6 mm tall is placed (*a*) 20 mm (*b*) 100 mm in front of it.

(*a*) $u = 20$ mm

$v = ?$

$f = +40$ mm (convex lens has a REAL focus, hence +)

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

$$\frac{1}{v}+\frac{1}{20}=\frac{1}{40}$$

$$\frac{1}{v}=\frac{1}{40}-\frac{1}{20}$$

$$v = -40 \text{ mm}$$

The negative sign indicates the image is virtual. This means it is 40 mm IN FRONT OF the lens. Also being virtual the image must be erect.

Also

$$m = \frac{v}{u}$$

$$= \frac{-40}{20}$$

$= -2$ the negative sign merely indicating the image is virtual

The linear magnification is 2. The image is 2× the height of the object i.e. 2× 6 mm = 12 mm tall.

Summarizing there is a magnified, virtual, erect image 12 mm tall in front of the lens.

(*b*) $u = 100$ mm

$v = ?$

$f = +40$ mm

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} + \frac{1}{100} = \frac{1}{40}$$

$$\frac{1}{v} = \frac{1}{40} - \frac{1}{100}$$

$$v = +66^2/_3 \text{ mm}$$

This time the image is REAL and therefore inverted.

$$M = \frac{v}{u}$$

$$= \frac{66^2/_3}{100}$$

$$= \frac{2}{3}$$

The image size is $\frac{2}{3} \times 6$ mm = 4 mm

Summarizing there is a diminished, real, inverted image 4 mm tall behind the lens.

WORKED EXAMPLE 5

Repeat the calculations of Worked Example 4 for a concave lens of focal length 40 mm.

(*a*) $u = 20$ mm

$v = ?$

$f = -40$ mm (concave lens has a VIRTUAL focus, hence $-$)

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} + \frac{1}{20} = \frac{1}{-40}$$

$$\frac{1}{v} = -\frac{1}{40} - \frac{1}{20}$$

$$v = -13^1/_3 \text{ mm}$$

The negative sign indicates the image is virtual and therefore erect.

$$M = \frac{v}{u}$$

$$= -\frac{13^1/_3}{20}$$

$$= -\frac{2}{3}$$

The image size is $\frac{2}{3} \times 6$ mm $= 4$ mm

Summarizing there is a diminished, virtual, erect image 4 mm tall in front of the lens.

(*b*) $u = 100$ mm

$v = ?$

$f = -40$ mm

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} + \frac{1}{100} = \frac{1}{-40}$$

$$\frac{1}{v} = \frac{1}{-40} - \frac{1}{100}$$

$$v = -200/_7 \text{ mm } (28{\cdot}57 \text{ mm})$$

Again the image is virtual and therefore erect.

$$M = \frac{v}{u}$$
$$= \frac{200}{7} \div 100$$
$$= \frac{2}{7}$$

The image size is $\frac{2}{7} \times 6\text{ mm} = {}^{12}/_{7}$ (1·714 mm)

Summarizing there is a diminished, virtual, erect image 1·7 mm tall in front of the lens.

EXERCISE D

1. A convex lens of focal length 40 mm has an object 10 mm tall placed in front of it. Calculate the position, size and nature of the image formed when the object is placed (*a*) 60 mm (*b*) 80 mm (*c*) 40 mm in front of the mirror.
2. Repeat question 1 for a concave lens of the same focal length.

27.27 PRINCIPLE OF ASTRONOMICAL TELESCOPE

The optical system of an astronomical telescope consists of two convex lenses,

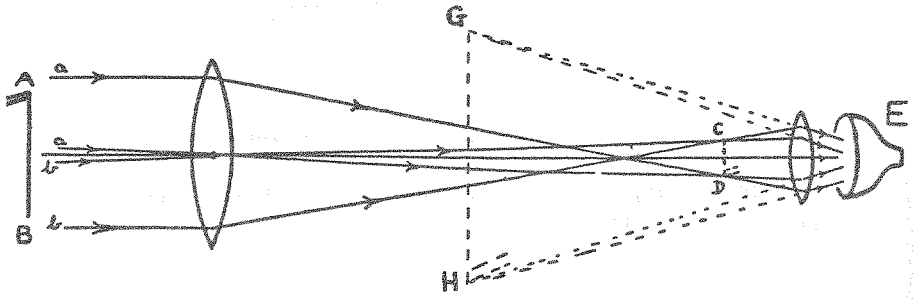
(*a*) the object glass, which produces an inverted image of a distant object, and

(*b*) the eyepiece, which produces an enlarged image of the first image.

The arrangement is illustrated in the diagram below. It should be borne in mind that the object, *AB*, is at infinity; both rays marked *a* come from *A*, and both marked *b* from *B*. The first image is at *CD*, and being viewed through the eyepiece at *E*, is seen enlarged, but still inverted, at *GH*.

The advantage of such a telescope lies in the absence of any prisms or lenses other than the two shown. Thus maximum illumination (since every lens the light has to pass through absorbs a little of it) is obtained, combined with clear definition. It is much

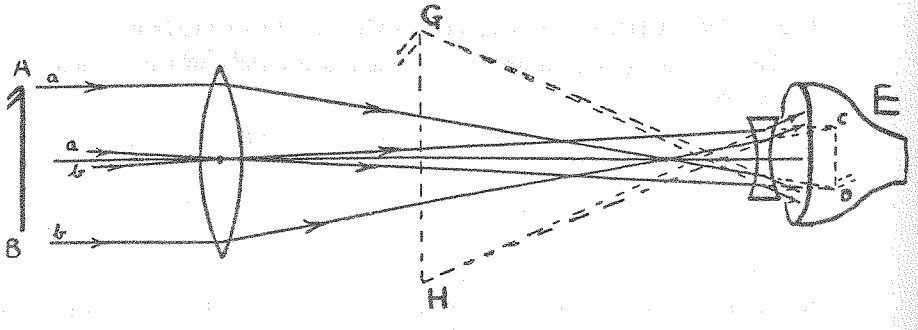
used for astronomical work. The inversion is too great a disadvantage for terrestrial work however.



Astronomical telescope.

27.28 GALILEO'S TELESCOPE

This was devised by Galileo early in the 17th century, about the same time as Kepler devised the astronomical telescope. The essential requirement was to avoid the inversion which was such a drawback to Kepler's instrument. The diagram below shows how this was done.

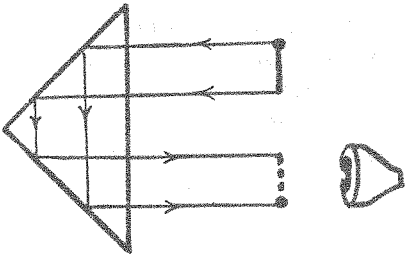


Galileo's telescope.

If the rays from the object glass were allowed to come to a focus, then an inverted image would be formed at *CD*. Before this can happen however, the rays are made to fall upon a concave lens, which diverges them as shown, and an eye at *E* would see a magnified, erect image at *GH*.

The field of such a telescope is smaller than that of the astronomical and also the illumination is not so good. It is, however, used for all normal terrestrial purposes where an erect view is important. Opera glasses and binoculars consist of two such telescopes side by side.

27.29 THE INVERTING PROPERTY OF A PRISM



Inverting effect of single prism.

We have already shown how a ray may be totally reflected by a 45° prism. The diagram opposite shows the same principle, this time applied to an object, however, instead of a single ray from a point.

Clearly, by passing through one prism, inversion of the object takes place, but in one direction only — in this case, top to bottom, but *not* left to right.

In order to complete the inversion a second prism is needed, placed at 90° to the first. This is illustrated in the diagram below.

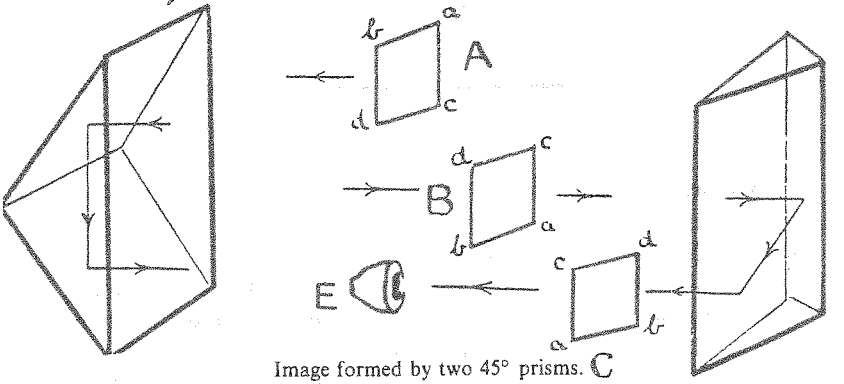


Image formed by two 45° prisms.

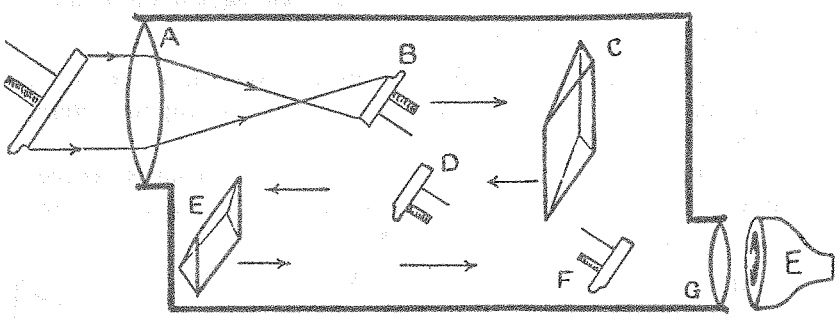
The object is at *A*. After passing through the first prism, the image at *B* is shown inverted top to bottom. The second prism inverts it left to right. This is shown at *C*, as it would be seen by the eye *E*. This is the principle employed in the prism binoculars.

27.30 THE PRISM BINOCULARS

These were developed by Zeiss towards the end of the 19th century. There was a great need for binoculars that would retain the advantages of the astronomical telescope, without its disadvantages. Galileo's instrument, it is true, got rid of the inverted image, but had a more restricted field of view and there was also some loss in illumination and in clear definition.

In any case, both telescopes had to be long if any worthwhile degree of magnification was to be achieved, which made it impracticable to devise binoculars on Galileo's principle which would be powerful enough for all requirements.

The basic idea of the prism binoculars is to use the optical system of the astronomical telescope, with its optical length "folded up" by means of prisms. The diagram below shows how this was done.



Principle of prism binoculars.

No attempt has been made to trace the individual rays through the optical system. Enough has been said in the preceding paragraphs to make the following facts clear.

(*a*) The convex lens at *A* will form an inverted image at *B*.

(*b*) By means of the prism *C*, this image is reversed left to right, as at *D*. It is still upside down.

(*c*) At *E*, the prism will reverse it top to bottom, so that the image is now erect, as at *F*.

(*d*) The convex lens at *G* will produce a magnified image of the image *F*. This magnified image is seen by the eye placed against *G*.

The prism binoculars thus retain the wide field of view of the astronomical telescope and also its high degree of magnification.

Binoculars are classified in terms of two numbers such as 7×50. The "7" is the linear magnification of the binoculars, this being the size of the final image obtained compared to the size of the image in the naked eye i.e. without using the binoculars. The "50" is the diameter of the objective lens A and is quoted in millimetres. This diameter controls the amount of light entering the binoculars. Obviously the bigger the diameter the better is the light-collecting ability of the binoculars.

There is some loss of illumination due to the additional lenses the rays now have to pass through. To offset this, however, they do possess the additional advantage over the telescope in that binocular ("two-eyed") vision is provided. As we know the fact that the eyes are set somewhat apart enables us to judge the relative distances of objects — stereoscopic effect, as it is called.

In the prism binoculars, the two object lenses are set even wider apart than the eyes, thus improving our stereoscopic vision still more.

TEST PAPER 27
Light

Draw a careful diagram in every case.

1. Give the formula for determining μ in terms of the angle of incidence and the angle of refraction.

   Taking μ air into glass as $\frac{3}{2}$, calculate the angle of refraction for a ray incident on the surface of a glass block at 32°.
2. A ray from the bottom of a tank of water is incident on the water surface at 36°. If μ (water into air) is $\frac{3}{4}$, find the angle of refraction. Calculate also the critical angle for water.
3. Why is it that a tank containing clear water appears less deep than it really is?

   A ray from a small object on the bottom of a tank 2·4 m deep is incident on the surface at 5°. If μ water into air is 0·75 calculate the (apparent) depth of water as it would appear to an eye so placed as to receive the refracted ray and show by

this numerical example that for a ray incident near the normal,

$$\text{Apparent depth} = \mu \times \text{Real depth.}$$

4. *(a)* State the meaning of the term 'centre of curvature' of a spherical mirror.
   (b) A concave spherical mirror has a focal length of 50 mm. State whether it is possible to project an image on to a screen and, if so, how far the screen should be placed from the mirror, when an object is placed on the principal axis:
   (i) 90 mm in front of the mirror;
   (ii) 20 mm in front of the mirror.
   (SCOTVEC December 1990).
5. The image of an object in a convex mirror is 30 mm from the mirror. If the mirror has a radius of curvature of 100 mm, calculate:
   (a) the position of the object;
   (b) the magnification.
   (SCOTVEC July 1990).
6. *(a)* State the meaning of the term a virtual image.
   (b) An object 10 mm high is placed in front of a concave spherical mirror of radius of curvature 60 mm. Find the size, nature and position of the image produced if the object is placed:
   (i) 80 mm from the mirror;
   (ii) 20 mm from the mirror.
   (SCOTVEC July 1993).
7. An object is 30 cm distant from a thin concave lens of focal length 15 cm. Find the position of the image and state its type and magnification.
8. Define the focus of a lens. A small source of light and a screen are placed 3·0 m apart. Where should a convex lens of focal length 40 cm be placed so as to form a real image of the source on a screen.
9. A convex lens placed 24 cm from an object forms a virtual image 72 cm from the lens. Find the focal length of the lens and the magnification.
10. A convex lens of focal length 20 cm is used as a magnifying glass to produce a virtual image of an object, enlarged four times. Where must the object be placed?

11. A convex lens has a focal length of 80 cm. Show by diagrams how it may be used to produce *(a)* a virtual image *(b)* a real image, in each case with a magnification of 4 and calculate the exact position of the object in each case.
12. A man whose eye is 1·75 m from the gound faces a vertical plane mirror standing on the ground. Find the height of the mirror if he can only just see his feet in the mirror.

    If he now stands 3·0 m from a wall to which a mirror is attached, find the vertical length of the mirror if he can just see his whole image from feet to top of hat, which is 30 cm above eye level.

CHAPTER 28

GENERAL WAVE PHENOMENA — SOUND — ELEMENTS OF RADIO

28.1 SOUND WAVES

Mention has more than once been made to light energy, heat energy — and now, to sound energy, electro-magnetic (or radio) energy, and to the phenomenon of this energy being transmitted through some intervening space.

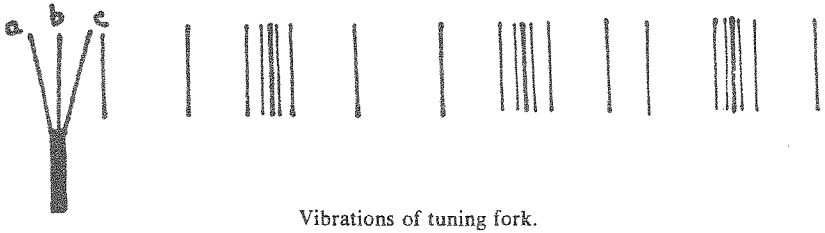
There is one thing, amongst others, which distinguishes the transmission of sound energy from the other forms, namely, **there must be some intervening medium**. Thus, although light, heat, radio can be transmitted over limitless distances in outer space, sound cannot be transmitted through a vacuum.

Usually the intervening medium is air, but sound may be transmitted through liquids — again, water is the commonest — or through solids, such as a long steel rod.

28.2 SENSATION OF SOUND

The sensation known as "sound" is caused by vibrations in the sounding body, or "source of sound".

Consider a tuning fork vibrating as in the diagram below.



Vibrations of tuning fork.

It may be noted in passing that the vibrations are S.H.M.

Hence the tuning fork is at rest at *a*, has maximum velocity at *b*, is at rest at *c* — and the motion is repeated back to *a*.

Each complete to and fro motion is called a vibration, or **cycle**. The number of such cycles per second is called the **frequency**.

The vibrations of the sounding body will cause alternately **compressions** and **rarefactions** in the medium (e.g., air) with the same frequency as the sounding body. This is called the **frequency** (symbol f) **of this particular sound sensation.**

The shock waves produced by the sounding body travel through the medium with a velocity called the **velocity of sound** (in that medium). Except for slight variations with the temperature or density of the medium, the velocity of sound is about **constant** for any given medium (air about 330 m/s).

The SI unit of frequency is the **hertz (Hz)**, and by definition.

1 Hz is a frequency of 1 cycle per second . N.B.

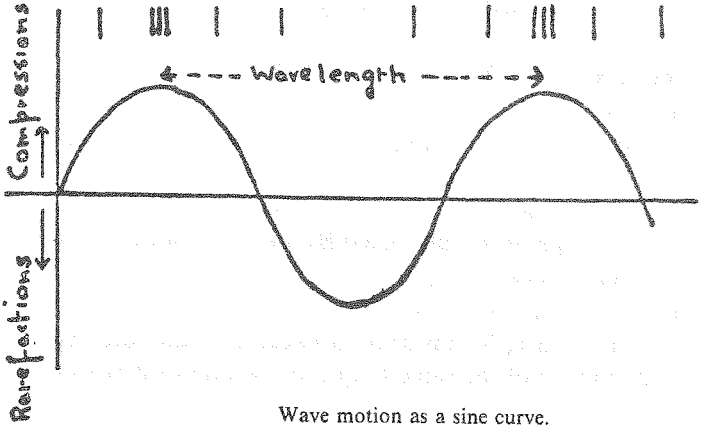
This definition applies to all forms of oscillatory motion — not merely to sound.

Multiples are the kilohertz (kHz) and megahertz (MHz).

28.3 TERMS USED — WAVELENGTH (SYMBOL λ)

The series of compressions and rarefactions shown in the diagram above, and which radiate uniformly outward from the source are called **longitudinal vibrations** because the vibration, the to-and-fro motion of the medium, is taking place in the same direction as the wave itself is progressing. (This is in contrast, for example, to electromagnetic waves, where the vibrations are transverse — at right angles to the line of progress of the wave).

Sound waves may conveniently be illustrated by a sine curve as in the diagram below.



Wave motion as a sine curve.

Wavelength is defined as the distance between two successive compressions, or, in general, between any two adjacent positions which are in the same phase.

Relationship between velocity, wavelength and frequency

Since the velocity is constant (very nearly) consider the velocity of sound in air, i.e., 330 m/s.

It is clear that if the frequency is, for example, 110 then there are 110 compressions within 330 m and so

$$\text{wavelength} = \frac{330}{110} = 3 \text{ m}$$

This is quite general — in fact, the terms used here in connection with sound waves apply to all forms of wave motion, and clearly,

$$\text{wavelength} = \frac{\text{velocity}}{\text{frequency}}, \text{ or as it is more usually expressed,}$$

$$\textbf{velocity} = \textbf{frequency} \times \textbf{wavelength}$$

$$\text{or } V = f \times \lambda \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{ I}$$

EXERCISE A

1. Calculate the velocity of a sound wave which has a frequency of 500 Hz and a wavelength of 0·664 m.
2. Calculate the wavelength of a sound wave whose velocity is 340 m/s and whose frequency is 1000 Hz,
3. The wavelength of a radio wave is 1500 m. Its velocity is 3×10^8 m/s. Calculate its frequency.

28.4 VARIATION IN TYPES OF SOUND

(*a*) **Pitch**

This depends solely on the frequency. High frequency causes a high pitched note, and conversely.

The lowest frequency to which the human ear is sensitive is about 50 Hz, the highest about 15,000 Hz though naturally these vary between individuals.

(*b*) **Quality**

This is the difference, for instance, between a violin note and a trumpet. Both may be of the same frequency, yet they differ to the ear.

This is due to the purity or otherwise of the note. The **fundamental frequency** determines the pitch. If other frequencies, called **overtones**, are mixed in with it, then the note is not so pure.

(*c*) **Loudness**

This depends on the **energy** contained in the sound waves, which in turn depends on the **amplitude** of the vibrations of the sounding body. The greater the amplitude, the louder the note, though the pitch and quality should remain the same.

28.5 EFFECT OF PRESSURE, TEMPERATURE AND HUMIDITY ON VELOCITY OF SOUND

It can be shown that the velocity of sound in air is given by

$$V = \sqrt{\frac{1{\cdot}41p}{\rho \text{ (rho)}}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{II}$$

where p = pressure in N/m^2
ρ (rho) = density in kg/m^3
and V will be in m/s.

Although not possible to give proof of this formula here, it should be studied carefully, as this will enable us to appreciate more clearly what follows.

Effect on velocity of sound

(*a*) **Pressure**

No effect by varying pressure, because density is directly proportional to pressure, so in II, p/ρ is constant.

Velocity of sound in air is not affected by changing atmospheric pressure ..... **III**

(*b*) **Temperature**

If temperature increases, density decreases. From II, velocity varies inversely as $\sqrt{d}$, that is directly as $\sqrt{\text{temperature}}$. Hence,

Velocity of sound increases with rise in temperature ..... **IV**

More exactly, velocity varies directly as the square root of the absolute temperature.

i.e. $$\frac{V_1}{V_2} = \sqrt{\frac{T_1}{T_2}}$$

(*c*) **Density**

Again it may be seen by inspection of II that the **velocity varies inversely as the square root of the density.**

In conditions of exceptional **humidity**, as in fog, since the density of water vapour is less than that of dry air, it follows that the density of damp air is less than that of dry air.

Velocity of sound is greater in damp air than in dry air V

WORKED EXAMPLE 1

At 0° C the velocity of sound was found to 330 m/s. Calculate its velocity at 15° C.

$V_1 = 330$ m/s $\qquad T_1 = 0°\text{C} = 273$ K

$V_2 = ?$ $\qquad T_2 = 15°\text{C} = 288$ K

$$\frac{V_1}{V_2} = \sqrt{\frac{T_1}{T_2}}$$

$$\frac{330}{V_2} = \sqrt{\frac{273}{288}}$$

$$\frac{330}{V_2} = 0{\cdot}9736$$

$$V_2 = \frac{330}{0{\cdot}9736} \text{ m/s}$$

$$V_2 = 338{\cdot}9 \text{ m/s}$$

EXERCISE B

1. A sound wave of frequency 600 Hz produces waves of wavelength 0·55 m in air at a temperature of 27° C.
   Calculate (*a*) the velocity of sound at 27° C.
   (*b*) the velocity of sound at 10° C.
   (*c*) the wavelength of the sound waves at 10° C.
2. The velocity of sound was measured to be 331 m/s at 0° C. At what temperature would its velocity be 350 m/s?

28.6 VELOCITY OF SOUND IN SEA WATER

The velocity of sound in sea water depends on the temperature, pressure and salinity of the water.

In general the greater the temperature, the pressure or the salinity the greater is the speed of sound.

The sea temperature drops quickly in the first 800 m or so and the fall in temperature outweighs the increase in pressure as far as the velocity of sound goes and it too falls. Below this depth the fall in temperature is very gradual and the increase in pressure becomes the dominant factor. At a particular place in the ocean the salinity can be regarded as constant and therefore does not affect the above discussion. The velocity of sound in sea water varies between about 1450 m/s and 1500 m/s.

28.7 DOPPLER EFFECT

This is the change in frequency due to the movement of source or observer or both.

Remembering that the sensation of sound from a source depends on the number of compressions per second reaching an observer, if the source is also moving towards the observer, or the observer towards the source, then **more compressions will pass the observer's ear in any given second.**

In other words, the **frequency is increased** and the **pitch of the note is higher.**

The converse is the case if observer and/or source are moving **away** from each other — the frequency is diminished and the pitch is lower.

There are many other applications of Doppler's principle. For example, the velocity of a star along its line of approach relative to the Earth may be determined from exact measurement of the observed wave-length of light waves from the star. An increased wave-length (or in other words, a decreased frequency) indicates that the star is receding from the observer, and conversely.

The principle of the Doppler Effect, using ultrasonic (high frequency) waves, is also used in the Doppler Log for measuring the speed of a vessel. Police radar speed checks also use the Doppler Effect.

Set out below are the formulae for four possible Doppler Effect situations:

(*a*) Source moving towards a stationary observer with a velocity U_S.

It can be shown that

$$f' = \left(\frac{V}{V - U_S}\right) f$$

where V is the velocity of the sound waves.

f is the frequency of the sound waves.

and f' is the apparent frequency heard by the observer.

(*b*) Source moving away from a stationary observer.

$$f' = \left(\frac{V}{V + U_S}\right) f$$

(*c*) Observer moving towards a stationary source with a velocity U_O.

$$f' = \left(\frac{V + U_O}{V}\right) f$$

(*d*) Observer moving away from a stationary source.

$$f' = \left(\frac{V - U_O}{V}\right) f$$

WORKED EXAMPLE 2

A source of sound approaches a stationary observer at 20 m/s. The sound has a frequency of 5000 Hz. What is the apparent frequency to the observer if the velocity of sound is 330 m/s?

$U_S = 20$ m/s $\quad V = 330$ m/s $\quad f = 5000$ Hz

Source moving towards observer

$$f' = \left(\frac{V}{V - U_S}\right) f$$

$$= \left(\frac{330}{330 - 20}\right) 5000 \text{ Hz}$$

$$= 5323 \text{ Hz}$$

EXERCISE C

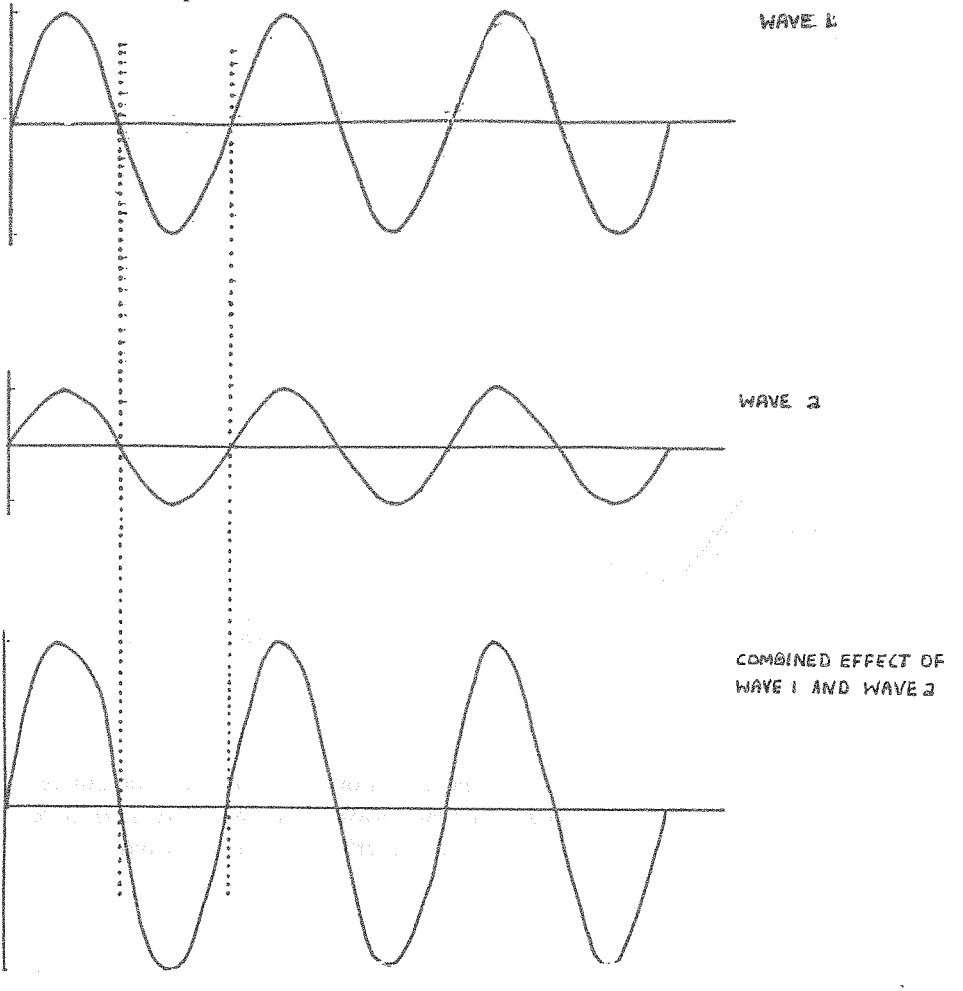
In the following questions assume the velocity of sound is 330 m/s.

1. Vessel A is stationary whilst vessel B is moving at 8 m/s. Vessel B sounds a whistle of frequency 1000 Hz. Calculate the apparent frequencies heard on vessel A when B is
   (*a*) approaching A.
   (*b*) sailing away from A.

2. If in question 1 vessel A rather than vessel B was sounding the whistle what would be the apparent frequencies heard on B as B?
   (a) approached A.
   (b) sailed away from A.

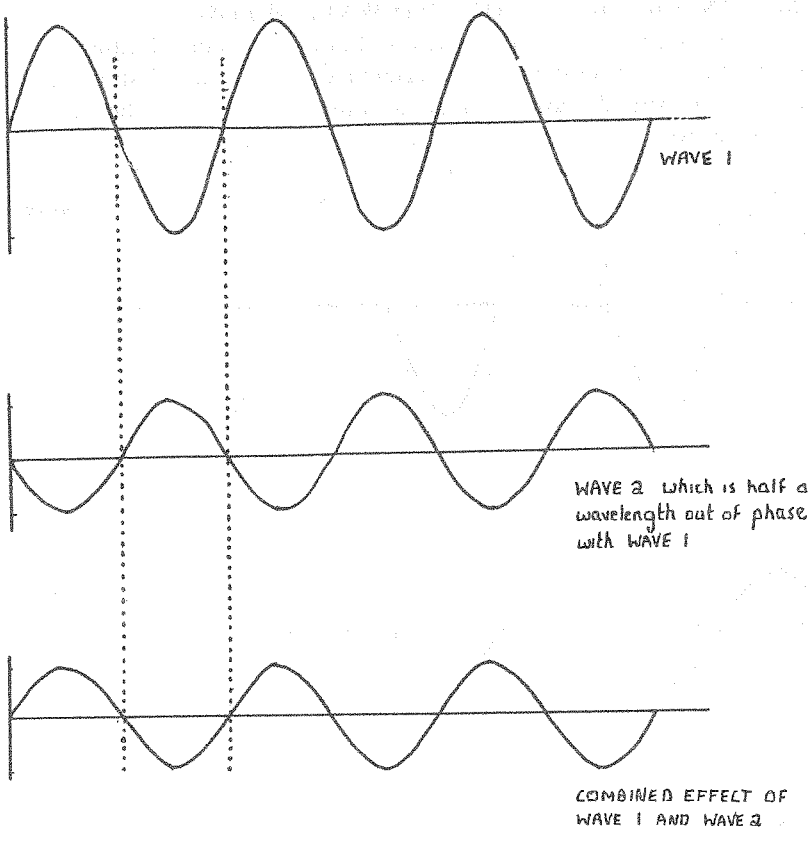
28.8 INTERFERENCE OF TWO WAVE MOTIONS

This occurs when two waves have the same frequency. Constructive or destructive interference can occur. Using the example of sound waves suppose an observer is exactly the same distance from two sound waves of the same frequency i.e. the two waves are in phase as shown below.



In this case constructive interference has occurred and the observer will hear a loud note. This phenomenon is called RESONANCE.

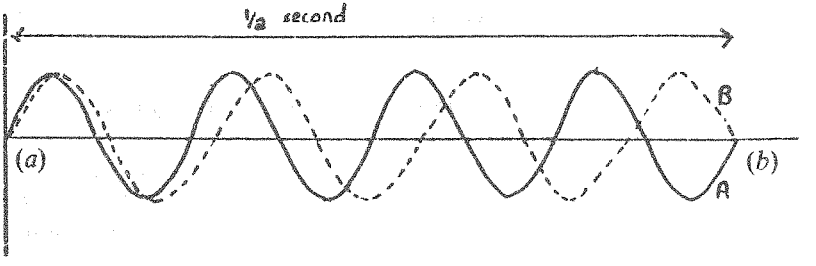
If one of the sound sources is now moved up by half a wavelength then destructive interference will occur as shown below.



Waves 1 and 2 tend to cancel each other out and a much quieter sound is heard. Indeed if the two waves had the same amplitude silence would prevail! Maximum interference has occurred.

28.9 BEATS

This is the term given to a rising and falling in loudness — a pulsation — in the sound. It occurs when two sounding bodies have *nearly* the same frequency.



Beats.

Take a simple case,

A . . . , 8 cycles per sec (8 Hz)

B . . . , 7 cycles per sec (7 Hz)

In ½ second, a stationary observer would receive

4 cycles of A

3½ cycles of B.

Thus, as point (a) passed his ear, he would receive compression waves of both.

Half a second later, 4 waves of A but only 3½ waves of B, will have passed, and he will experience maximum interference, as at (b).

Half a second later he again receives both compressions simultaneously.

Thus, he receives 1 beat per second, the difference between their frequencies.

This is quite general:

No of beats per sec = difference between frequencies . . . . . **VI**

28.10 PHASE COMPARISON — THE DECCA PRINCIPLE

Let us consider two radio transmitters, each continuously sending out an electro-magnetic wave, each exactly in phase with the other.

From what has been said earlier, it is clear that at any point equidistant from each transmitter the electro-magnetic waves would reach a receiver exactly in phase with one another. If this condition could be accurately measured (electronically) it would give us a position line — since there will be many such points — a P.L. upon which the receiver must lie, and on which the signals are received **in phase**.

Similarly, at a point half a wavelength nearer to one than the other, the signals would be **half a cycle, 180°, out of phase**, which gives another position line.

These two position lines would be quite a distance apart of course — and let us repeat that they represent two sets of conditions at the receiver, one in which the signals are in phase, the second in which the signals are 180° out of phase.

In between these two lines, there must be many other position lines upon which the signals are 20° out of phase, 40° out of phase and so on. If the **phase-difference** at any place can be measured, therefore, we would at once have a means of deciding which position line the receiver is on. This is the Decca principle.

28.11 DISTANCE BY ECHO

It seems clear that if a pulse of energy in wave form can be sent out from a source, so that it strikes some object such as a vertical cliff and is reflected back to the source, then if we can measure the time taken by the pulse on its journey, we shall have a ready means of estimating the distance of the object. This principle is used in the echo sounder and in radar. It should be noted that for the human ear to be able to distinguish between a sound and its echo the two must be at least $^1/_{10}$th of a second apart.

The echo sounder transmits a sound signal to the sea bed and the reflected echo returns to the ship, where it is received. The time elapsed is measured electronically and converted into depths for convenience in direct reading, since the velocity of sound in sea water is known.

Similarly, in marine radar a pulse of energy in the form of an electro-magnetic wave is sent out from a transmitter, is reflected from some object and returns to the transmitter. The time elapsed is measured electronically.

A fuller treatment of both these instruments, and of Decca, may be found in any textbook on electronic aids to navigation.

WORKED EXAMPLE 3

Find the depth of water under a vessel's keel if an echometer signal is received after an interval of 3·6 seconds. Assume the velocity of sound in the water to be 1450 m/s.

$$\text{velocity} = \frac{\text{distance}}{\text{time}}$$

$$1450 = \frac{x}{3{\cdot}6}$$

$$x = 5220 \text{ m}$$

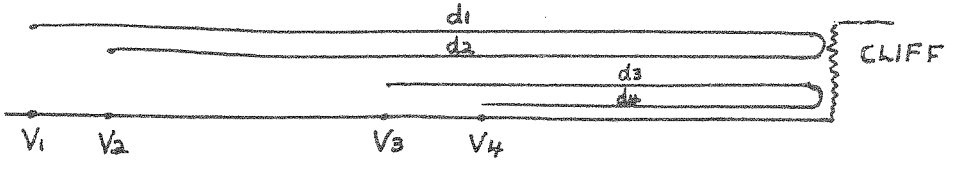
The sound waves travel a distance of 5220 m from the vessel to the sea-bed AND BACK AGAIN. Therefore the depth of water is $^{5220}/_{2}$ m = 2610 m.

WORKED EXAMPLE 4

A launch approaching a cliff at reduced speed sounds a short blast and the echo is heard after 10 seconds. Five minutes after sounding the first blast she again sounds a blast and the echo is heard after 8 seconds. What is the speed of the launch? (Velocity of sound = 330 m/s).

This is quite an involved problem.

The launch's positions are shown on the diagram below.



The launch sounds the first blast at V_1 and receives the echo at V_2.

The launch sounds the second blast at V_3 and receives the echo at V_4.

Let the launch be travelling with a velocity v m/s.

Consider the first echo and employ velocity $= \frac{\text{distance}}{\text{time}}$

for the sound wave and for the launch.

Sound wave $\qquad 330 = \frac{d_1 + d_2}{10}$

$$3300 = d_1 + d_2 \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Launch $v = \dfrac{d_1 - d_2}{10}$

$10\,v = d_1 - d_2$ ..... (2)

Adding (1) and (2)

$3300 + 10\,v = 2d_1$

$1650 + 5\,v = d_1$ ..... (3)

For the second echo:

Sound wave $330 = \dfrac{d_3 + d_4}{8}$

$2640 = d_3 + d_4$ ..... (4)

Launch $v = \dfrac{d_3 - d_4}{8}$

$8\,v = d_3 - d_4$ ..... (5)

Adding (3) and (4)

$2640 + 8\,v = 2d_3$

$1320 + 4\,v = d_3$ ..... (6)

Now to travel from V_1 to V_3 the launch takes 5 minutes i.e. 300 s.

Again using velocity $= \dfrac{\text{distance}}{\text{time}}$ we have

$v = \dfrac{d_1 - d_3}{300}$

$300\,v = d_1 - d_3$

Substituting from (3) for d_1 and from (6) for d_3 we have

$300\,v = 1650 + 5\,v - (1320 + 4\,v)$

$300\,v = 1650 + 5v - 1320 - 4\,v$

$300\,v = 330 + v$

$299\,v = 300$

$v = 1{\cdot}1$ m/s.

EXERCISE D

1. A vessel is approaching a cliff and gives a short blast when 1 kilometre distant by radar. The echo reaches the vessel after 5·7 seconds. If the speed of sound is 340 m/s what is the vessel's speed?
2. A launch is approaching a cliff at reduced speed. She gives a short blast and the echo is heard after 10 seconds. She proceeds at constant speed for 5 minutes when the engine is put full astern and she anchors. A second blast is now sounded and the echo is heard after 7 seconds. If the speed of sound is 332 m/s, how far is she now from the cliff and what was her speed between blasts?

28.12 HEAT AND LIGHT WAVES — REFRACTION

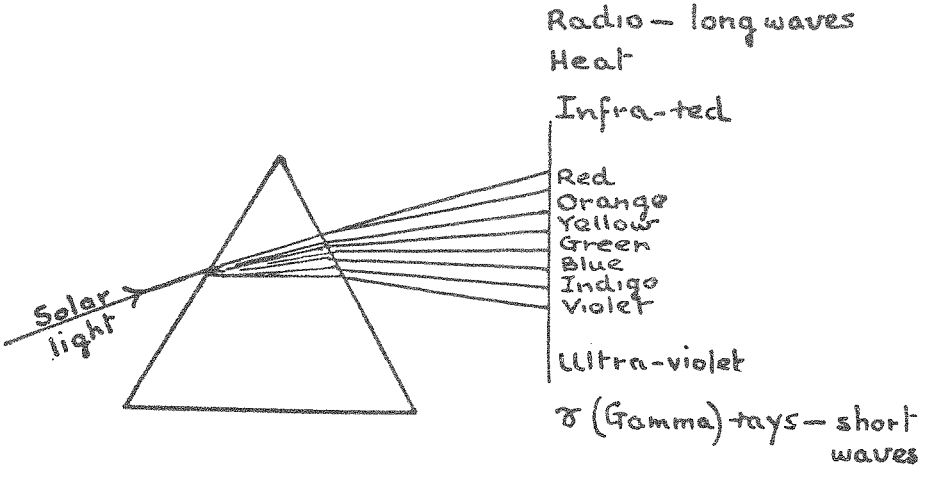
If pure white light is allowed to fall on to a prism, or even on to globules of water vapour in the atmosphere, it will be split up into separate colours, called the **spectrum**. The rainbow is a common example of this.

The sketch illustrates the manner in which the spectrum may be demonstrated in a laboratory. The order in which the colours appear is always the same, and is sometimes remembered by "Read Out Your Good Book In Verse". This phenomenon is explained by the theory that different colours of light have different wavelengths and so are differently refracted by the prism. The red rays have the longest wavelength, (or lowest frequency, if preferred, since Velocity is constant and $V = f \times \lambda$, so if λ is long, f is smaller, being about 0·01 cm).

The violet rays have the shortest wavelength of about 0·00001 cm (10^{-7}m).

Since light is the result of a body being heated to incandescence, light and radiant heat have the same character.

The whole spectrum is in fact enormously greater than the visible part, and it (the complete spectrum) consists of rays of different frequencies, which in turn determines many of their different behaviourisms.



Visible spectrum, with extensions.

The hottest part of the spectrum is in the dark space just beyond the red end of the visible spectrum, shown as the **infra-red** in the diagram above. The shortest wavelengths are beyond the violet, the **ultra-violet** rays, which are important in photography, and beyond that again to the gamma rays, with wavelengths as small as 10^{-10} cm.

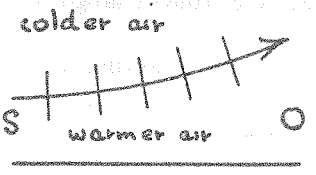
In the other direction, we pass through the heat spectrum and into the radio spectrum where the longest wave-lengths may be 10^{7} cm.

Thus, the range of known wavelengths of wave forms all having the same character as light waves extends continuously from about 10^{7} cm for the longest radio waves to about 10^{-10} cm for the shortest gamma rays. The range is known as the ELECTROMAGNETIC SPECTRUM. All the waves in it, including light, travel at 3×10^{8} m/s.

28.13 REFRACTION OF SOUND WAVES

By this, we mean a change in the direction of sound waves, due usually to atmospheric conditions.

By Day

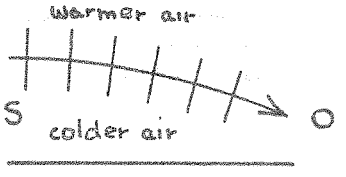


S is the source, *O* the observer.

Air is warmer near the surface.

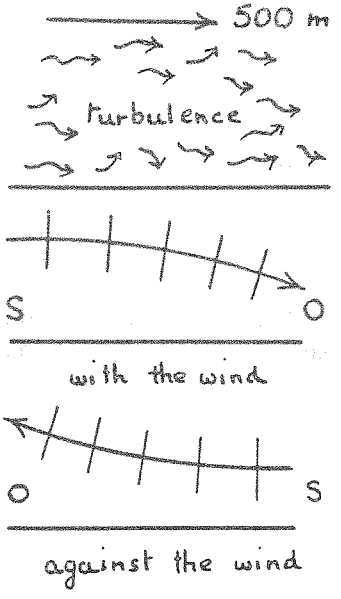
Sound travels faster with increase in temperature; hence the waves tend to be refracted away from the surface and **sounds do not carry so far by day.**

By night



The reverse conditions may be found, and the waves are refracted towards the surface and so **sounds carry further by night.**

Wind Effect



The friction of the air stream with the surface causes turbulence and reduced wind velocity in the lower layers. Hence, the sound waves are refracted as shown and sound reaches an observer more clearly. **Sounds carry better down-wind.**

The reverse is the case if an observer is up-wind from the source. The sound waves are refracted away from the surface and he **hears the sound less distinctly.**

28.14 THE RADIO SPECTRUM

Reference has already been made to "radio" waves as electromagnetic waves. The behaviour of such waves is largely dependent upon the frequency of the transmission, and for this reason the entire range is broken up into the **frequency spectrum** (analogous to the light spectrum).

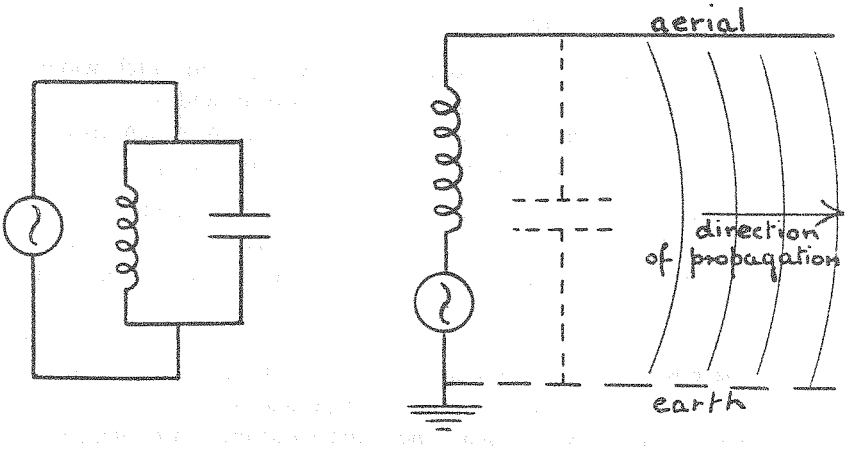
These frequency bands are summarised in the following table.

Frequency	*Band*	*Behaviour*
10-30 kHz	V.L.F.	Long range over the ground — some sky waves at very long range.
30-300 kHz	L.F.	Medium ground wave — some sky waves at night.
300-3000 kHz	M.F.	Fairly short ground wave. Good sky waves at night, few by day.
3-30 MHz	H.F.	Very short ground wave. Good sky waves by day and night.
30-300 MHz	V.H.F.	No ground wave — no sky wave. Direct ray only.
300-3000 MHz	U.H.F.	Ultra High Frequency. Direct ray only.
3000-30,000 MHz	S.H.F.	Super High Frequency. Direct ray only. Easily beamed for radar.
30,000 MHz and above.	E.H.F.	Extra High Frequency. Direct ray only. Very narrow beams for radar.

It should be noted that in practice, the bands merge somewhat and the behaviour of the top frequencies of one band is similar to the lower frequencies of the band above. The ground waves and the sky waves referred to in the above table are explained later in this chapter.

28.15 PRINCIPLE OF RADIO TRANSMISSION

We have already seen in Chapter 25 that a circuit such as the one illustrated in the left-hand diagram below provided with an A.C. input, will produce continuous oscillatory currents within the circuit. We referred to these as forced oscillations.



(*a*) Oscillatory circuit (elevation). (*b*) Transmitter principle (elevation)

Principle of the radio transmission.

It was shown by Lord Kelvin that if a capacitor is continuously charged and discharged, as are the plates of the capacitor in (*a*), the charge does not simply disappear. One must appreciate that at any given instant, one of the plates will be + ve'ly charged, and the other − ve'ly charged. Hence lines of flux will fill the space between the plates. And this flux is forming — collapsing — forming, and so on, with the same frequency as the alternating quantity.

The whole of this energy does not however, disappear back into the plate. This is because, however short, some time is needed for the lines to collapse entirely, and before this can be completed the next pressure is building up and the lines are growing again. The surplus energy is pushed out from between the plates in the form of electromagnetic waves, called so because as we have seen many times, it is virtually impossible to have a changing electric quantity (e.m.f. or current) without some accompanying magnetic effect — and of course the converse also applies.

All this takes place when the alternating quantity changes slowly, though the word "slowly" must not be thought of in the ordinary sense. We have spoken already of frequency of 50 Hz; but even if a complete reversal of current takes place 1000 times a second it is still slow enough for the changes to occur as described.

In the right-hand diagram above however, two important changes have been made.

(1) The plates of the capacitor have been placed much wider apart. One of them is placed high in the air and is known as the aerial; the other is formed by the Earth, which always contains some moisture and is a good conductor.

(2) Because the capacitance of the aerial-Earth capacitor is very small indeed, the resonant frequency is increased enormously to many kilocycles per second and indeed to megacycles per second (MHz).

The effect of these two changes is that the flux lines cannot possibly collapse in time before the reverse field is created. Some of them are "cut off" so to speak and radiate out into space with the same frequency as the alternating quantity, and with a velocity of propagation of 300×10^6 m/s.

These radiations are electromagnetic waves or radio waves.

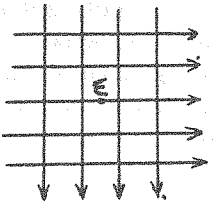
When the "capacitor" is fully charged there will be a system of electro-lines vertically, between aerial and Earth and associated with them, a system of horizontal magnetic lines.

These electric and magnetic lines (or fields) are at right angles to each other and are in time phase with each other. That is, the forming — collapsing — forming process is going on, with the frequency of the transmission, in two directions, mutually at right angles, but both in the vertical plane. This is therefore **transverse wave motion**, since the actual direction of propagation of the wave front, as indicated in the right-hand diagram shown in previous page is outwards, i.e., at right angles to *both* sets of vibrations.

The radiation, of course, flows out in all directions around the transmitter (unless we employ some special type of aerial system) and may be received by a conductor suitably placed.

28.16 POLARISATION

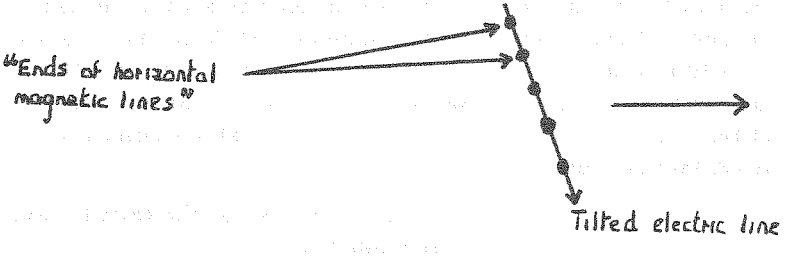
An electromagnetic wave of this type, that is, one in which the electric and magnetic vibrations in the wave front are mutually at right angles, and both occur in one plane which itself is at right angles to the direction of propagation is said to be **plane polarised**, or **normally polarised.**



Normally polarised wave front, advancing out of the paper towards the observer's eye, focused on E.

Thus an observer near the Earth's surface might see a wave front advancing towards him, with the vertical (electro) lines alternately travelling up then down, and the horizontal (magneto) lines alternately left then right.

Above the Earth's surface, and particularly after reflection by the ionosphere, the magnetic lines are still horizontal but the electro-lines are no longer vertical but are tilted. Such a wave front is said to be **abnormally polarised.**

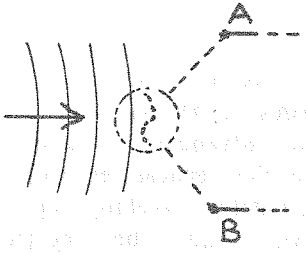


Section through abnormally polarised wave front, advancing across the page from left to right.

By convention it is the electric field to which we refer when deciding polarization. Hence a vertically polarized wave has a vertical electric field.

28.17 PRINCIPLE OF RADIO RECEIVER

Reference has already been made in Chapter 26 to the manner in which a weak signal may be amplified and it is hardly necessary to go over it again here.



Receiver principle.

The effect of the advancing alternating wave front meeting a conductor (again an aerial — this time, a receiving aerial) is simply to create oscillatory currents in that aerial of the same frequency as the transmission.

Referring to the diagram above, it is just as if a generator were feeding A.C. into the terminals *A* and *B* of the amplifying circuit. The rest follows from what has already been said.

28.18 MODULATION

Radio waves are inaudible to the human ear since the frequency of transmission is so high. They are modified before transmission in order to "carry" a low frequency signal such as a voice, or audio, frequency. The intelligence to be communicated, be it voice (audio), or picture (video) — such as the facsimile for a weather chart — is superimposed upon the basic wave transmission and in a sense, is carried along with it. The modification to the basic transmission is called **modulation**.

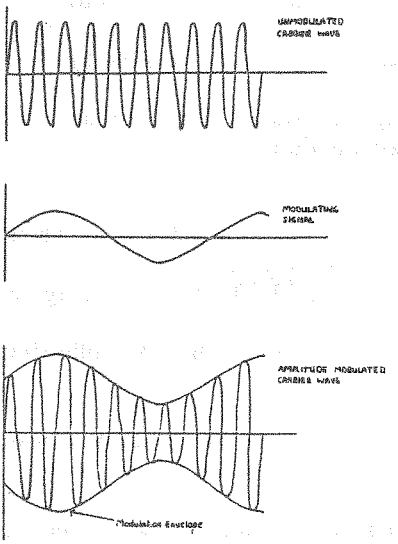
The basic radio frequency signal is known as the **carrier wave** and its frequency as the **carrier frequency**.

The frequency of the intelligence (audio or video) is the **modulation frequency** and the resultant modulated carrier wave conveys the intelligence to the receiver.

Modulation is normally effected in one of two ways:

(a) Amplitude Modulation

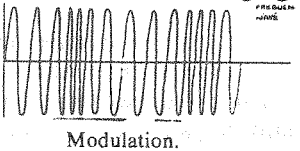
This consists of changing the *amplitude* of the carrier wave with a periodicity such as to create a superimposed wave whose frequency can be well within the audio-video range. In other words (referring to the diagrams) the audio frequency, as a result of the modulation of the carrier wave, is that of the "bulges", which correspond to changes in amplitude, in the basic signal. The received signal must be demodulated by the receiver before being fed to the loud-speaker.



The amplitude of the signal can be affected by noise and interference. High quality signals are difficult to achieve. However the range is considerable.

(b) Frequency Modulation

This consists of changing the *frequency* of the carrier wave with a periodicity such as to bring the superimposed wave within the range of audio frequencies.



Modulation.

This is also illustrated in the diagrams. This method of modulation is limited to V.H.F. and higher frequencies and is widely used for its low noise and high quality communication properties. (For example, F.M., i.e., Frequency Modulation is often used in V.H.F. Marine R/T). However because of the high frequencies involved in F.M. the range is more limited than A.M.

28.19 GENERAL PROPERTIES OF RADIO WAVES

Irrespective of frequency or other characteristics, all radio waves have the following properties:

(*a*) They travel in great circle paths.
(*b*) Their speed of propagation is 300×10^6 m/s.
(*c*) They are reflected by objects of a size commensurate with their wavelength.

The use of radio as a navigational aid is made possible by these characteristics.

(*d*) Like all other electromagentic waves, they can undergo **refraction, interference, attenuation.**

28.20 GROUND WAVE

This is the name given to that portion of a radio wave which travels over the Earth's surface to the receiver. It suffers loss of energy into the surface over which it travels. This loss of energy is called **attenuation.**

The amount of attenuation is influenced by the following factors:

Frequency

The higher the frequency, the greater the attenuation. Above 30 MHz the range of the ground wave is very short.

Nature of the surface

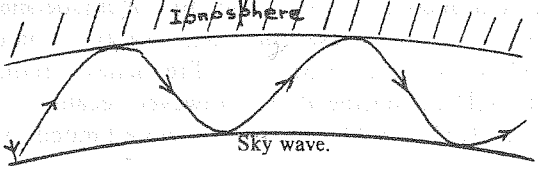
Dry arid land will cause rapid attenuation. Sea water on the other hand is a good conductor. The range of Consol may be of the order of 600 miles over land and 1200 miles over sea.



Ground wave.

28.21 SKY WAVE

Ranges far in excess of the ground wave may be obtained from sky waves. These are waves which are reflected from the ionosphere, a number of layers of ionised air in the upper atmosphere. Sky waves may travel in a series of multi-hops around the Earth. Attenuation is caused only by passage through the ionosphere or, as already stated, by contact with the ground.



Sky wave.

28.22 IONOSPHERE

In the upper atmosphere, the ultra-violet and higher frequency rays from the sun cause the ejection of free electrons from the atoms and molecules of the attenuated atmosphere. These are negative ions and atoms which have lost electrons are positive ions. The electron density may reach nearly $10^6/cm^3$, compared with $10/cm^3$ at ground level.

These ions have a marked effect on the behaviour of radio waves, causing: **Refraction, Reflection, Attenuation.**

The ionosphere can be affected by such factors as the time of day or night, the seasons, sunspots and solar flares. Despite these influences the use of sky waves for international radio communication has been a major development of the twentieth century.

28.23 SKY WAVE PROPAGATION FACTORS

The factors which must be taken into account in deciding whether a sky wave is returned to Earth or not are as follows:

1. The angle at which the wave reaches the layer

For a given frequency and density of ionisation, a radio wave will be returned, depending upon the angle at which it strikes the ionosphere. As the wave strikes, it suffers both attenuation and bending. If the angle of entry is right and the degree of bending sufficient then the wave is reflected. Otherwise it may bend, (i.e., is refracted) but by an insufficient amount, and will pass through the ionosphere.

2. The density of ionisation

For a given angle of entry, and frequency, the amount of refraction is governed by the density of ionisation. The greater the density, the greater is the bending effect. The density itself is governed by the **time of day** — greater density during daylight hours when the sun's rays are on the layers — **latitude**, being more intense in equatorial regions, and by **sunspot activity**.

3. The frequency of the radio wave

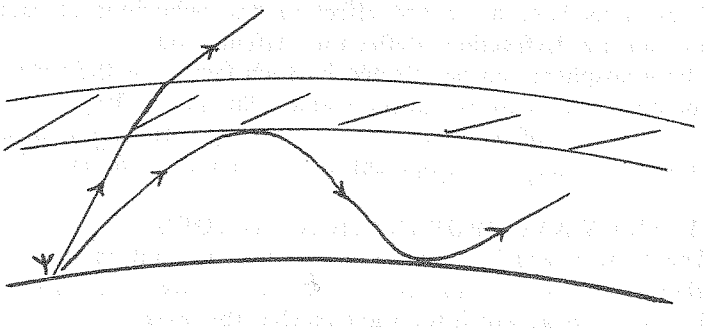
For a given density, and angle of entry, the final factor is the frequency itself.

Low frequencies are generally absorbed by the ionosphere and fail to return.

Medium frequencies penetrate further into the ionosphere and suffer a greater degree of refraction as a result but are only returned to Earth at night when the ionosphere is less dense and less attenuation takes place.

High frequency signals are easily returned — both day and night — and this frequency band is the most suitable for long range communications.

Very high frequency signals penetrate the ionosphere completely and therefore fail to return.



Reflection and refraction at the ionosphere.

28.24 DEAD SPACE AND SKIP DISTANCE

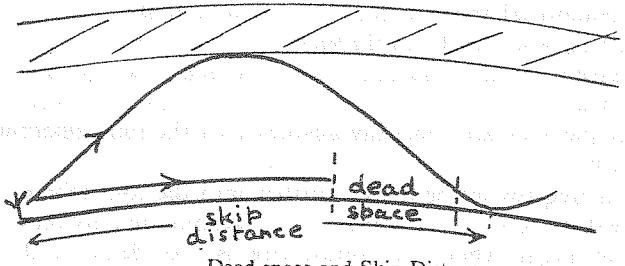
A **dead space** is an area where no signals are received. It may occur:

(*a*) Between the limit of the ground wave and reception of the first hop sky wave.

(*b*) Between reception of the first hop and second hop sky wave signals.

Skip Distance

This is the distance between the point of reception of the first sky wave and the transmitter.



Dead space and Skip Distance.

28.25 MAXIMUM RANGE OF FIRST HOP

This is governed by the height of the reflecting layer, and the geometry of the Earth's surface. It occurs for a tangential take off at the transmitter and a tangential return to Earth. Its value is between 1150 and 1480 miles.

28.26 FADING

This is an erratic change in signal strength caused by three factors:

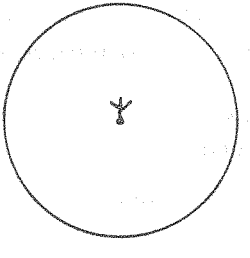
(*a*) Mixing of ground and sky waves at the receiver. If the sky wave is out of phase with the ground wave the signals mix and fade. Since the ionosphere is not stable the phase of this signal will change with ionospheric movement and hence the signal changes in strength.

(*b*) At the limit of ground wave signal the strength will fluctuate due to variations in the degree of attenuation.

(*c*) Abnormal polarisation. This has already been discussed.

28.27 AERIALS — POLAR DIAGRAMS

In order to represent pictorially the directional properties of a given aerial system, a diagram of field strengths, called **polar diagrams**, for various bearings about the aerial may be drawn. This may be considered in two ways:



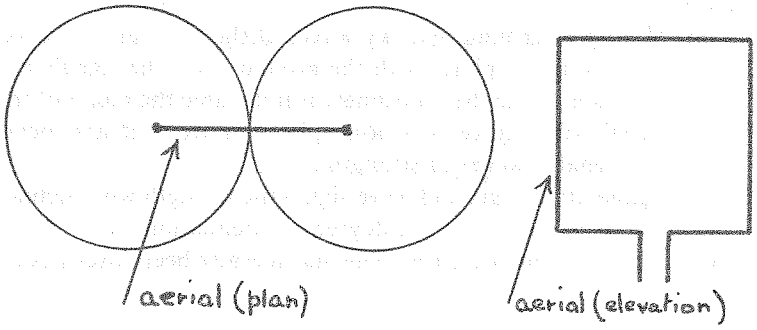
Omni-directional polar diagram about a transmitter.

(*a*) A receiver moves about a transmitting aerial at a constant distance, and signal strengths received are plotted as vectors about the aerial. If these are equal, the polar diagram will be a circle as in the diagram.

This would be the diagram for example, for a straight vertical transmitting aerial or similar receiving aerial. Such an aerial is said to be **non-directional**, or **omni-directional**; that is, it radiates signals in all directions at the same strength, or, in a receiver, receives from any direction at the same strength.

(*b*) A transmitter with constant non-directional output moves around the receiver aerial at a constant distance and strengths received are plotted as vectors about the aerial. If the receiver is a straight aerial, the result of course would be the same as (*a*).

But if the receiving aerial is in the form of a loop, for example, as in the typical D/F receiver, it receives strongly in the end-on position but not at all in the broadside position.



Polar diagram for directional receiving aerial.

In this case the polar diagram would be as in the diagram above, which is often termed a "Figure Eight" diagram.

An aerial which beams transmission in one direction is said to be **directional**. Such an aerial is used in radar. Similarly, a receiver which only accepts from one direction — or from more than one but *not* from *all* directions — is referred to as a **directional receiver**.

TEST PAPER 28
Wave Motion, Sound

1. If the velocity of sound in air at 16°C is found to be 338 m/s, calculate the velocity at 0°C.
2. State a formula for calculating the velocity of sound in air and explain the meaning of the terms in it.
   Calculate the velocity of sound in air if the barometer pressure that day is 75·4 cm mercury and the density of air is 1·30 kg/m^3.
   (R.D. mercury 13·6 and g = 9·81 m/s^2).

3. With the velocity as found in the previous question, what is the wavelength of a note with a frequency of 320 Hz?
4. The velocity of sound in air is found to be 342 m/s when the temperature is 28°C. What will it be when the temperature is 12°C?
5. Calculate the velocity of sound in air if the barometer pressure is 1016 mb and the density of air 1·295 kg/m^3.
6. On a day when the corrected barometer reading was 1012·6 mb, the velocity of sound by experiment was found to be 334·5 m/s. Calculate the density of the air at the time.
7. An observer a certain distance from a cliff fires a shot and hears the echo after 3 seconds. He walks 165 m nearer to the cliff and this time the corresponding interval is 2 seconds. Calculate the velocity of sound and his original distance from the cliff.
8. Taking the velocity of sound in water to be 1450 m/s, find the depth of water under a vessel's keel if an echometer signal is received after an interval of 3·6 seconds. What is the wavelength if the frequency of transmission is 2600 kHz?
9. *(a)* A sonar pulse of frequency 50 kHz travels through water at a velocity of 1500 m/s. Calculate its wavelength.

   (b) A ship is heading directly towards a lighthouse and hears a fog signal of 500 Hz. If the true frequency of the lighthouse fog signal is known to be 495 Hz and the velocity of sound through air is 660 knots, calculate the speed of the vessel. (SCOTVEC March 1990).
10. *(a)* State the effect on the velocity of sound through the atmosphere if:
    (i) the humidity of the air increases;
    (ii) the atmospheric pressure decreases;
    (iii) the temperature of the air rises.

    (b) A pilot boat is stopped, awaiting a ship's arrival. The ship is travelling directly towards it at 15 knots. The ship emits a whistle blast at a frequency of 700 Hz. Calculate the frequency of the sound heard at the pilot boat. Take the velocity of sound through the air to be 660 knots. (SCOTVEC July 1990).

11. *(a)* A buoy sounds a signal of frequency 700 Hz. Calculate the frequency of the signal received aboard a vessel steaming at 15 knots:

(i) directly towards the buoy;
(ii) directly away from the buoy.

(b) Calculate the wavelength of the signal the buoy sounds. (Velocity of sound in air = 340 m/s, 1 nautical mile = 1852 metres).
(SCOTVEC December 1990).

12. *(a)* Taking the velocity of sound to be 340 m/s, find the wavelength of the highest and lowest notes audible to the human ear if their frequencies are 20 kHz and 40 Hz respectively.

(b) The sound of an approaching helicopter is heard aboard a stationary ship at one and a quarter times its true frequency. Assuming the speed of sound through the air to be 340 m/s, calculate the approach speed of the helicopter in knots.
(Take one nautical mile to be 1852 metres).
(SCOTVEC March 1991).

ANSWERS

CHAPTER 1

Exercise A

1. 5000 kg/m^3 2. 8000 kg 3. $4{\cdot}145\times 10^{-8}$m^3

Exercise B

1. 4 2. 1·020

Exercise C

1. 5734·4 kgf 2. 3087·52 kgf 3. 2·333

Test Paper 1

1. 48000 tf 2. 4·39 m 3. 700 kgf
4. 749·6 kgf 5. 4·08 tf 6. 21·48 cm

CHAPTER 2

Exercise A

1. 2453 N/m^2 2. 56737 N

Exercise B

1. 320 N 2(*a*) $1{\cdot}273\times 10^6$ N/m^2 (*b*) 40000 N

Exercise C

1. $5{\cdot}629\times 10^5$ N
2. (*a*) $8{\cdot}493\times 10^4$ N (*b*) $2{\cdot}987\times 10^5$ N
3. $2{\cdot}415\times 10^4$ N, $1{\cdot}699\times 10^5$ N
4. $9{\cdot}319\times 10^4$ N
5. $1{\cdot}844\times 10^4$ N/m^2

Test Paper 2

1. (*a*) 64 (*b*) 1024 kgf
2. 50 cm, 100 3. 11·94 MN
4. (*a*) 4·025 kg (*b*) 3·578 MN
5. 22·2 m 6. 139·2 m 7. 79·5 m 8. 105 kN
9. (*a*) 48·5 kN/m^2 (*b*) 20·59 m (*c*) 34·87 kN
10. (*a*) 40·42 kN/m^2 (*b*) 2·102 MN (*c*) 231·2 kN

CHAPTER 3

Exercise A

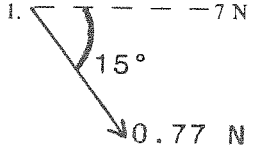
1. (*a*) 13 N (*b*) 57·96 N (*c*) 42 N
2. 9·644 tf 3. 2·828 kgf South West

Exercise B

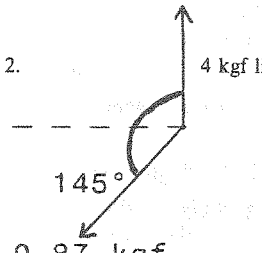
1. 1·5 tf, 2·0 tf 2. 240 kgf 3. 1·134 tf
4. 3·857 tf, 4·596 tf 5. 1·2 tf, 2·4 tf

Exercise C

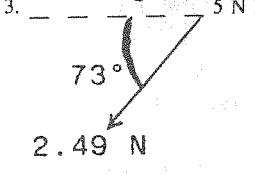
1. 7 N line of action (0·7685 N, 15·23°)



2. 4 kgf line of action (0·8671 kgf, 54·97°)



3. 5 N line of action (2·490 N, 72·93°)



Exercise D

As for Exercise C

Test Paper 3

1. 385·2 kgf, 336·1°
2. 3·356 tf, 5·222 tf
3. 26·04 tf
4. 400 kgf
5. Resultant 1000 N, 141·6°T, Equilibrant 1000 N, 321·6°T
6. 788·3 N

Chapter 4

Exercise A

1. 96·07 kgf, 133·93 kgf
2. 30·18 kgf, 5·82 kgf
3. 6 kg
4. 0·87 m from 5 kg towards 7 kg

Exercise B

1. 18·48 cm from left hand end
2. 22·24 cm from left hand end

Exercise C

1. 0·9127 m
2. 6·08 m

Test Paper 4

1. 60 kgf m, 98 kgf m, 4·647 m from A
2. 89·3 kgf (A), 202·7 kgf (C)
3. 44 kg
4. 14 kg, 3 m
5. 0·25 m from O on axis of symmetry
6. 0·7383 m from the side of the square as base, on the axis of symmetry
7. 4·850 m
8. Tension = 43·19 kgf, Reaction = 43·19 kgf

CHAPTER 5

Exercise A

1. 14·62 kgf 2. 20·48 kgf

Exercise B

1. Stress $= 6{\cdot}366 \times 10^6$ kgf/m^2 or $6{\cdot}245 \times 10^7$ N/m^2
   Strain $= 0{\cdot}001$, Youngs Modulus $= 6{\cdot}245 \times 10^{10}$ N/m^2
2. (*a*) 1×10^9 N/m^2 (*b*) 0·026 m (*c*) 4×10^5 N (400 kN)

Exercise C

(*a*) 0 (*b*) -40 N rising to $+80$ N
(*c*) 0 (*d*) -80 N rising to $+40$ N (*e*) 0

Exercise D

(*a*) 0 (*b*) 20 Nm (*c*) -60 Nm
(*d*) 20 Nm (*e*) 0

Exercise E

Distance from left hand end (m)	0	1	2	3	4	5	6
Shearing Force (N)	0	-40 -70	-110 $+90$	50	10	-30 $+40$	0
Bending Moment (Nm)	0	20	110	40	10	20	0

Test Paper 5

1. 35·29 kgf, 400 kgf
2. 12 kgf parallel to AC, distant 0·8333 m from AC towards B
3. Stress $= 1{\cdot}9075 \times 10^7$ N/m^2, Strain $= 0{\cdot}004695$, Y.M $= 4{\cdot}063 \times 10^9$ N/m^2
4. (*a*) Strain $= 0{\cdot}001333$ (*b*) Stress $= 9{\cdot}021 \times 10^7$ N/m^2 (*c*) Force $= 63{\cdot}77$ kN
5. 1177 N

6.

Distance (m)	0	2	4	6	8	10
S.F. (N)	0 to 600	360	$+120$	-120	-360	-600 to 0
B.M. (Nm)	0	-960	-1440	-1440	-960	0

7.

Distance (m)	3	6
S.F. (N)	600	0
B.M. (Nm)	2700	3600

8.

Distance (m)	3	6
S.F. (N)	800	200
B.M. (Nm)	-3300	-4800

CHAPTER 6

Exercise A

1. 0·3143 m, 1·222

Exercise B

1. $1{\cdot}172\times10^{-4}$ m³, $1{\cdot}2\times10^{-4}$ m³, 0·056 m
2. 1·286, 0·1779 m
3. $8{\cdot}571\times10^{-6}$ m³
4. 0·875
5. $2{\cdot}25\times10^{-4}$ m³, $1{\cdot}5\times10^{-4}$ m³, h = 16·67 cm
6. 0·4331
7. 0·7857, 1·222 m
8. 0·03925 m
9. 0·6 m, 0·6487 m

Exercise C

1. 36 mm above W mark
2. 3·546 m
3. 110·2 mm above W and S marks respectively
4. 8·2 m
5. 8·965 m
6. 7·745 m, 7·883 m
7. 1·822 m

Test Paper 6

1. 53·3 mm above W mark
2. 7·853 m
3. 29·33 cm
4. $1{\cdot}2\times10^{-5}$ m³
5. $1{\cdot}951\times10^{5}$ m³
6. 29·27 m³, 0·3
7. 4·854 tf
8. 140 mm, 39·2 mm

CHAPTER 7

Test Paper 7

1. (*a*) 101129 N/m² (*b*) 1011·29 mb
2. 761 mm
3. (*a*) 1014·1 mb (*b*) 997·2 mb (*c*) 1006·9 mb

CHAPTER 8

Exercise A

1. 3352 N
2. 198·6 N
3. 3·620 MN On the vertical axis of symmetry, 4 m below the waterline.

Exercise B

1. 1120 mb
2. 7101 N

Exercise C

1. 8·267
2. 1141 mb
3. 10673 N/m²
4. 8973 N
5. 1445 mb

Exercise D

1. 14·5 m
2. 6046 mb, 28·71 cm
3. 6365 N outwards
4. 2·8364 bar, 4·286 m³

Exercise E

1. 3·885 m

Test Paper 8

1. 423164·16 N/m²
2. 1317·7 mb
3. 98100 N/m²
4. 18·13 m
5. 7471 N/m²
6. 99969 N/m², 10·19 m
7. 77911 N, 70 m
8. 1·23 m

CHAPTER 9

Exercise A

1. 11·11 m/s
2. 135 s
3. 0·7143 m/s, 0·4167 m/s
4. 108 km/h
5. $-0{\cdot}2778$ m/s^2
6. 5·556 s

Exercise B

1. 20·39 m
2. $-0{\cdot}1111$ m/s^2, 1800 m
3. 7·143 s
4. 50 s, 8 m/s, 25 s, 250 m after point where speed was reduced.

Exercise C

1. 120 m
2. (*b*) 720 m
3. (*b*) 0·6667 m/s^2, 2 m/s^2 (*c*) 12 m

Exercise D

1. 8 knots, 122°T
2. 22·4 m/s, 170°T
3. 15·6 knots, 131°T, closest distance approx 8 miles after approx 3 hours.

Exercise E

1. 23·33 kN
2. 150 kN

Exercise F

1. 13·86 kN
2. 2·5 kN
3. 59·41 kN

Exercise G

1. 2 m/s
2. 11·25 m/s
3. 3·75 m/s

Test Paper 9

1. 10·51 km
2. (*b*)(i) 5 m/s^2 (ii) 960 m (iii) 80 m
3. (*a*) 19·81 m/s (*b*) 0·7033 s
4. (*a*) 0·01111 m/s^2 (*b*) 2880 m (*c*)(i) 180 s (ii) 1620 m
5. (*a*) 0·5 m/s (*b*)(ii)(1) 40 s (2) 1·125 m/s^2
6. (*a*) 10 m/s^2 (*b*) 4 s (*c*) 14 s, 1540 m
7. (*b*)(i) 32 m/s (iii) 6080 m
8. (*a*) 25·5 m/s, 051°T (*b*) 21·5 m/s, 052°T
9. 311°T, 9 knots
10. 90 m/s^2
11. 16·67 kN
12. 42·42 kN
13. 6·49 s
14. 761·6 kN
15. 5·23 m/s in the same direction as the 8 kg mass.
16. 407880 kg m/s, 4·635 kN

CHAPTER 10

Exercise A

1. 196·2 kJ
2. 3·637 MJ
3. 15·696 MJ
4. 5·097 kJ

Exercise B

1. 92·59 MJ
2. 3852 MJ
3. (*a*) P.E. = 0, K.E. = 90·26 J (*b*) P.E. = 90·26 J, K.E. = 0
(*c*) P.E. = 45·13 J, K.E. = 45·13 J

Exercise C

1. 8960 J, 162·9 W
2. 98·1 s
3. 99·19 kW
4. 57·552 kW
5. 23·93 kJ, 35·1 kW
6. 440·4 t/h
7. 43·6 kW
8. 137·6 km/h
9. 29 mins 26 s
10. 850·2 W
11. 16·37 kW

Exercise D

1. (*a*) 20 (*b*) 25 (*c*) 80%
2. (*a*) 6 (*b*) 3·6 (*c*) 40 N
3. (*a*) 10·67 (*b*) 0·075 m (*c*) 3360 N

Exercise E

1. 14·33 kgf
2. 13·95 kgf
3. 343·6 kgf
4. 8·571 kgf
5. 9·174 kg
6. 126 kgf
7. 54 kgf

Test Paper 10

1. 46·08 kW
2. 23·54 kW
3. 2·158 kW
4. 4·588 kJ, 152·9 W
5. 20·34 kgf
6. 13·55 m/s
7. 50·03 kJ, 208·5 W
8. 509·7 m, 91·74 s
9. 70·51 N, 56·02 kJ

CHAPTER 11

Exercise A

1. 0·2
2. 10·4 m/s²

Exercise B

1. 70·8 N
2. (*a*) 0·644m N (*b*) 4·434m N (In both cases "m" is the mass of the lifeboat).

Exercise C

1. 18·73 kW, 28·81 kW
2. 72·92%

Exercise D

1. 0·845 tf
2. 1·12 tf
3. 2·13 tf
4. 76·9%
5. 1·8 tf
6. 0·585 tf
7. 80%

Test Paper 11

1. 0·2, 11·31°
2. 1·313 m, 361 J
3. (*a*) 11·31° (*b*) 641·2 N
4. (*a*) 11·81 kN (*b*) 23·01 kN
5. (*b*)(i) 21·89 MN (ii) 20·28 MN (iii) 2·9 m/s²
6. 140 kgf
7. 384·3 m

CHAPTER 12

Exercise A

1. 18·85 rad/s
2. 286·5 revs/min
3. 463 N
4. 0·324 N
5. 1451 N, 83°
6. 1·895 N

Exercise B

1. 0·54 s
2. (*a*) 79·5 cm/s², 0 (*b*) 39·5 cm/s², 5·4 cm/s (*c*) 0, 6·3 cm/s
3. 1 rad/s. 6·3 s

Test Paper 12

1. 2·75 N
2. 2·094 m/s, 43·86 m/s²
3. 0·062 m
4. 0·5591 m
5. 66·88 rpm
6. 4·914 s, 0·1279 m/s, 0·1635 m/s²
7. 189
8. 22·74 kN
9. 3·948 m
10. 3·77 s

CHAPTER 13

Exercise A

1. (*a*) 293 K	(*b*) 408 K	(*c*) 193 K
2. (*a*) 25°C	(*b*) −146°C	(*c*) 128°C
3. (*a*) 25°C	(*b*) −20°C	(*c*) 116·4°C
4. (*a*) 77°C	(*b*) −4°F	(*c*) 233·6°F
5. (*a*) 293 K	(*b*) 268 K	(*c*) 324·1 K
6. (*a*) 95°F	(*b*) −31°F	(*c*) 44·6°F

Test Paper 13

1. (*a*) (i) 68°F	(ii) 173 K	(iii) 423 K
2. 77°F, 25°C	3. −40°	
5. (*a*) 10°C, 283 K	(*b*) 149°F, 338 K	(*c*) 55°C, 131°F

CHAPTER 14

Exercise A

1. 228 kJ	2. 40 kg	3. 140 J/kg°C

Exercise B

1. 66·8°C	2. 81·94°C

Exercise C

1. 38·74 m	2. 0·56°C

Test Paper 14

1. 896 kJ, 4·27 K	2. 16·5 MJ, 28·03 t	3. $1{\cdot}3265 \times 10^9$ J, 150·7 kg
4. 416·1 J/kg°C	5. 4905 J/kg°C	6. 2467 J/kg°C
7. 38·88 MJ/kg	8. 18·38°C	

CHAPTER 15

Exercise A

1. 8·01368 m	2. 4·3 mm	3. $66{\cdot}67 \times 10^{-6}$/°C
4. 2·766 m^2	5. 9·011 cm	

Exercise B

1. 2·220 bar	2. 62·93 litres	3. 337·3 K
4. 1361 kN/m^2	5. 4·690 m^3	

Exercise C

1. 287 J/kg/K

Exercise D

1. P = 508·0 mm Hg, T = 243·3 K
2. P = 2486 mm Hg, T = 334·9 K

Test Paper 15

1. (*b*) (i) 2 bar (ii) 2·639 bar		
2. (*c*) 2·497 mm	3. 1·424	4. (*b*) 10·001 m
5. (*b*) 1·102 litres	6. (*b*) 999·402 mm	7. (*b*) 0·1378 cm
8. 107·1 cc	9. (*a*) 4·21875 bar (*b*) 1·31	
10. 655°C	11. (*b*) (i) 1·5 bar (ii) 1·979 bar	

12. 4126·8 J/kg/K, 8·320 J/mol/k, 2·016 g

CHAPTER 16

Exercise A

1. 71·2°C
2. 19·2°C
3. 2·833 kg
4. 39·95°C
5. 57·9°C
6. 114·7 kJ/kg

Test Paper 16

1. 53·9°C
2. 61·1°C
3. 34·1°C

CHAPTER 17

Test Paper 17

1. 43%
7. (*a*) None (*b*) Decrease (*c*) None
8. (*a*) 13·5 g/m^3 (*b*) 45 g/m^3 (*c*) 30%

CHAPTER 18

Exercise A

1. 19·15 A/m, 16·07 A/m
2. 25 A/m, 73·74°
3. 24·42 A/m

Test Paper 18

1. 31·11 A/m
3. (i) 19·70 A/m (ii) 3·473 A/m
4. (i) 32·64 A/m (ii) 20·98 A/m (iii) South

CHAPTER 19

Test Paper 19

1. 4·5°
2. 0·2 m
3. 11·14 A/m
4. 2653
5. 12°43′
6. 5·208 A/m
7. 2·02 m
8. 0·361 m end on to the magnet
9. 2.520 m
10. (*a*) 96000 AT/m (*b*) 0·1206 Wb/m^2 (*c*) 434·3 Wb/m^2
11. 318·3 × 10^6 units
12. 0·5 A
13. 1·093 × 10^{-5} Nm
14. 5 × 10^{-5} N
15. 0·2264 A/m
16. 0·4073 m

CHAPTER 20

Exercise A

1. 5·33°
2. (*a*) 8° (*b*) 20°
3. 18
4. 4° E

Test Paper 20

1. (*a*) −P,Q (*b*) +P, −Q (*c*) −P, −Q (*d*) +P, +Q
2. 7·5° E
3. N — Directive force weakened
   E — Westerly deviation
   S — Directive force strengthened
   W — Easterly deviation
4. 18·8° W
5. (*a*) 8·8° W (*b*) 12·1° W
6. 29·52 A/m
7. 4·03° E
8. 9·9° E

CHAPTER 21

Exercise A

1. 6 μF, charges are 6 μC (1 μF), 12 μC (2 μF), 18 μC (3 μF)
2. 1 μF, p.d.s. are 6 V (2 μF), 4 V (3 μF), 2 V (6 μF)
3. (*a*) 4 V (*b*) 8 μC (*c*) $Q_2 = 4\ \mu C$, $Q_3 = 4\ \mu C$ (*d*) 20 μF (*e*) 0·4 V (*f*) 4·4 V (*g*) 1·818 μF
4. (*a*) 3 μF (*b*) 27 μC (*c*) $Q_1 = Q_2 = 18\ \mu C$ (*d*) 6 V
5. 3·4 μF

Test Paper 21

1. 5 A, 300 C
2. 576Ω, 0·347 A, 69·44 W
3. 0·225 A, 48·6 W
4. 5 μF
5. (*a*) 9·5 μF (*b*) 1·017 μF
6. (*a*) (i) Increases (ii) Increases (*b*) (i) 2 μF (ii) 100 μC
7. (*a*) 8 μF
   (*b*) 2 mC (C_1), $333^1/_3\ \mu$C (C_2), $666^2/_3\ \mu$C (C_3), 1000 μC (C_4)
   (*c*) $83^1/_3$ V (C_1), $166^2/_3$ V (C_2, C_3, C_4)
8. (i) $6^2/_3\ \mu$F (ii) $666^2/_3\ \mu$C
9. (i) 1·25 μF (ii) 125 μC (on each of them)
   (iii) 62·5 V (2 μF), 25 V (5 μF), 12·5 V (10 μF)

CHAPTER 22

Exercise A

1. (*a*) 73·7 kΩ (*b*) 3·392 mA (*c*) 159·4 V
2. 1·143Ω
3. (*a*) 19·39 kΩ (*b*) 12·89 mA
   (*c*) 5·319 mA (47 kΩ), 7·576 mA (33 kΩ)
4. (*a*) 12·5 kΩ (*b*) 7·5 kΩ

Exercise B

1. (*a*) 496·2Ω (*b*) 48·37 mA, 36·54 mA, 11·82 mA
   (*c*) 15·96 V, 8·039 V, 8·039 V
2. (*a*) 353·1Ω (*b*) 0·136 A, 0·102 A, 0·0337 A
   (*c*) 20·39 V, 27·61 V, 27·61 V
3. (*a*) 100Ω (*b*) 0·25 A (R_1, R_5), 0·125 A (R_2, R_3, R_4)
   (*c*) 5 V, 12·5 V, 10 V, 2·5 V, 7·5 V

Exercise C

1. 9 mΩ
2. 1·765 km
3. 4·664 mm²

Exercise D

1. External 8·5Ω, internal 1·5Ω
2. P.d. = 35·29 V, current = 1·47 A
3. (*a*) 4Ω (*b*) 2·286 A, 1·143 A, 0·571 A; Sum = 4 A

Test Paper 22

1. 0·55 A, 2Ω
2. 35·2 V, 44 V
3. 7·143Ω, 0·2188 A, 1·563 V
4. 12Ω, 1Ω, 4·5 V, 1·125 A, 0·375 A
5. 0·7059 A, 1·412 A
6. 0·1135 A, 1·589 V
7. 0·06048 kg
8. 50·61Ω
9. $1{\cdot}539 \times 10^{-8}\ \Omega m$
10. (*a*) 7Ω (*b*) 250 W
11. (*a*) $1^2/_3$ A (30Ω), $8^1/_3$ A (6Ω), 10 A (5Ω)
    (*b*) 50 V (across each of them)
12. (*a*) 12 V (*b*) 8 V (R_1, R_2), 4 V (R_3)
    (*c*) 0·4 A (R_1), 1·6 A (R_2), 2 A (R_3), (*d*) 3·2 W

CHAPTER 23

Exercise A

(*a*) Westerly (*b*) Nil but directive force strengthened
(*c*) Nil but directive force weakened

Exercise B

1. 0·404Ω 2. 27·78Ω, 25Ω

Exercise C

1. 29·9 kΩ 2. 990Ω 3. Fit 6850Ω in series
4. 252 V

Test Paper 23

1. Series, Fit shunt of 0·04Ω 2. Westerly deviation
3. 5420Ω 4. 0·052 A
5. Easterly deviation, Nil but directive force weakened
   Westerly deviation, Nil but directive force strengthened
6. 0·124 A 7. 0·0303 A 8. 0·060Ω

CHAPTER 24

Test Paper 24

8. 55 000 V

CHAPTER 25

Exercise A

1. 53·1Ω, 4·52 A 2. 56·8 Hz 3. 50·9 μF

Exercise B

1. 1880Ω, 0·127 A 2. 38·2 Hz 3. 1·59 H

Exercise C

1. 144 V, 48°22′ (Lags) 2. 100 V, 53°8′ (Leads) 3. 247 V, 75°58′ (Lags)
4. 4210 V, 85°55′ (Leads)

Exercise D

1. E = 151·3 V, Z = 60·53Ω, ϕ = 7·59° (Leading), Is = 2·5 A
2. E = 120·9 V, Z = 60·46Ω, ϕ = 34·22° (Leading), Is = 2 A,
   C = 48·23 μF, L = 0·3183 H

Exercise E

1. 0·6 2. 0·99

Test Paper 25

1. (*a*) 0·3142Ω, 31·83 A (*b*) 31·42Ω, 0·3183 A
2. 1592Ω, 0·1257 A 3. 424·3 V, 21·21 A
4. 19·4 mA (Lagging the supply voltage by 32°), 3·78 W
5. 2513Ω, 95·5 mA 6. 729Ω, 0·3635 A (leading by 46·7°)
7. 78 mA (78·8° lagging), 3·042 W 8. 0·12 A (72·6° leading)
9. 3194Ω, 62·6 mA (48·9° lagging), 8·23 W,~10 Hz, 2100Ω
10. (*a*) 24·8 V (*b*) 198·5 V 11. 0·118 A (45·2° lagging), 16·6 W
12. V_R = 118 V, V_C = 5·87 V, V_L = 150·1 V,~10 Hz, 1200Ω
13. 1165Ω, 0·172 A 14. 12·4 Hz, 4000Ω
15. I_C = 1·885 A, I_L = 53 mA, Z = 109·2Ω
16. 8·39 Hz, zero 17. (*a*) 186·5 V (*b*) 65·9 V (*c*) 65·9 mA
18. I_C = 0·22 A (90° leading), I_L = 0·22 A (90° lagging), zero
19. 125·7 V, r.m.s. 88·86 V; 62·8 V; 0 V
20. 26 Hz, 5000Ω, 40 mA

CHAPTER 27

Exercise A

	Nature	*Position*	*Size*
1. (*a*)	Virtual and erect	40 mm behind	20 mm
(*b*)	Image at infinity		
(*c*)	Real and inverted	120 mm in front	20 mm
(*d*)	Real and inverted	80 mm in front	10 mm
(*e*)	Real and inverted	$66^2/_3$ mm in front	$6^2/_3$ mm
2. (*a*)	Virtual and erect	$13^1/_3$ mm behind	$6^2/_3$ mm
(*b*)	Virtual and erect	20 mm behind	5 mm
(*c*)	Virtual and erect	24 mm behind	4 mm
(*d*)	Virtual and erect	$26^2/_3$ mm behind	$3^1/_3$ mm
(*e*)	Virtual and erect	$28^4/_7$ mm behind	$2^6/_7$ mm

Exercise B

1. (*a*) As for question 1 (*c*) of Exercise A
   (*b*) As for question 1 (*d*) of Exercise A
   (*c*) As for question 1 (*b*) of Exercise A
2. (*a*) As for question 2 (*c*) of Exercise A
   (*b*) As for question 2 (*d*) of Exercise A
   (*c*) As for question 2 (*b*) of Exercise A

Exercise C

	Nature	*Position*	*Size*
1. (*a*)	Virtual and erect	40 mm in front	20 mm
(*b*)	Image at infinity		
(*c*)	Real and inverted	120 mm behind	20 mm
(*d*)	Real and inverted	80 mm behind	10 mm
(*e*)	Real and inverted	$66^2/_3$ mm behind	$6^2/_3$ mm
2. (*a*)	Virtual and erect	$13^1/_3$ mm in front	$6^2/_3$ mm
(*b*)	Virtual and erect	20 mm in front	5 mm
(*c*)	Virtual and erect	24 mm in front	4 mm
(*d*)	Virtual and erect	$26^2/_3$ mm in front	$3^1/_3$ mm
(*e*)	Virtual and erect	$28^4/_7$ mm in front	$2^6/_7$ mm

Exercise D

1. (*a*) As for question 1 (*c*) of Exercise C
   (*b*) As for question 1 (*d*) of Exercise C
   (*c*) As for question 1 (*b*) of Exercise C
2. (*a*) As for question 2 (*c*) of Exercise C
   (*b*) As for question 2 (*d*) of Exercise C
   (*c*) As for question 2 (*b*) of Exercise C

Test Paper 27

1. 20·7° 2. 51·6°, 48·6° 3. 1·795 m
4. (*b*) (i) POSSIBLE, 112·5 mm (ii) NOT POSSIBLE ($-33^1/_3$ mm)
5. (*a*) 75 mm (*b*) 0·4
6. (*b*) (i) 6 mm high, Real and inverted, 48 mm in front of mirror
   (ii) 30 mm high, Virtual and erect, 60 mm behind mirror
7. -10 cm, Virtual and erect, $M = {}^1/_3$
8. 0·48 m or 2·52 m from object 9. 36 cm, $M = 3$
10. 15 cm from the lens 11. 60 cm, 100 cm 12. 0·875 m, 1·025 m

CHAPTER 28

Exercise A

1. 332 m/s
2. 0·34 m
3. 200 kHz

Exercise B

1. (*a*) 330 m/s (*b*) 320·5 m/s (*c*) 0·534 m
2. 305·2 K (32·2°C)

Exercise C

1. (*a*) 1024·8 Hz (*b*) 976·33 Hz
2. (*a*) 1024·2 Hz (*b*) 975·76 Hz

Exercise D

1. 10·88 m/s
2. 1162 m, 1·63 m/s

Test Paper 28

1. 328·5 m/s
2. 330·3 m/s
3. 1·032 m
4. 332·8 m/s
5. 332·6 m/s
6. 1·276 kg/m^3
7. 330 m/s, 495 m
8. 2610 m, 0·558 mm
9. (*a*) 0·03 m (*b*) 6·67 knots
10. (*b*) 716·3 Hz
11. (*a*) (i) 715·9 Hz (ii) 684·1 Hz (*b*) 0·4857 m
12. (*a*) 0·017 m, 8·5 m (*b*) 132·2 knots.

INDEX

INDEX

PAGE

A

Absolute Humidity 269
—curve of 271
—scale 215
—temperature 242
Acceleration 122
Accumulator 344, 400
Aclinic line 295
Adiabatic expansion 249
Aerials 569
Alternating current 347, 442
Alternating quantities, addition of 448
Ampère 350, 373,
—hours 402
—per metre 305, 306
—turns per metre 306
Amplifier 490
Amplitude modulation 565
Aneroid barometer 102
—corrections 102
Angle, critical 526
—of incidence 508, 523
—of refraction 523
—of rotation of mirror 508
Angular frequency 445
Anode 341
—in a valve 482
Arc Lamp 392
Archimedes' Principle 5
Armature 346
Atmospheric pressure 14, 98
Atomic number 494
Attenuation 567, 569
Attraction and repulsion, law of 287
Avogardo's Law 247
Azimuth mirror 529

B

Bar 11, 101
Barograph 105

PAGE
Barometer, aneroid 102
—corrections 102
—priniciple of 100
Barrier field 497
Battery 340, 400, 404
Beats 553
Bending moment 73
Bias, forward 497
—grid 491
—reverse 497
Black Box 484, 486
Boyle's Law 114
—departures from 251
Brake power 179
Brushes 347

C

Cantilever 72
Capacitance 361
Capacitive circuit 454
Capacitive reactance 455
Capacitor, charge on 361
—in battery or cascade 364
Capacitors, parallel 364
—series 364
Cargo, movement of 51
Carnot's cycle 228
Cathode 341
—in a valve 480
Cathodic protection 399
Cathode Ray Tube 501, 502
Cells, electric 340
—in parallel 405
—in series 404
Celsius, scale of temperature 215
Central heating 210
Centre, of curvature 514
Centre, of gravity 40
—of pressure 108
Centrifugal force 187
Charles's Law 242
Circuit symbols 427
Circuits, D.C. 372
—A.C. 452
Circular motion 187

PAGE
Cloud 280
Coefficient λ 323
—of friction 171
Commutator 347
Compass card 319
—bowl 320
Compressive stress 64
Concave lens 531
Concave mirror 514
Condensation 260
Conduction 201
Conductor 202, 340
Conical governor 189
Contraction 207, 234
Convection 201
Convex lens 531
Convex mirror 514
Coulomb 350
Couples 59
Covalent 494
Critical angle 526
—temperature 265
Current, electric 339
—root mean square (RMS) 446
—sinusoidal 445

D

Dalton's Law 264
Daniell cell 342
D.C. 347, 372, 407
—motor 438
Dead space— 568
Deflection, of a beam 65
Density 1
—of fresh water 2
—relative 3
Derived units xiii
Dynamo 345
Deviation 301
Dew 270
Dew-point 273
Diesel fuel 231
Diode 482
Dip, angle of 295
Direct current 347, 372, 407
Directive force 297, 322

PAGE
Distance/Time graph 129
Domain Theory 284
Doppler effect 549
Draught, calculations 95
Drizzle 279
Dry Cell 344

E

Echoes 554
E.m.f. 339,372
—equation of 446
—induced in a coil 430
Earth fault 387
Eclipse 506
Efficiency 178
Elasticity 64
Electric bell 416
—cell 340
—current 339
Electricity, nature of 339
—static 356
Electro-chemical series 395
—equivalent 399
Electrolysis 398
Electro-magnet 415
Electron 339
—valence 494
Emitter 498
Energy, conservation of 153
—electrical 352
—kinetic 149
—potential 149
Equilibrium, general conditions of 55
—stable and unstable 55
Evaporation 269
Expansion 234
—coefficients of 234
—of gases 242

F

Fahrenheit scale 215
Farad 362
Ferro-magnetic 283

PAGE
Field, horizontal component 296
—magnetic 285
—strength 306
—vertical component 296
Figure Eight diagram 570
Firepoint 231
Flashpoint 231
Fleming's R.H. Rule 444
—L.H. Rule 439
Flotation, law of 6
Flux, magnetic 285, 303
—linkage 433
Focal point 515
Focus 515, 531
Fog 278
Force 10, 137
—centrifugal 187
—components of 36
—coplanar 56
—equilibrant 26
—moment of 40
—parallelogram of 24
—polygon of 32
—resolution of 36
—resultant 23
—triangle of 26
—units of 137
Frequency 443, 544
—angular 445
—modulation 565
Friction 170
—coefficient of, Dynamic 172
—coefficent of, Static 171
—force of 170
—inclined Plane 174
Frontal cloud and rain 281
Fuels, calorific value 230
—coal gas 232
—natural gas 230
—petroleum 230
—uranium 232
Full wave rectification 483

G

Galvanic action 396
Galvanometer 418
—principle of 394

PAGE
Gas constant 245
—equation 245
—Laws 241
Gas oil 231
Gasoline 231
Generator 345
Gram-molecule 247
Grid 488
—bias 491
Ground wave 566

H

H (horizontal component) 296
Half wave rectification 483
Hard iron 288
Haze 279
Heat, nature of 200
—as energy 200, 222
—capacity 222
Helical spring 195
Henry 309
Hooke's Law 65
Humidity, absolute 269
—relative 272
Hydraulic press 13
Hygrometer 274
Hypsometer 214

I

Image in a plane mirror 512
—in a curved mirror 516
—through a prism 527
—through a lens 532, 534
—real and virtual 518, 533
Impedance 467
Inductance 429
—in A.C. circuits 457
—self and mutual 433, 434
Induction coil 436
Inductive circuit 457
Inductive reactance 458
Insulator 340
Intensity of magnetisation 228, 308

PAGE
Interference 551
International ampère 373
—ohm 351
—volt 374
Inverse square law 304
Ionisation 567
Ionosphere 566, 567
Isoclinic lines 295
Isogonal 300
Isothermal expansion 249

J

Joule of work 148
—of energy 150
—of heat 200, 222

K

Kelvin Scale 215
Kerosene 231

L

Lag (of A.C. current) 550
Land and sea breezes 209
Latent heat 256
Lead (of A.C. current) 450
Leclanchè cell 343
Lens 531
—formula 534
Lenz's Law 432
Lever 160
Loadlines 93
Loudspeaker 492

M

Magnetic circuit 315
—domain theory 284
—equator 294
—field 305
—coil 292
—end-on and broadside 312-313
—straight wire 291
—surrounding a conductor 291, 412
—flux 285, 303
—force 285
—induction 288, 309
—meridian 299

PAGE
—moment 306
—poles 284
—variation 299
Magnetisation 288, 308
Magnetising force 305
Magnetomotive Force 315
Magnets, method of making 289
Magnification 521, 534
Manometer 113
Mass 1
Maximum and minimum thermometers 218
Mechanical advantage 158
Millibar 101
Mist 279
Modulation 564
Mole 247
Molecular theory 283
Moment of a force 40
—of a couple 59
—magnetic 306
Momentum 140
—conservation of 142
Motor principle D.C. 438, 439

N

N type material 496
Natural gas 230
Neutral axis 72
Newton 137
Newton's Laws 137, 140, 142

O

Ohm 351
Ohm's Law 352
Oil cargo, expansion of 239
—fire hazard 358
Orographic cloud 280
Oscillation, S.H.M. 191
—free and forced 476, 477
Oscilloscope 501

P

P type material 496
Pendulum, simple 197
Pentavalent 495
Penumbra 506

PAGE
Periscope ... 527
Permanent magnetism ... 289
Permeability ... 309
Petroleum ... 230
Phase ... 447, 551, 553
—difference ... 454
—vector addition ... 450
Pitch, of a screw ... 165
Polar diagram ... 569
Polarisation of a cell ... 341
—of an e.m. wave ... 563
Pole strength ... 304
Potential difference ... 340, 372
Power ... 155, 179
—in electric circuits ... 352, 471
Power pack ... 484
Pressure ... 11
—atmospheric ... 14
—in a liquid ... 15
—on a tank top ... 12
—of a gas ... 241
Principal axis ... 514
Prism ... 526
—binoculars ... 540
Proton ... 339
Pulley ... 161
Pump, principle of ... 117

R

Radiation ... 201
Radio reception ... 564
—spectrum ... 560
—transmission ... 561
Rain ... 277
Rating, of appliances ... 354
Reactance, capacitive ... 455
—inductive ... 458
Rectification half wave ... 484
—full wave ... 485
—valve ... 483
Reflection, of light ... 508, 513
Refraction, of light ... 522
—of sound waves ... 559
Refractive index ... 523
Refrigeration ... 266

PAGE
Rejector circuit 476
Relative Density 3
Relative humidity 270
Relative motion 133
Reluctance 316
Resistance, parallel 375
—in series 374
—measurement of 425
Resistivity 384
Resonance 469, 552
Restoring couple 307
Rope, strength of 183

S

Saturation 268
Screw jack 164
Searchlight 392
Secondary cell 344, 397
Secular change 300
Semiconductor 493
Sextant, principle of 508
Shadows, formation of 504
Shearing force 73
Simple Harmonic Motion 191
Skip distance 568
Sky wave 566
Slip rings 346
Smoothing circuit 486
Soft iron 288
Solenoid 292, 413
Sound 544
—pitch, quality 546
Sound, refraction of 559
Sounding tube 115
Space charge 481
Specific Heat Capacity 222
Specific Latent Heat 256
Specific Resistance 384
Spectrum, visible 557
—electromagnetic 558
State, change of 256
Strain 69
Stress 69
Susceptibility 310
Symbols, in D.C. circuits 426, 427
—in A.C. circuits 427

PAGE

T

Tackles, S.W.L. of 181, 183
Telescope, astronomical 537
—Galileo's 538
Temperature 201
—conversion between scales 215
—critical 265
—scales 215
—standard 246
Tensile stress 64
Testing of materials 67
Tetravalent 494
Thermal capacity 262
—energy 222
Thermionic emission 480
Thermodyamics 227
Thermometers 211
Thermostat 237
Thrust 11
—on a lamina 17
—on tank top 19
Torricellian vacuum 100
Torsion 65
Trade winds 209
Transformer 435
Transistor, emitter, base, collector 498
Triode 488
Trivalent 495
Turbulence cloud 280

U

Ullage 239
Ultimate strength 67
Umbra 506

V

Valence, electrons, orbit 494
Vaporisation 260
Vapour pressure 263
Variation 299
Vectors 23
—in A.C. current 450
Velocity 121
—angular 186
—of sound in air 547
—of sound in sea water 549

PAGE
—ratio 158
—relative 133
—time graph 130
Vertical soft iron 298
Volatility 230
Volt 351, 374
Voltaic cell 341
Voltmeter 394, 421

W

Watt 352
Wave length 545
Weber 303
Weight 4
Welding 392
Weston purchase 166
Wheel and axle 160
Winch, simple hand 160
Wind, causes of 208
Work, units of 148
—in D.C. circuits 352
Working stress 67

X

X plates 502

Y

Y plates 502

Z

Z, importance of 298